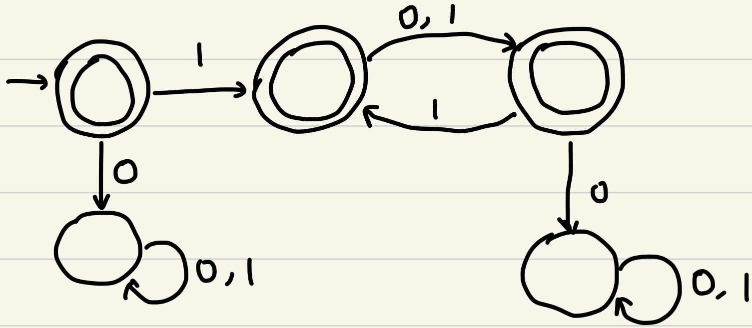
Note: some parts of answers are omitted, as underlined: ____ .

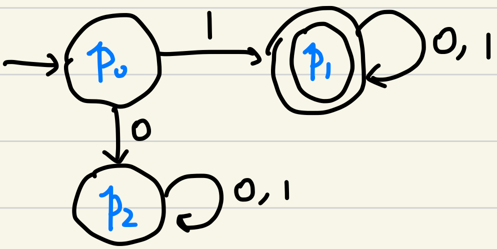
Assignment 3, 2022, part 1

1. Reg exp : $(1(0+1))^*(1+\varepsilon)$

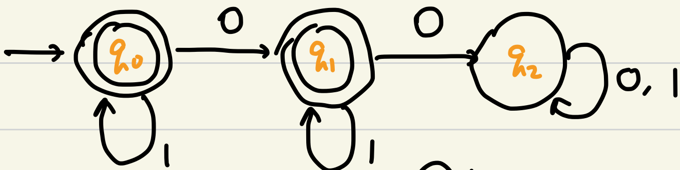
DFA :



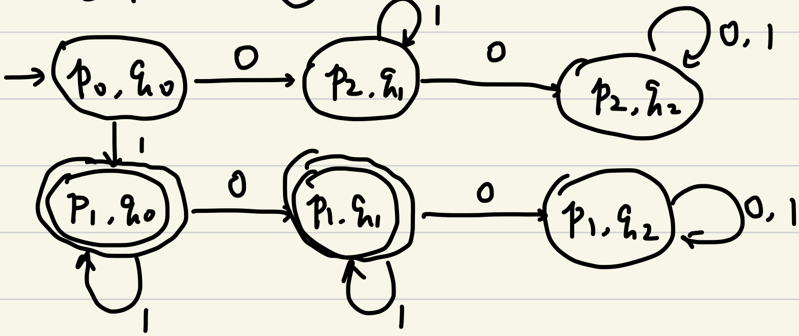
2. L_1 :



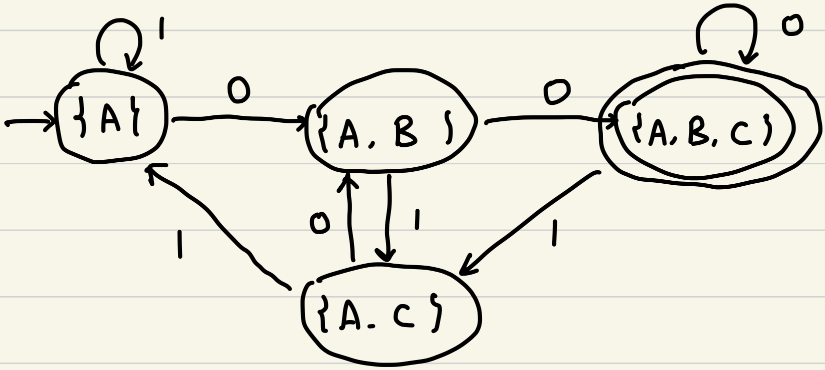
L_2 :



$L_1 \cap L_2$:



3.



4. L(G) is a union of two languages.

The first language L_1 consists of strings of the form

$$0^a \# 0^b \# 0^c, \quad a, b, c \in \mathbb{N}$$

The second language L_2 consists of strings of the form

$$0^\alpha \# 0^{2\alpha}. \quad \alpha \in \mathbb{N}$$

To show L(G) not regular. we use pumping lemma.

Suppose it is. Then $\exists$ an integer P, s.t.

$$0^P \# 0^{2P} \in L$$

And $0^P \# 0^{2P} = uvw$, s.t. $|uv| < P$. $|v| > 0$ and $uv^iw \in L$. then $uvvw$ cannot be $0^\alpha \# 0^{2\alpha}$

This is a contradiction.

5. Design a context-free grammar:

$S \to R \mid T$

$R \to 00R1 \mid 001$

$T \to 0T11 \mid 011$

6 To show L is not context-free, use the pumping lemma for context-free grammars.

Suppose it is. Then $\exists$ an integer P, s.t.

$w = 0^P 1^{P^2} \in L$

$w = uvxyz$, $|vxy| < P$. $|vy| > 0$

s.t. $uv^ixy^iz \in L$.

Case 1: vxy is in 0-segment or 1-segment

then uv^2xy^2z is of the form $0^Q 1^{P^2}$, $Q > P$.

or $0^P 1^Q$, $Q > P^2$. so not in L.

Case 2: vxy must straddle 0's and 1's.

2.1: if v contains only 0 and y contains only 1

Suppose $|v| = C$ and $|y| = D$.

Then $P^2 + D = (P+C)^2$.

(for $uv^2xy^2z \in L$)

This yields $D = 2CP + C^2$ (1)

And $P^2 - D = (P-C)^2$

This yields $D = -2CP + C^2$ (2)

(1) and (2) give $C = D = 0$, a contradiction.

2.2. if v or y contains both 0 and 1 ... $\blacksquare$

Assignment 3, part 2, Some parts of answers are omitted

1. See the slides or the textbook :)

2. Let F be a boolean formula with n variables $x_1, \cdots, x_n$. Let $C := 0$.

For every $\vec{b} = (b_1, \cdots, b_n) \in \{0,1\}^n$,

let $x_i = b_i$ for $i = 1, \cdots, n$.

Evaluate F on this assignment. If the result is true, then $C := C+1$.

If $C = 5$, then return yes. Otherwise return no

The above algorithm terminates in finite time because the For loop runs in 2^n rounds. Other steps run in poly time. So it terminates in finite time $\Rightarrow$ decidable. $\blacksquare$

3. Use Rice's theorem. L is a property of TMs.

(1) Nontrivial: M accepting everything $\in L$.

M rejecting everything $\notin L$.

(2) Semantic: Suppose M_1 and M_2 are two TMs.

Suppose their languages are the same.

If $\{1, 11, 111\} \in L(M_1) = L(M_2)$

$\Rightarrow$ M_1 and $M_2 \in L$.

If $\{1, 11, 111\} \notin L(M_1) = L(M_2)$

$\Rightarrow$ M_1 and $M_2 \notin L$.

Therefore, by Rice's thm, L is undecidable.

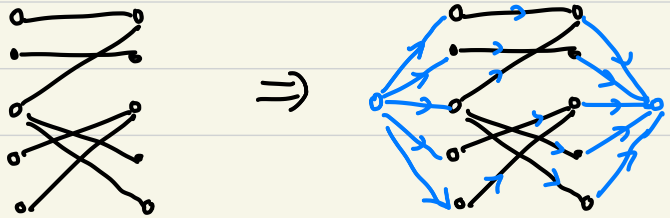
4. Let $G = (L \cup R, E)$, $E \subseteq L \times R$ be a bipartite graph.

We construct a weighted directed graph $H = (V, E, w), v \in \mathbb{N}$

that G has a p.m. $\Leftrightarrow$ H has a max flow value of v

Sketch: you will need to describe below properly

by setting $V = \{s, t\} \cup L \cup R$, etc.



w: every edge has wt 1

$v = n$.

5. (1) LEC-ARRANGE

$= \{(U, L_1, \cdots, L_n, P_1, \cdots, P_{m-n}) \mid L_i, P_j \subseteq U, \; i=1,\cdots,n$

such that $\exists\, a_i \in L_i$, letting $A = \{a_1, \cdots, a_n\}$. $\forall j \in [m-n]$,

$A \cap P_j \neq \phi \}$

(2) In NP: guess $a_i \in L_i$ and verify $A \cap P_j \neq \phi$

(3) NP-hard: we reduce Set Cover to LEC-ARRANGE

Set Cover $= \{(W, S_1, \cdots, S_m, k) \mid S_i \subseteq W, \; \exists T \subseteq [m]$

$|T| = k, \; \bigcup_{i \in T} S_i = W\}$

We construct a LEC-ARRANGE instance as follows.

$U = \{S_1, \cdots, S_m\}$

$L_1 = \cdots = L_k = U$

$\forall w \in W, \; P_w = \{S_i \mid w \in S_i\}$.

Claim. $(W, S_1, \cdots, S_m, k) \in$ Set Cover

$\Leftrightarrow (U, L_1, \cdots, L_k, P_w : w \in W) \in$ LEC-ARR

Pf. $(\Rightarrow)$ $T \subseteq [m]$ is a solution for Set Cover

Let $T = \{i_1, \cdots, i_k\}$

this is just $S_{ij} \in P_w$

That is, $\forall w \in W$, $\exists j \in [k]$, $w \in S_{ij}$.

So we select S_{ij} from L_j for $j \in [k]$,

we have $S_{ij} \in P_w$.

This means that $\{S_{i_1}, \dots, S_{i_k}\}$ is a solution for LEC-ARR.

($\Leftarrow$) ___

State the construction, correctness proof, and running time (poly-time) based on the above. $\boxtimes$