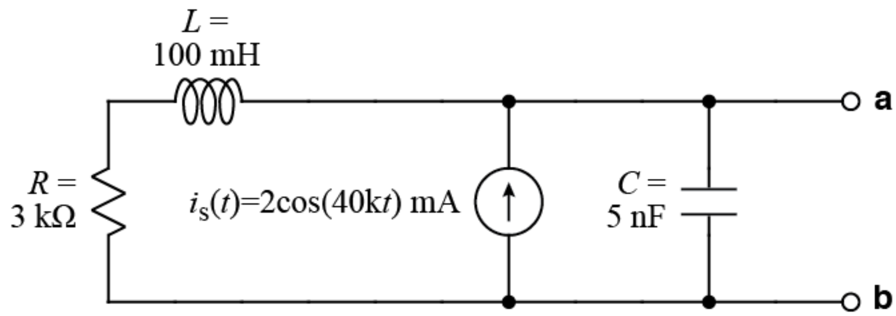
**EX:**




Find the phasor for the Thevenin equivalent voltage of the above circuit in the frequency domain. That is, find the numerical value of the phasor, $\mathbf{V}_{Th}$, which is the voltage drop from **a** to **b** (with the + sign at **a** and the – sign at **b**).

Express the value of $\mathbf{V}_{Th}$ in polar form.

**SOL'N:** We analyze the circuit the same way we would if we had a DC source and resistors, except that we use a phasor for $i_s(t)$ and impedance values for the $R$, $L$, and $C$.

Converting $i_s(t)$ to a phasor, $\mathbf{I}_s$, involves nothing more than identifying the magnitude and phase shift for the phasor transform:

$$P[A\cos(\omega t+\phi)] = Ae^{j\phi}.$$

There is no phase shift here, so $\phi = 0°$.

$$P[i_s(t)] = P[2\cos(40\text{k}t+0°)\text{mA}] = 2e^{j0°}\text{ mA} = \mathbf{I}_s$$

**NOTE:** We could omit the $e^{j0°}$ in $\mathbf{I}_s$ since $e^{j0°} = 1$, but we retain the $e^{j0°}$ so we remember that we are dealing with a phasor that represents a sinusoidal source as opposed to a DC source.

Our impedance values are calculated with the usual formulas:

$$z_R = R \qquad z_L = j\omega L \qquad z_C = \frac{1}{j\omega C}$$

The value of $\omega$ is found in $i_s(t)$ to be 40 krad/s.

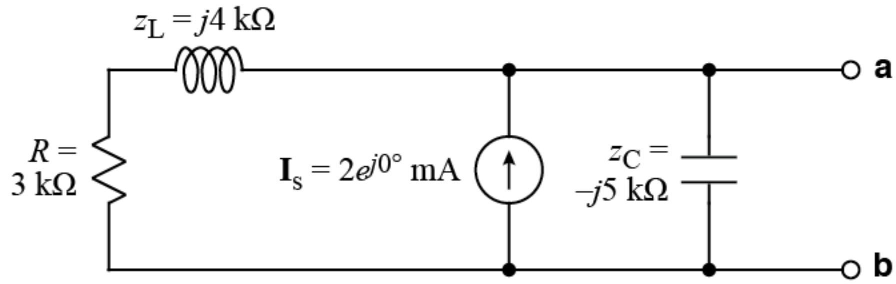
$$z_R = 3\text{k}\Omega$$

$$z_L = j40\text{k}\cdot 100\text{mH} = j4\text{k}\Omega$$

$$z_C = \frac{1}{j40\text{k}\Omega \cdot 5\text{nF}} = \frac{-j}{200\mu}\Omega = -j5\text{k}\Omega$$

**NOTE:** $\frac{1}{j} = -j$

Our frequency-domain circuit representation is shown below.




The three branches of the circuit are in parallel. That is, we have a current source in parallel with two impedances: $z_L + R$ on the left, and $z_C$ on the right. Thus, our voltage from **a** to **b** to is given by Ohm's law:

$$\mathbf{V}_{Th} = \mathbf{I}_s \cdot (R + z_L) \,\|\, z_C$$

We substitute the complex numbers for the phasor $\mathbf{I}_s$ and the impedances and simplify. First, we compute the parallel impedance value.

$$(R + z_L) \,\|\, z_C = (3\text{k}\Omega + j4\text{k}\Omega) \,\|\, -j4\text{k}\Omega$$

The sum of inverses is often the best way to compute parallel impedances. However, when the impedances are not purely real or purely imaginary the product-over-sum formulation is unavoidable.

$$(R + z_L) \,\|\, z_C = \frac{(3\,\text{k}\Omega + j4\,\text{k}\Omega)(-j5\text{k}\Omega)}{3\,\text{k}\Omega + j4\,\text{k}\Omega - j5\,\text{k}\Omega)} = \frac{(3 + j4)(-j5\text{k}\Omega)}{3 - j}$$

Here, rationalizing the denominator is a practical approach to calculating the quotient. An alternative is to convert all quantities to polar form. The advantage of rationization is that it will yield exact values, at least temporarily. Note that we keep the multiplying factor –*j*5 kΩ intact, as it may cancel out before we need to multiply it through.

$$(R + z_L) \,\|\, z_C = \frac{(3 + j4)(-j5\text{k}\Omega)}{3 - j} \cdot \frac{3 + j}{3 + j} = \frac{[3 \cdot 3 - 4 \cdot 1 + j(3 + 4 \cdot 3)](-j5\text{k}\Omega)}{3^2 + 1^2}$$

or

$$(R+z_L)\parallel z_C=\frac{(5+j15)(-j\not{5}\,\text{k}\Omega)}{\not{10}\,2}=\frac{(5+j15)(-j)}{2}\text{k}\Omega$$

$$=\frac{15-j5}{2}\text{k}\Omega=7.5\text{k}\Omega-j2.5\text{k}\Omega$$

Now we multiply by $\mathbf{I}_s$.

$$\mathbf{V}_{Th}=\mathbf{I}_s\cdot(R+z_L)\parallel z_C=2\,\text{mA}(7.5\text{k}\Omega-j2.5\text{k}\Omega)$$

or

$$\mathbf{V}_{Th}=15-j5\,\text{V}$$

Now we convert the answer to polar form using the following identity:

$$a+jb=\sqrt{a^2+b^2}\,e^{j\tan^{-1}(b/a)}.$$

Thus,

$$\mathbf{V}_{Th}=15-j5\,\text{V}=5\sqrt{3^2+1^2}\,e^{j\tan^{-1}(-5/15)}\,\text{V}$$

or

$$\mathbf{V}_{Th}=5\sqrt{10}e^{-j18.43^\circ}\,\text{V}\approx 15.8\text{e}^{-j18.4^\circ}\text{V}.$$

An alternate approach that yields a clean solution is to use a current divider first and then multiply by $z_C$.

$$\mathbf{I}_C=\mathbf{I}_s\frac{R+z_L}{R+z_L+z_C}=2\text{mA}\frac{3\not{\text{k}\Omega}+j4\not{\text{k}\Omega}}{3\not{\text{k}\Omega}+j4\not{\text{k}\Omega}-j5\not{\text{k}\Omega}}=2\text{mA}\frac{3+j4}{3-j}$$

Now rationalize the denominator, as before.

$$\mathbf{I}_C=2\text{mA}\frac{3+j4}{3-j}\cdot\frac{3+j}{3+j}=2\text{mA}\frac{9-4+j(3+12)}{3^2+1^2}=2\text{mA}\frac{5+j15}{10}$$

or

$$\mathbf{I}_C=1+j3\,\text{mA}$$

Now multiply by $z_C$ to get $\mathbf{V}_{Th}$.

$$\mathbf{V}_{Th}=\mathbf{I}_C z_C=(1+j3\text{mA})(-\text{j}5\text{k}\Omega)=15-j5\,\text{V}=15.8\text{e}^{-j18.4^\circ}\,\text{V}$$

Our answer is the same as before.