48510 LEC 11 – AC CIRCUIT ANALYSIS

Topic 11-1: AC circuit analysis startup

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11-1 AC CIRCUIT ANALYSIS STARTUP

Giving $V_s = 10\,cos(2000t)$, $R = 5\ \Omega$, $C = 2 \times 10^{-4}\ F$, find out the current and voltages.

Let's try to solve this problem in time-domain only

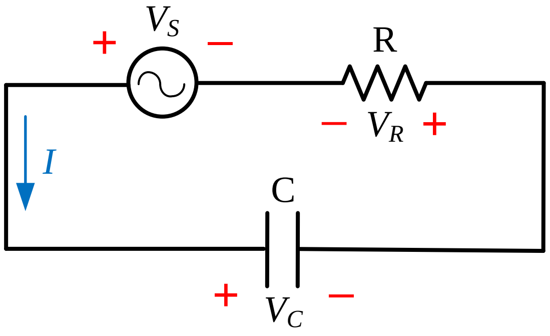
Recall how we conducted the transient analysis,

$$V_c + V_R = V_s$$

$$I = C\frac{dV_c}{dt}$$

$$V_c(t) + RC\frac{dV_c(t)}{dt} = V_s(t)$$

First-order differential equation



This one is actually more complicated than our previous case presented in Lec 7 because $V_s(t)$ is a time-variant function rather than a constant.

11-1 AC CIRCUIT ANALYSIS STARTUP

Giving $V_s = 10\,cos(2000t)$, $R = 5\ \Omega$, $C = 2 \times 10^{-4}\ F$, find out the current and voltages.

Using the transformation method (phasor)

1. Convert the circuit from time-domain to frequency-domain

Identify the angular frequency $\omega = 2000$.

Find the phasor for the sources: $V_s = 10\angle 0^0$

Find the complex impedance:

$$Z_R = R = 5$$

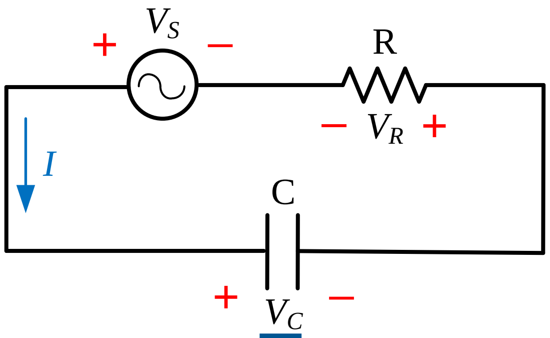
$$Z_C = -j\frac{1}{\omega C} = -j\frac{1}{2000 \times 2 \times 10^{-4}} = -j2.5$$

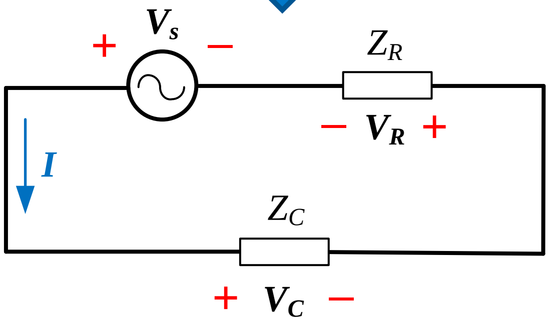
2. Redraw the circuit and analysis it as if it is a DC resistive circuit.

$$Z_T = Z_R + Z_C = 5 - j2.5$$

$$I = \frac{V_s}{Z_T} = \frac{10\angle 0^0}{5 - j2.5}$$

Need to convert it to polar form as well

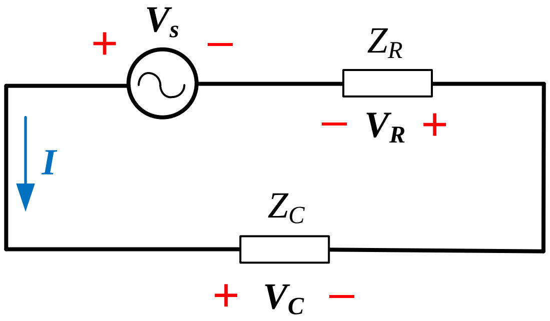




11-1 AC CIRCUIT ANALYSIS STARTUP

Giving $V_s = 10\,cos(2000t)$, $R = 5\ \Omega$, $C = 2 \times 10^{-4}\ F$, find out the current and voltages.

2. Redraw the circuit and analysis it as if it is a DC resistive circuit.



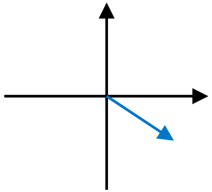
$$Z_T = Z_R + Z_C = 5 - j2.5$$

convert it to polar form

$$|Z_T| = \sqrt{5^2 + 2.5^2} = \sqrt{25 + 6.25} = \sqrt{31.25} = 5.59$$

$$\angle(Z_T) = arctan\frac{-2.5}{5} = -26.6^0$$

Check whether the phase is correct in complex plane



It is correct!

$$Z_T = 5 - j2.5 = 5.59\angle -26.6^0$$

$$I = \frac{V_s}{Z_T} = \frac{10\angle 0^0}{5.59\angle -26.6^0} = \frac{10}{5.59}\angle(0^0 + 26.6^0) = 1.79\angle 26.6^0$$

$$V_R = IZ_R = 1.79\angle 26.6^0 \times 5 = 8.95\angle 26.6^0$$

$$V_C = IZ_C = 1.79\angle 26.6^0 \times (-j2.5) = 1.79\angle 26.6^0 \times 2.5\angle -90^0 = 4.48\angle -63.4^0$$

11-1 AC CIRCUIT ANALYSIS STARTUP

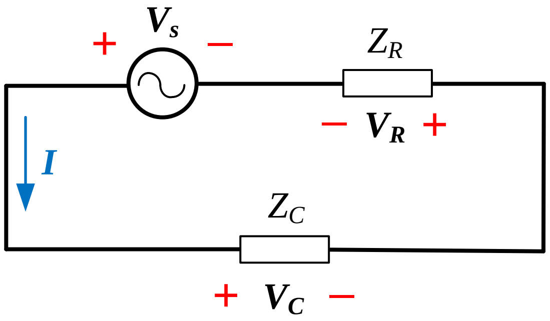
Giving $V_s = 10\,cos(2000t)$, $R = 5\,\Omega$, $C = 2\times10^{-4}\,F$, find out the current and voltages.

3. Convert the phasor to waveform, convert the circuit from frequency-domain to time-domain.

$I = 1.79\angle 26.6^0$ → $I = 1.79\cos(2000t + 26.6^0)$

$V_R = 8.95\angle 26.6^0$ → $V_R = 8.95\cos(2000t + 26.6^0)$

$V_C = 4.48\angle -63.4^0$ → $V_C = 4.48\cos(2000t - 63.4^0)$



Once could also use voltage division principle to find out VR and VC directly.

$$V_R = V_s\frac{Z_R}{Z_R + Z_C} = V_s\frac{Z_R}{Z_T} \Rightarrow = 10\angle 0^0 \times \frac{5}{5.59\angle -26.6^0} = \frac{50\angle 0^0}{5.59\angle -26.6^0} = 8.95\angle 26.6^0$$

$$V_C = V_s\frac{Z_C}{Z_R + Z_C} = V_s\frac{Z_C}{Z_T} \Rightarrow = 10\angle 0^0 \times \frac{-j2.5}{5.59\angle -26.6^0} = \frac{25\angle -90^0}{5.59\angle -26.6^0} = 4.48\angle -63.4^0$$

$$Z_T = 5 - j2.5 = 5.59\angle -26.6^0$$

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Topic 11-2: Node Voltage Analysis (complex version)

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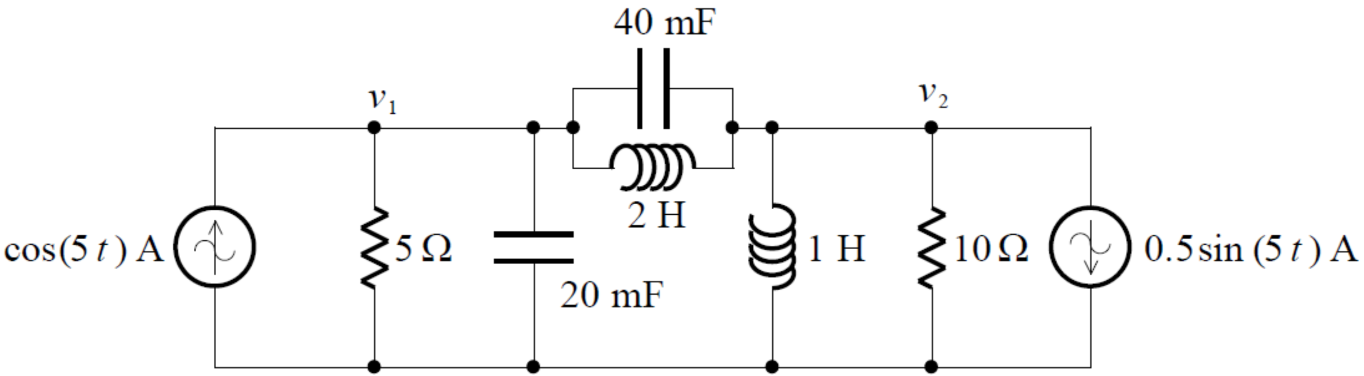
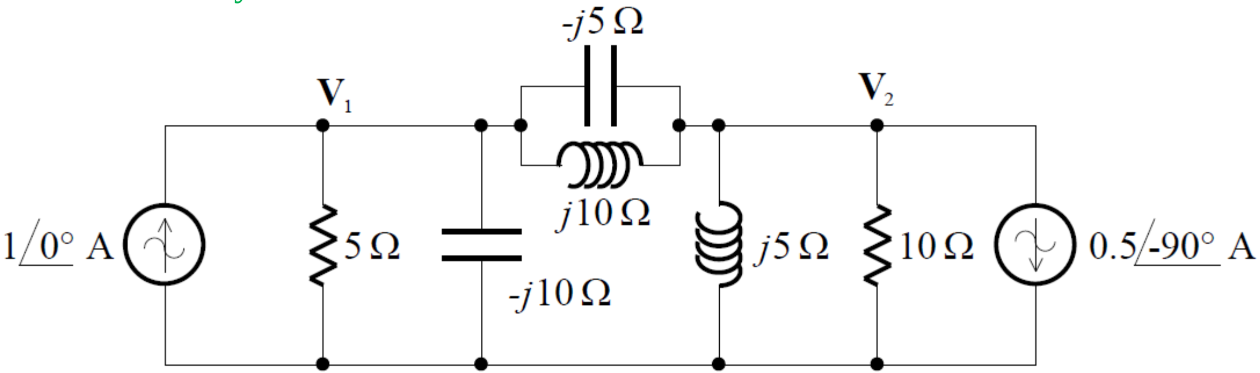
11-2 NODE VOLTAGE ANALYSIS (COMPLEX VERSION)



1. Convert the circuit from time-domain to frequency-domain

Identify the angular frequency $\omega = 5$. Find the phasor for the sources: Find the complex impedance:

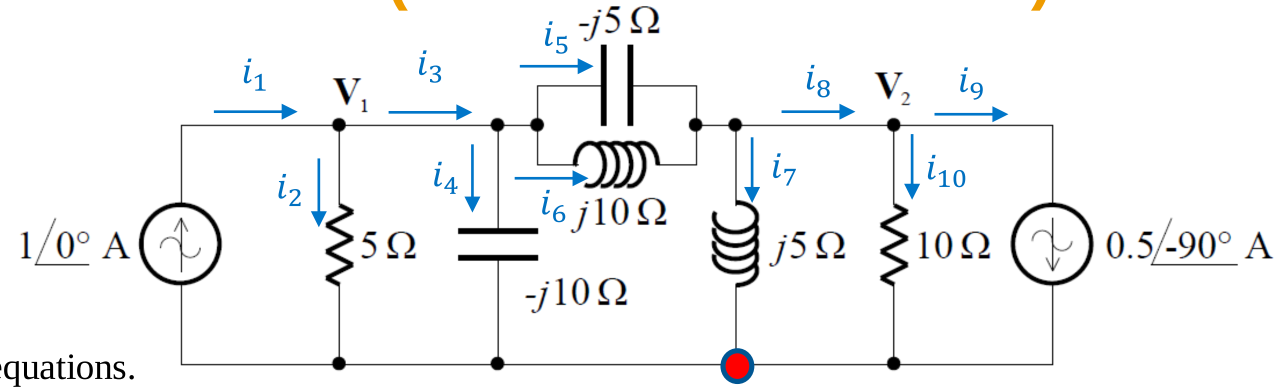
2. Redraw the circuit and analyse it as if it is a DC resistive circuit.



11-2 NODE VOLTAGE ANALYSIS (COMPLEX VERSION)

2. Redraw the circuit and analysis it as if it is a DC resistive circuit.

Use node voltage analysis to solve this circuit



We have 2 nodes, therefore we need 2 equations.

Apply KCL at the left node: *Current_In = Current_Out*

$$i_1 = i_2 + i_3 = i_2 + i_4 + i_5 + i_6 \Rightarrow 1\angle 0^0 = \frac{V_1}{5} + \frac{V_1}{-j10} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j10} \Rightarrow (0.2 + j0.2)V_1 - j0.1V_2 = 1$$

Apply KVL at the right node:

$$i_8 = i_9 + i_{10} = i_5 + i_6 - i_7 \Rightarrow 0.5\angle -90^0 + \frac{V_2}{10} = \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j10} - \frac{V_2}{j5} \Rightarrow -j0.1V_1 + (0.1 - j0.1)V_2 = j0.5$$

Use Cramer's Rule (the matrix method) to solve the equation set:

11-2 NODE VOLTAGE ANALYSIS (COMPLEX VERSION)

$$(0.2 + j0.2)\boldsymbol{V_1} - j0.1\boldsymbol{V_2} = 1$$

$$-j0.1\boldsymbol{V_1} + (0.1 - j0.1)\boldsymbol{V_2} = j0.5$$

Use Cramer's Rule (the matrix method) to solve the equation set:

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 1 & -j0.1 \\ j0.5 & (0.1 - j0.1) \end{vmatrix}}{\begin{vmatrix} (0.2 + j0.2) & -j0.1 \\ -j0.1 & (0.1 - j0.1) \end{vmatrix}} = \frac{0.1 - j0.1 - 0.05}{0.02 - j0.02 + j0.02 + 0.02 + 0.01} = \frac{0.05 - j0.1}{0.05} = 1 - j2 \text{ V}$$

$$\mathbf{V}_2 = \frac{\begin{vmatrix} (0.2 + j0.2) & 1 \\ -j0.1 & j0.5 \end{vmatrix}}{0.05} = \frac{-0.1 + j0.1 + j0.1}{0.05} = -2 + j4 \text{ V}$$

Convert to polar form

$$\mathbf{V}_1 = \sqrt{5}\angle -63.4^\circ \text{ V}$$

$$\mathbf{V}_2 = 2\sqrt{5}\angle 116.6^\circ \text{ V}$$

Convert to time-domain

$$v_1(t) = \sqrt{5}\cos(5t - 63.4^\circ) \text{ V}$$

$$v_2(t) = 2\sqrt{5}\cos(5t + 116.6^\circ) \text{ V}$$

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Topic 11-3: Mesh Current Analysis (complex version)

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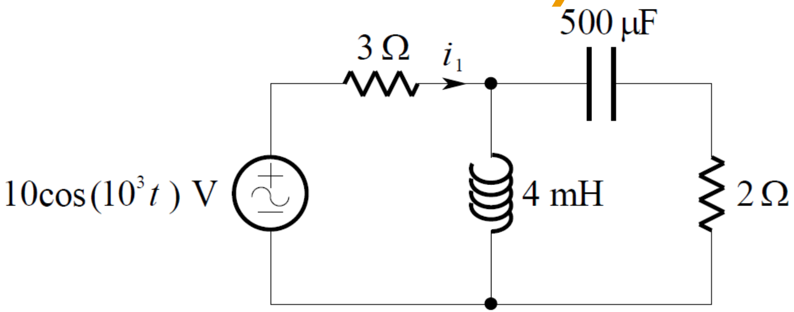
11-3 MESH CURRENT ANALYSIS (COMPLEX VERSION)



1. Convert the circuit from time-domain to frequency-domain

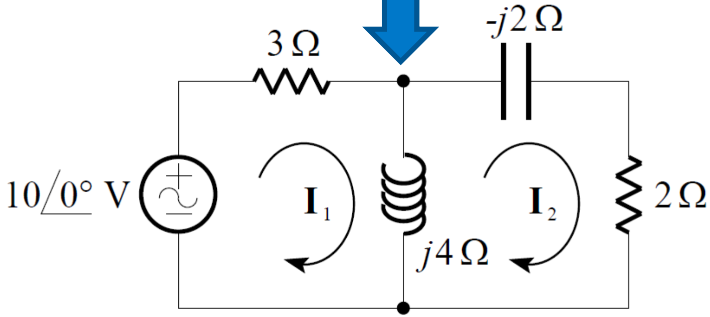
Identify the angular frequency $\omega = 10^3$.

Find the phasor for the sources:

Find the complex impedance:

2. Redraw the circuit and analyse it as if it is a DC resistive circuit.

Use mesh current analysis to solve this circuit



We have 2 meshes, therefore we need 2 equations.

Apply KVL around the left mesh:

$$10\angle 0^0 - 3\boldsymbol{I_1} - j4(\boldsymbol{I_1} - \boldsymbol{I_2}) = 0$$

Apply KVL around the right mesh:

$$-j4(\boldsymbol{I_2} - \boldsymbol{I_1}) - (-j2\boldsymbol{I_2}) - 2\boldsymbol{I_2} = 0$$

Simplify the two equations we have

$$(3 + j4)\mathbf{I}_1 - j4\mathbf{I}_2 = 10$$
$$-j4\mathbf{I}_1 + (2 + j2)\mathbf{I}_2 = 0$$

Use matrix method

$$\mathbf{I}_1 = \frac{10}{7} = \frac{10}{7}\angle 0^\circ \text{ A}$$

$$\mathbf{I}_2 = \frac{10}{7}(1 + j) = \frac{10\sqrt{2}}{7}\angle 45^\circ \text{ A}$$

$$i_1(t) = \frac{10}{7}\cos\left(10^3 t\right) \text{ A}$$

$$i_2(t) = \frac{10\sqrt{2}}{7}\cos\left(10^3 t + 45^\circ\right) \text{ A}$$

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Topic 11-4: Superposition Principle (complex version)

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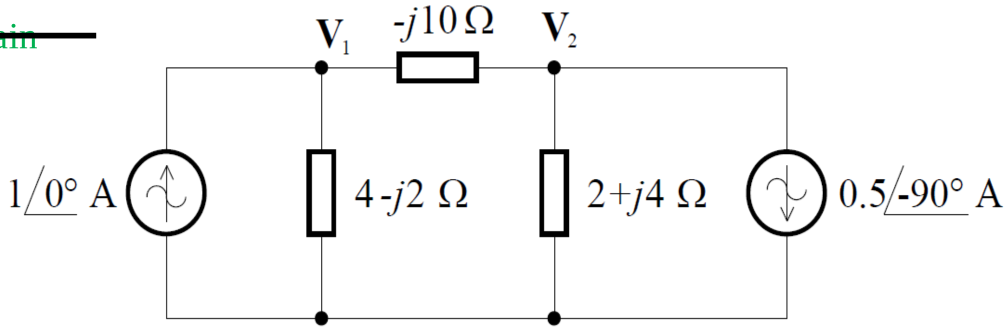
11-4 SUPERPOSITION PRINCIPLE (COMPLEX VERSION)

~~1. Convert the circuit from time-domain to frequency-domain~~

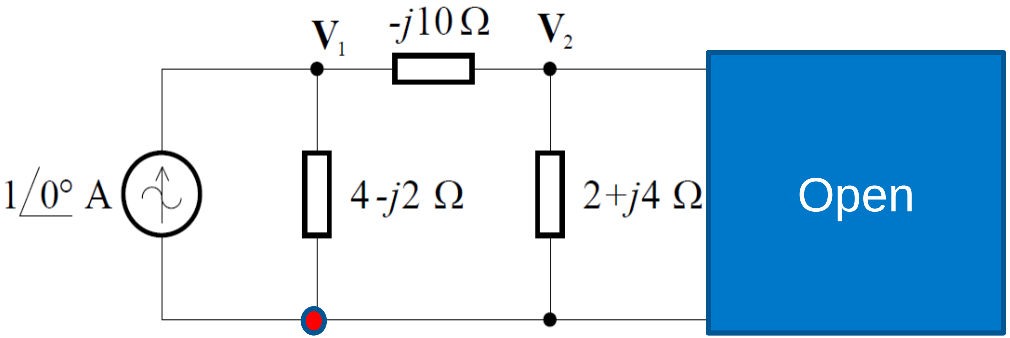
We don't need to make the conversion as this circuit is already in frequency domain.

~~2. Redraw the circuit and~~ analysis it as if it is a DC resistive circuit.

Use superposition to solve this circuit



Only activating the left source:



Reference point V=0

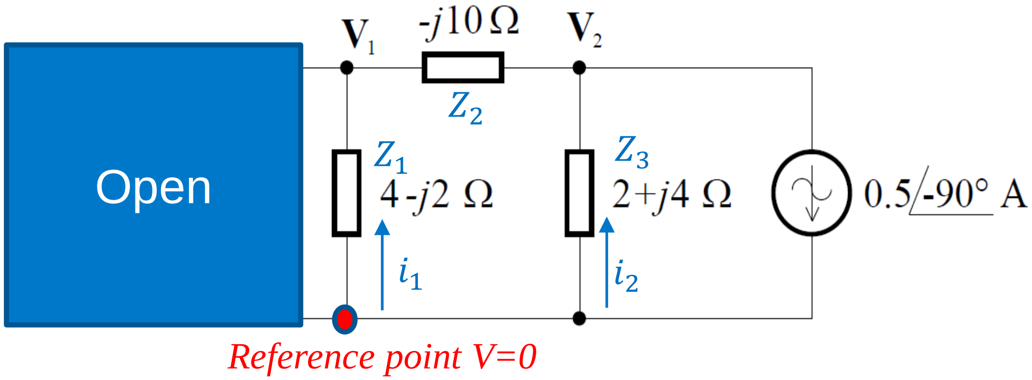
Total voltage = current * total impedance

$$\mathbf{V}_{1L} = 1\angle 0° \frac{(4-j2)(-j10+2+j4)}{4-j2-j10+2+j4}$$

$$= \frac{-4-j28}{6-j8} = 2-j2$$

11-4 SUPERPOSITION PRINCIPLE (COMPLEX VERSION)

Only activating the right source:



Use current division principle

$$i_1 = i_T \frac{Z_3}{Z_3 + Z_1 + Z_2}$$

$$V_{1R} = -i_1 Z_1 = -i_T \frac{Z_1 Z_3}{Z_3 + Z_1 + Z_2}$$

$$\mathbf{V}_{1R} = (-0.5\angle -90°)\left(\frac{2 + j4}{4 - j2 - j10 + 2 + j4}\right)(4 - j2) = \frac{-6 + j8}{6 - j8} = -1$$

Previously we obtained

$$\mathbf{V}_{1L} = 2 - j2$$

Sum V_{1L} and V_{1R}

$$\mathbf{V}_1 = 2 - j2 - 1 = 1 - j2 \text{ V}$$

In similar way, we can get

$$\boldsymbol{V_2} = -2 + j4$$

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Topic 11-5: Thévenin’s and Norton’s Theorem (complex version)

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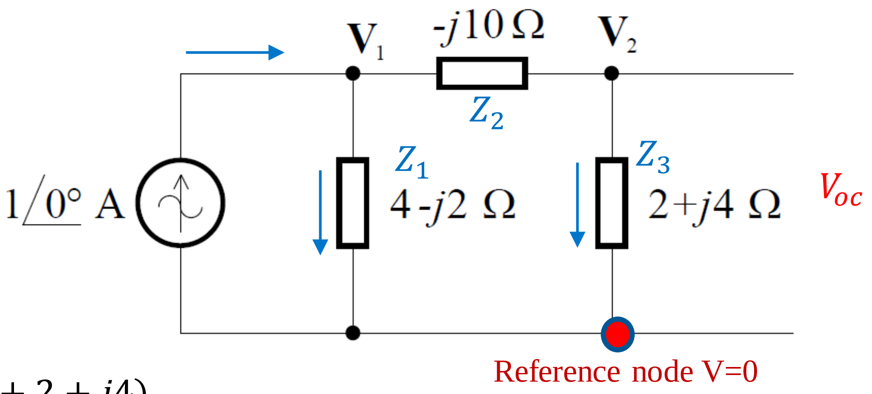
11-5 THÉVENIN'S AND NORTON'S THEOREM (COMPLEX VERSION)

1. Convert the circuit from time-domain to frequency-domain

2. Redraw the circuit and analyse it as if it is a DC resistive circuit.



Finding V_{th}

$$V_{th} = V_{oc} = V_2 - 0$$

Total voltage V_t = current * total impedance

$$V_t = I_t Z_t = 1\angle 0^0 \frac{Z_1(Z_2+Z_3)}{Z_1+(Z_2+Z_3)} = \frac{(4-j2)(-j10+2+j4)}{4-j2-j10+2+j4} = 2-j2$$

According to voltage division principle

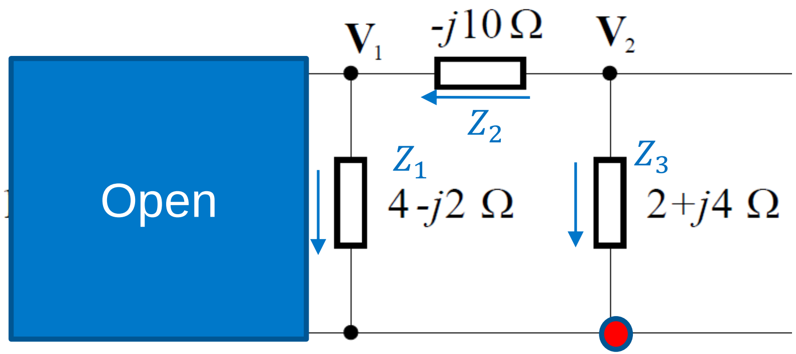
$$V_2 = V_t \frac{Z_3}{Z_2+Z_3} = (2-j2)\frac{2+j4}{-j10+2+j4} = \mathrm{j}2 = V_{th}$$

$$= 2\angle 90^0$$

11-5 THÉVENIN'S AND NORTON'S THEOREM (COMPLEX VERSION)

1. Convert the circuit from time-domain to frequency-domain

2. Redraw the circuit and analyse it as if it is a DC resistive circuit.



Finding V_{th}

$$V_{th} = j2 = 2\angle 90^0$$

Finding Z_t

$$Z_t = \frac{Z_3(Z_1+Z_2)}{Z_3+(Z_1+Z_2)} = \frac{(2+j4)(4-j2-j10)}{4-j2-j10+2+j4} = 4+j4$$

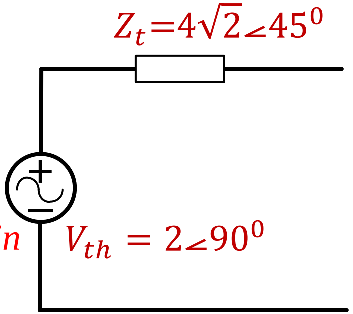
$$= 4\sqrt{2}\angle 45^0$$

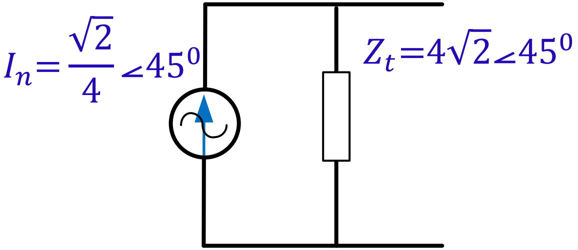
Finding I_n

$$I_n = \frac{V_{th}}{Z_t} = \frac{j2}{4+j4}$$

$$= 0.25 + 0.25j$$

$$= \frac{\sqrt{2}}{4}\angle 45^0$$





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Topic 11-6: Maximum Power Transfer (Complex version)

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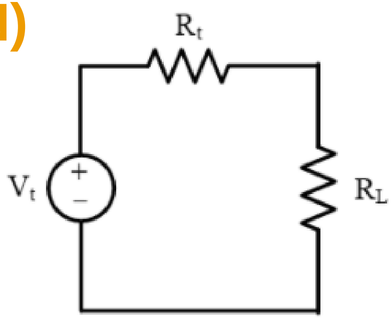
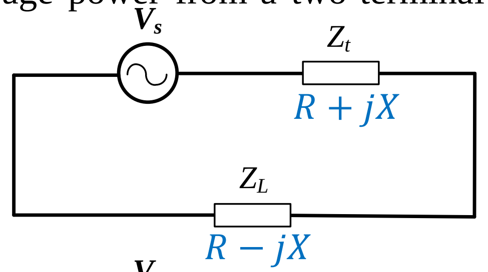
11-6 MAXIMUM POWER TRANSFER (COMPLEX VERSION)



In pure resistive circuit, maximum Power Transfer occurs when the resistive value of the load is equal in value to that of the voltage source's internal resistance **(R_L=R_s)** allowing maximum power to be supplied.

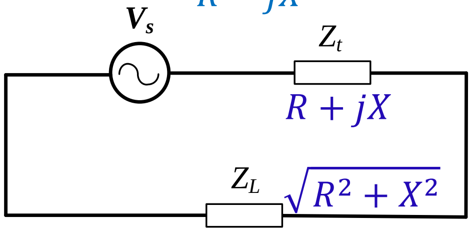
The maximum power received by the load is $P_L = \frac{V_t^2}{4R_t}$, which is half of the total power generated by the source.

In comple circuit, how to adjust a load impedance $\boldsymbol{Z_L}$ to extract the maximum average power from a two-terminal circuit? (Note $\boldsymbol{Z_t}$ is the inner impedance or Thevenin equivalent impedance.)



1) **If the load can take on any complex value**, maximum power transfer is attained for a load impedance equal to the complex conjugate of the Thévenin impedance.

$$Z_{load} = Z_t^*$$

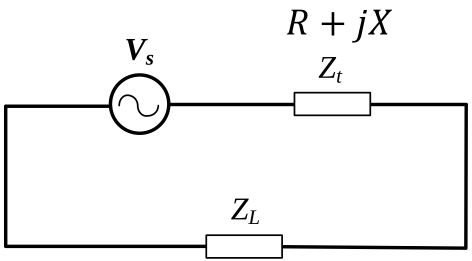


2) **If the load is required to be a pure resistance**, maximum power transfer is attained for a load resistance equal to the magnitude of the Thévenin impedance.

$$Z_{load} = |Z_t| = \sqrt{R^2 + X^2}$$

11-6 MAXIMUM POWER TRANSFER (COMPLEX VERSION)

In comple circuit, how to adjust a load impedance $\boldsymbol{Z_L}$ to extract the maximum average power from a two-terminal circuit? (Note $\boldsymbol{Z_t}$ is the inner impedance or Thevenin equivalent impedance.)



1) **If the load can take on any complex value**, maximum power transfer is attained for a load impedance equal to the complex conjugate of the Thévenin impedance.

$$Z_{load} = Z_t^*$$

2) **If the load is required to be a pure resistance**, maximum power transfer is attained for a load resistance equal to the magnitude of the Thévenin impedance.

$$Z_{load} = |Z_t| = \sqrt{R^2 + X^2}$$

Note that the power should be a pure real number. Only resistance can receive power. Reactance (capacitance and inductance) does not take power. The average AC power on capacitors and inductors are always 0. With the AC source, the capacitors and inductors keep charging and discharging, thus the energy comes and goes away but no real energy is consumed.

The maximum power delivered on the load is only determined by the effective RMS current and the resistance of the impedance:

$$P_{max} = I_{rms}^2 R_{load} \qquad \text{Where } R_{load} = \begin{cases} R_t \text{ if load is a complex impedance} \\ |Z_t| \text{ if load is a pure resistive load} \end{cases}$$