48510 Introduction to Electrical and Electronic Engineering

Exam Topics

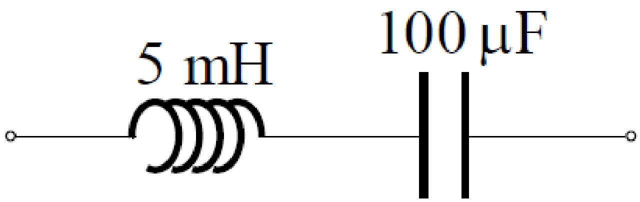
1. Equivalent impedance calculation;
2. Nodal or mesh analysis using Phasors;
3. Thévenin’s Theorem using Phasors;
4. Diodes
5. Capacitor or Inductor

1.Equivalent Impedance Calculation

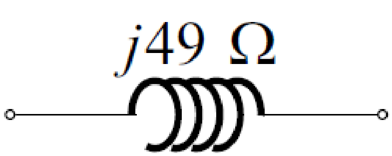
Time-domain	*Frequency-domain*
R	R
L	$Z_L = j\omega L$
C	$Z_C = 1/j\omega C$ $\boxed{= -j/(\omega C)}$

1.Equivalent Impedance Calculation

We have an inductor and capacitor in series:



At $\omega = 10^4$ rads^{-1}, the impedance of the inductor is $\boxed{\mathbf{Z}_L = j\omega L = j50\,\Omega}$ and the impedance of the capacitor is $\boxed{\mathbf{Z}_C = 1/j\omega C = -j1\,\Omega.}$ Thus the series combination is equivalent to $\mathbf{Z}_{eq} = \mathbf{Z}_L + \mathbf{Z}_C = j50 - j1 = j49\;\Omega$:



1.Equivalent Impedance Calculation

In rectangular form an impedance is represented by:

$$\boxed{\mathbf{Z} = R + jX}$$

The real part, R, is termed the *resistive component*, or *resistance*. The imaginary component, X, including sign, but excluding j, is termed the *reactive component*, or *reactance*. The impedance $100\angle -60^\circ\,\Omega$ in rectangular form is $50 - j86.6\,\Omega$. Thus, its resistance is $50\,\Omega$ and its reactance is $-86.6\,\Omega$.

It is important to note that the resistive component of the impedance is not necessarily equal to the resistance of the resistor which is present in the circuit.

1.Equivalent Impedance Calculation

Phasor Representations

Phasors can be represented in four different ways:

$\mathbf{X} = X_m \angle \phi$	polar form
$\mathbf{X} = X_m e^{j\phi}$	exponential form
$\mathbf{X} = X_m(\cos\phi + j\sin\phi)$	trigonometric form
$\mathbf{X} = a + jb$	rectangular form

Rectangular to Polar form

$$X_m = \sqrt{a^2 + b^2}$$

$$\phi = arctan\frac{b}{a}$$

Polar to Rectangular form

$$a = X_m cos\,\phi$$

$$b = X_m sin\,\phi$$

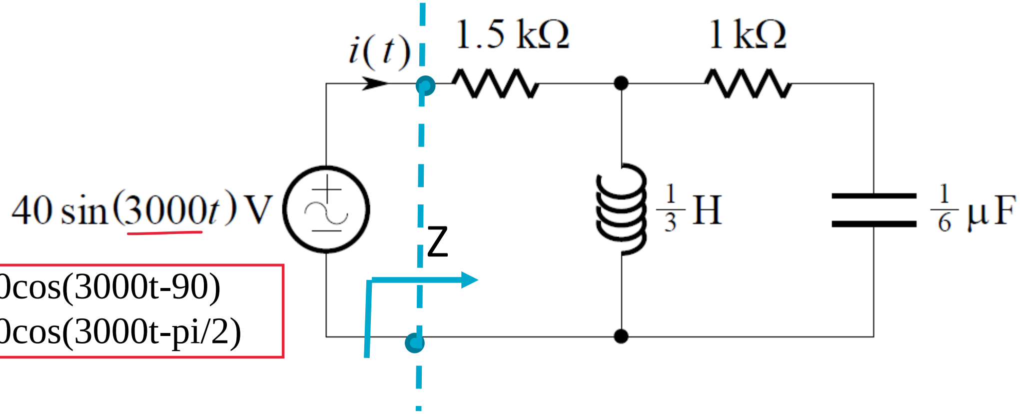
1.Equivalent Impedance Calculation

$$x(t) = A\cos(\omega t + \phi) \Leftrightarrow \mathbf{X} = Ae^{j\phi} = A\angle\phi$$

$$\text{amplitude} \Leftrightarrow \text{magnitude}$$

$$\text{phase} \Leftrightarrow \text{angle}$$

1.Equivalent Impedance Calculation

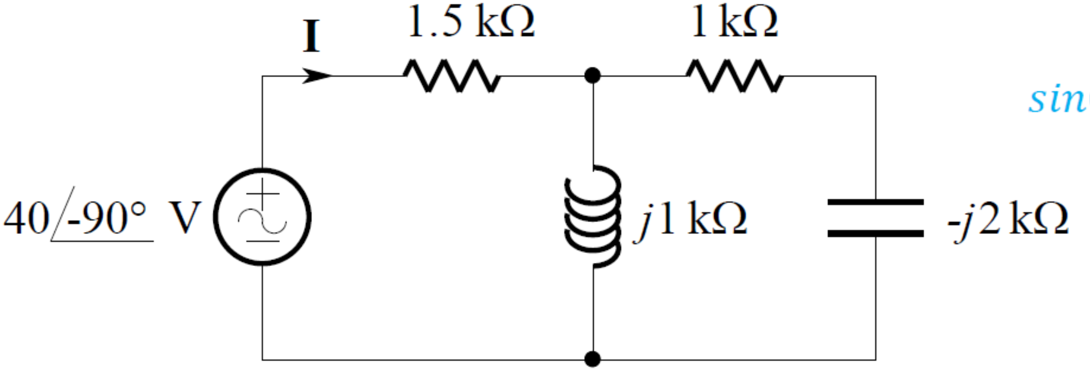


= 40cos(3000t-90)
= 40cos(3000t-pi/2)

Z_L = jωL

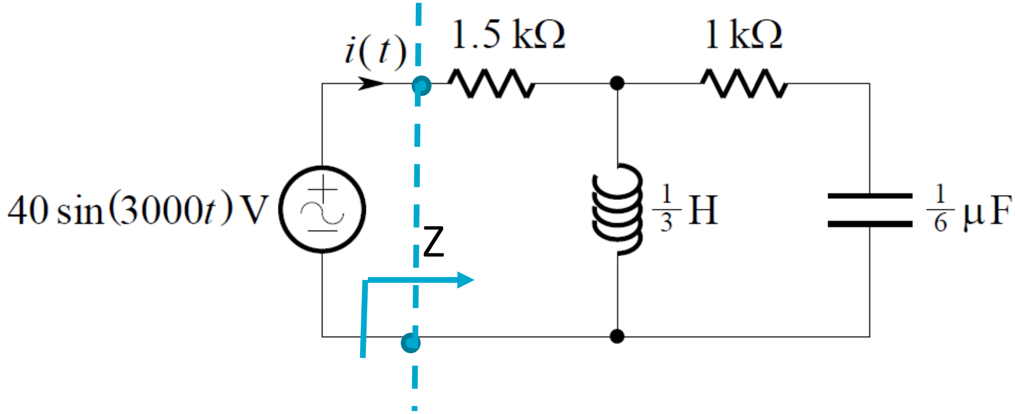
Z_C = 1/(jωC) = -j/(ωC)

The impedances of the inductor and capacitor, determined at ω =3000 rad/s, are j1KΩ and –j2K Ω , respectively.

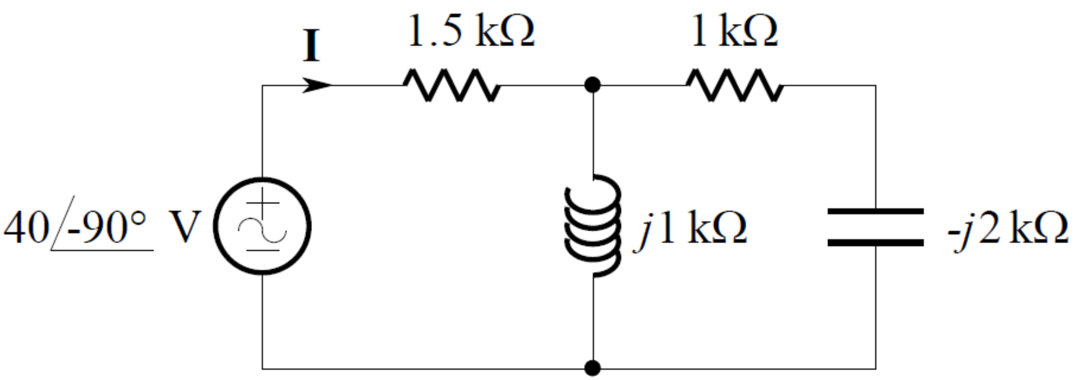


$$sin(x) = cos(90^0 - x) = cos(x - 90^0)$$

1.Equivalent Impedance Calculation



The impedances of the inductor and capacitor, determined at ω =3000 rad/s, are j1KΩ and –j2K Ω , respectively.

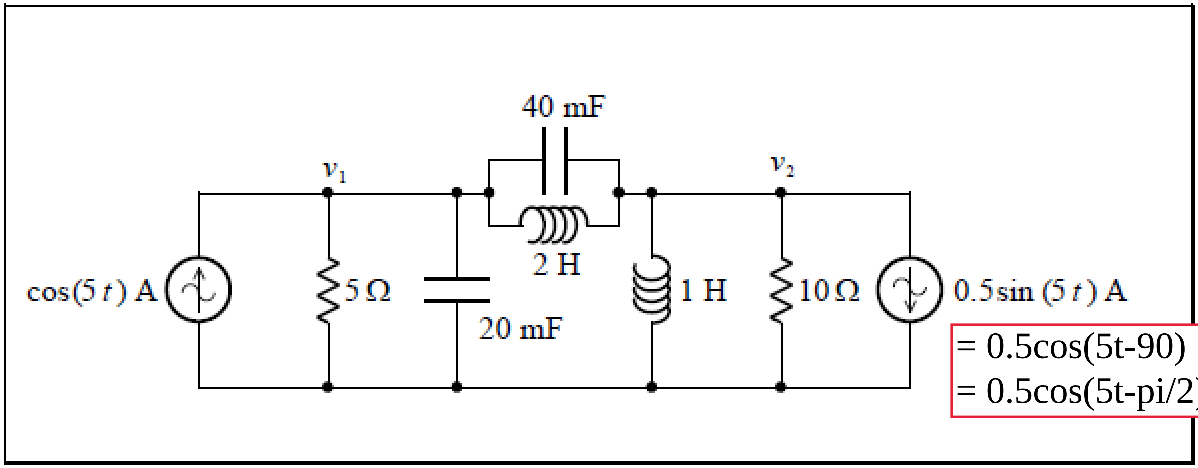


$$\mathbf{Z}_{eq} = 1.5 + \frac{(j1)(1-j2)}{j1+1-j2} = 1.5 + \frac{2+j}{1-j}$$

$$= 1.5 + \frac{2+j}{1-j}\frac{1+j}{1+j} = 1.5 + \frac{1+j3}{2}$$

$$= 2 + j1.5 = 2.5\angle 36.9^\circ \quad \text{k}\Omega$$

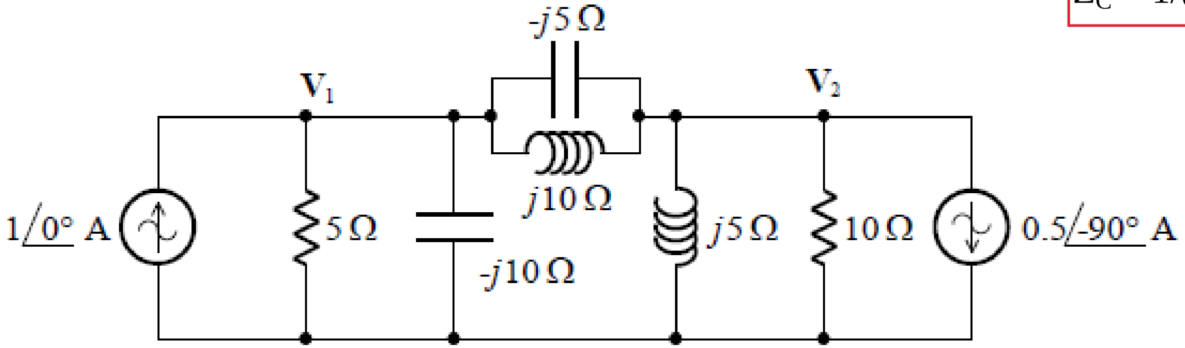
2. Nodal or mesh analysis using Phasors



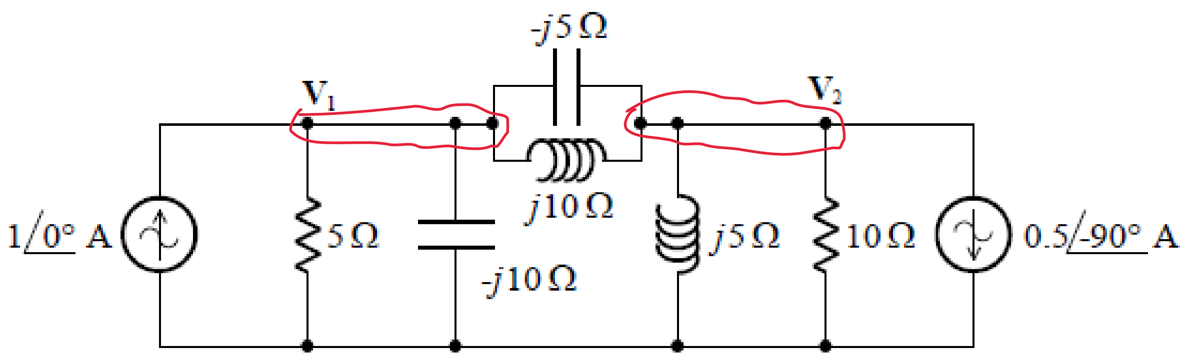
ω =5 rad/s

$Z_L = j\omega L$

$Z_C = 1/(j\omega C) = -j/(\omega C)$



2. Nodal or mesh analysis using Phasors



At the left node, we apply KCL and $\mathbf{I} = \mathbf{V}/\mathbf{Z}$:

$$\frac{\mathbf{V}_1}{5} + \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} = 1 + j0$$

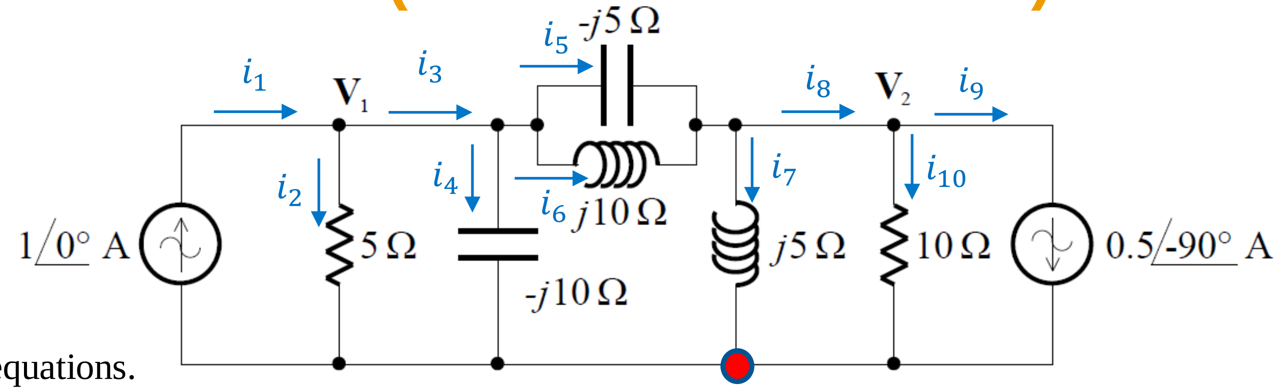
At the right node:

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j5} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{j10} + \frac{\mathbf{V}_2}{j5} + \frac{\mathbf{V}_2}{10} = -(-j0.5)$$

11-2 NODE VOLTAGE ANALYSIS (COMPLEX VERSION)

2. Redraw the circuit and analysis it as if it is a DC resistive circuit.

Use node voltage analysis to solve this circuit



We have 2 nodes, therefore we need 2 equations.

Apply KCL at the left node: *Current_In = Current_Out*

$$i_1 = i_2 + i_3 = i_2 + i_4 + i_5 + i_6 \Rightarrow 1\angle 0^0 = \frac{V_1}{5} + \frac{V_1}{-j10} + \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j10} \Rightarrow (0.2 + j0.2)V_1 - j0.1V_2 = 1$$

Apply KCL at the right node:

$$i_5 + i_6 = i_7 + i_8 = i_7 + i_9 + i_{10} \Rightarrow 0.5\angle -90^0 + \frac{V_2}{10} = \frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j10} - \frac{V_2}{j5} \Rightarrow -j0.1V_1 + (0.1 - j0.1)V_2 = j0.5$$

$$i_9 + i_{10} = i_5 + i_6 - i_7$$

Use Cramer's Rule (the matrix method) to solve the equation set:

11-2 NODE VOLTAGE ANALYSIS (COMPLEX VERSION)

$$\boxed{\begin{aligned}(0.2 + j0.2)\boldsymbol{V_1} - j0.1\boldsymbol{V_2} &= 1 \\ -j0.1\boldsymbol{V_1} + (0.1 - j0.1)\boldsymbol{V_2} &= j0.5\end{aligned}}$$

Use Cramer's Rule (the matrix method) to solve the equation set:

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 1 & -j0.1 \\ j0.5 & (0.1 - j0.1) \end{vmatrix}}{\begin{vmatrix} (0.2 + j0.2) & -j0.1 \\ -j0.1 & (0.1 - j0.1) \end{vmatrix}} = \frac{0.1 - j0.1 - 0.05}{0.02 - j0.02 + j0.02 + 0.02 + 0.01} = \frac{0.05 - j0.1}{0.05} = 1 - j2 \text{ V}$$

$$\mathbf{V}_2 = \frac{\begin{vmatrix} (0.2 + j0.2) & 1 \\ -j0.1 & j0.5 \end{vmatrix}}{0.05} = \frac{-0.1 + j0.1 + j0.1}{0.05} = -2 + j4 \text{ V}$$

Convert to polar form

$$\mathbf{V}_1 = \sqrt{5}\angle -63.4^\circ \text{ V}$$

$$\mathbf{V}_2 = 2\sqrt{5}\angle 116.6^\circ \text{ V}$$

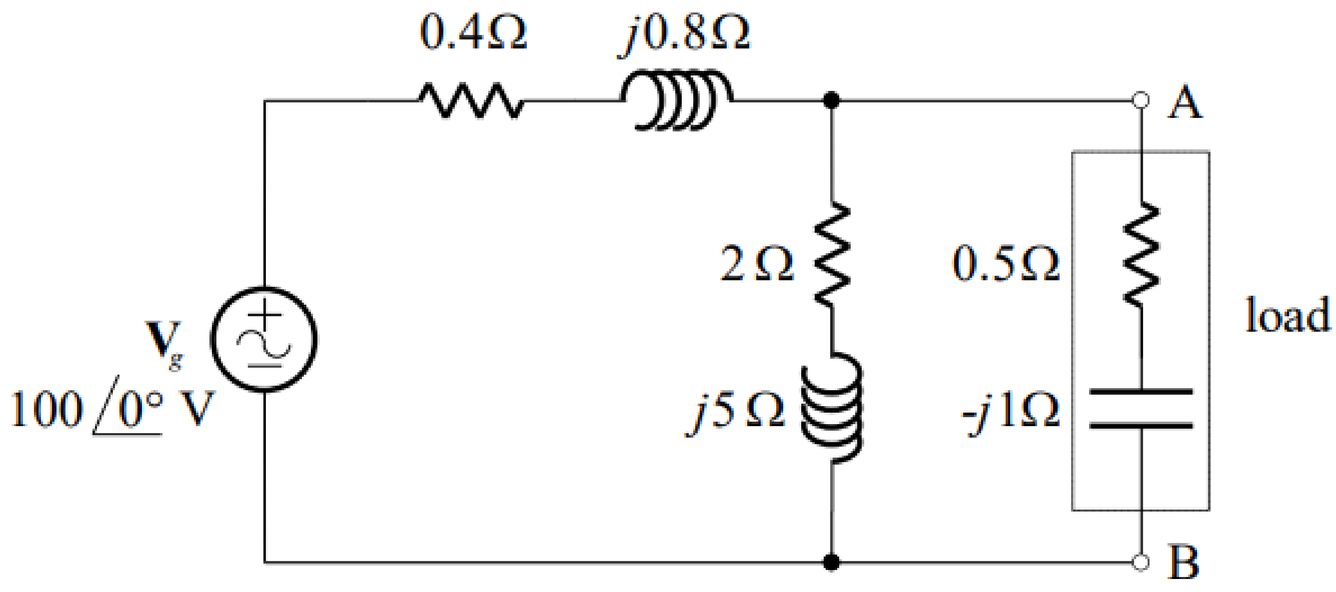
Convert to time-domain

$$v_1(t) = \sqrt{5}\cos(5t - 63.4^\circ) \text{ V}$$

$$v_2(t) = 2\sqrt{5}\cos(5t + 116.6^\circ) \text{ V}$$

3. Thévenin's Theorem using Phasors

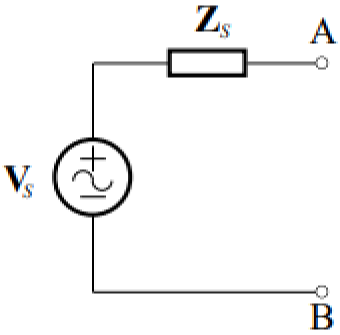
A generator, $\mathbf{V}_g = 100\text{ V}$ RMS, supplies power to a load which is connected between terminals A and B in the circuit below. $\boxed{\text{Tutorial 11, Qu. 2}}$



Calculate the current and voltage across the load

3. Thévenin's Theorem using Phasors

Simplify the circuit be applying Thévenin's Theorem to terminals *A* and B:



The Thévenin voltage $\mathbf{V}_S$ is the open-circuit voltage. Using the voltage divider rule we get:

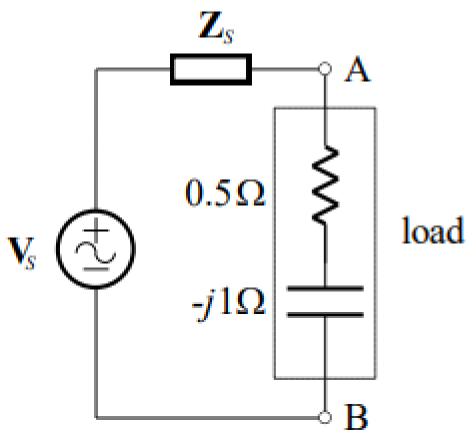
$$\mathbf{V}_S = \frac{2+j5}{2.4+j5.8}\times 100\angle 0^\circ = \frac{5.385\angle 68.20^\circ}{6.277\angle 67.52^\circ}\times 100\angle 0^\circ = 85.79\angle 0.678^\circ \text{ V}$$

Setting the independent voltage source to zero, looking into terminals *A* and B we see two impedances in parallel:

$$\mathbf{Z}_S = (0.4+j0.8)\,\|\,(2+j5) = \frac{-3.2+j3.6}{2.4+j5.8} = \frac{4.817\angle 131.6^\circ}{6.277\angle 67.52^\circ}$$
$$= 0.7673\angle 64.08^\circ\,\Omega = 0.3350 + j0.6904\,\Omega$$

3. Thévenin's Theorem using Phasors

The equivalent circuit is now:



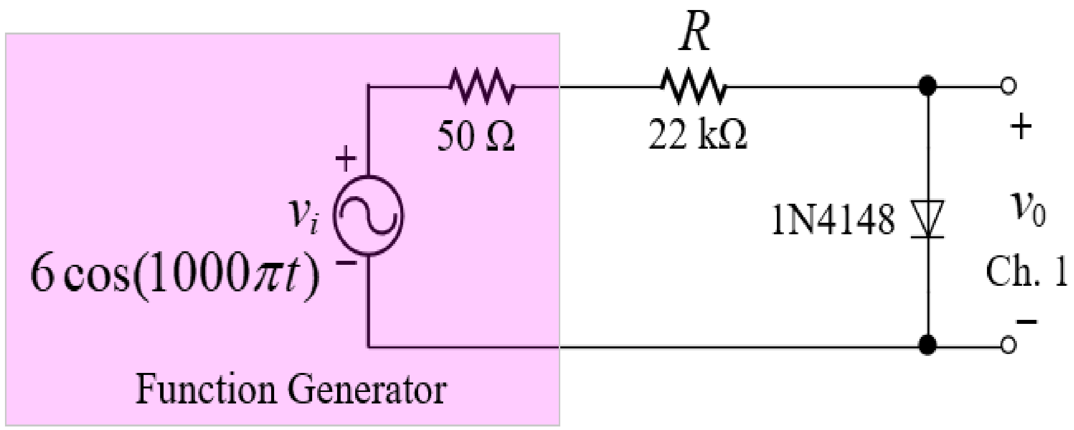
The load current is then given by Ohm's Law:

$$\mathbf{I}_L = \frac{\mathbf{V}_S}{\mathbf{Z}_S + \mathbf{Z}_L} = \frac{85.79\angle 0.678^\circ}{0.3350 + j0.6904 + 0.5 - j} = \frac{85.79\angle 0.678^\circ}{0.8905\angle -20.34^\circ} = 96.33\angle 21.02^\circ \text{ A}$$

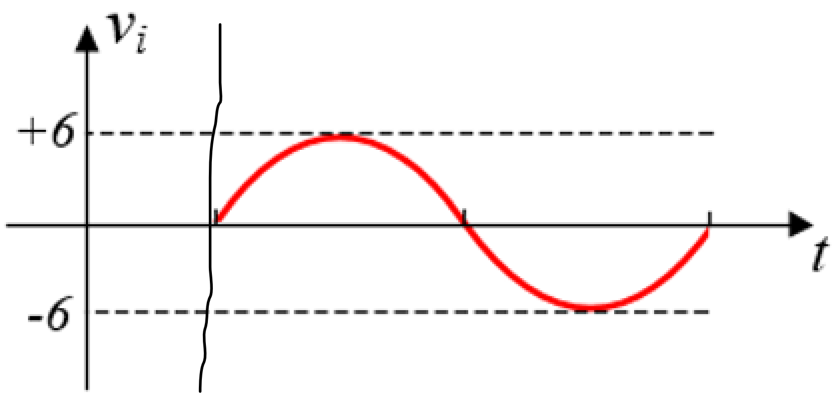
The voltage between terminals A and B is given by Ohms' Law:

$$\begin{aligned}\mathbf{V}_L &= \mathbf{Z}_L \mathbf{I}_L = (0.5 - j)\times 96.33\angle 21.02^\circ = 1.118\angle -63.43^\circ \times 96.33\angle 21.02^\circ \\ &= 107.7\angle -42.41^\circ \text{ V} = 79.52 - j72.64 \text{ V}\end{aligned}$$

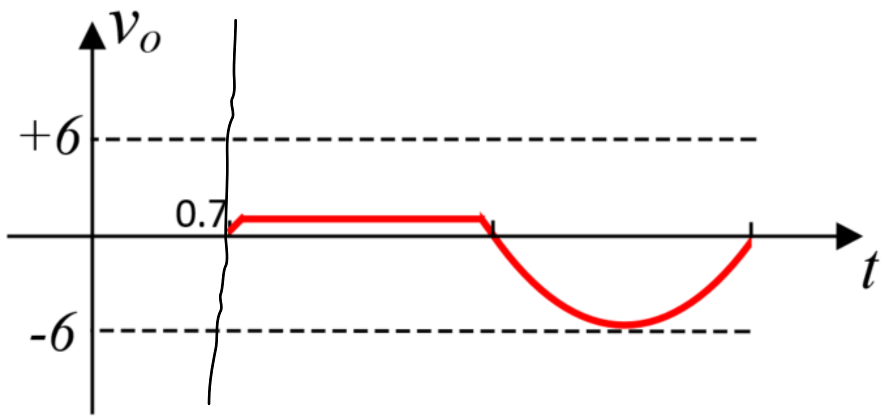
4. Diodes



R
50 Ω
22 kΩ
+
v_i
$6\cos(1000\pi t)$
−
1N4148
+
v_0
Ch. 1
−
Function Generator



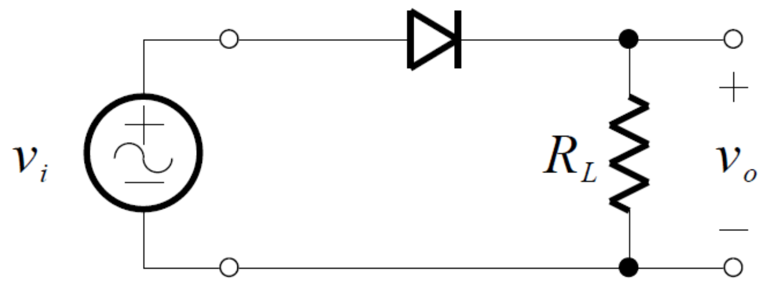
v_i
+6
-6
t



v_o
+6
0.7
-6
t

4. Diodes

Consider the circuit shown below: Week 8 Tutorial, Qu.2



Assume that the diode can be modelled using the "constant voltage drop model" with $e_{fd} = 0.7\ \text{V}$.

Given that $v_i(t) = 5\sin(500\pi t)\ \text{V}$ and load resistance $R_L = 1\ \text{k}\Omega$:

(a) Plot $v_i(t)$ and $v_o(t)$ on the same graph.

(b) What is the peak load current?

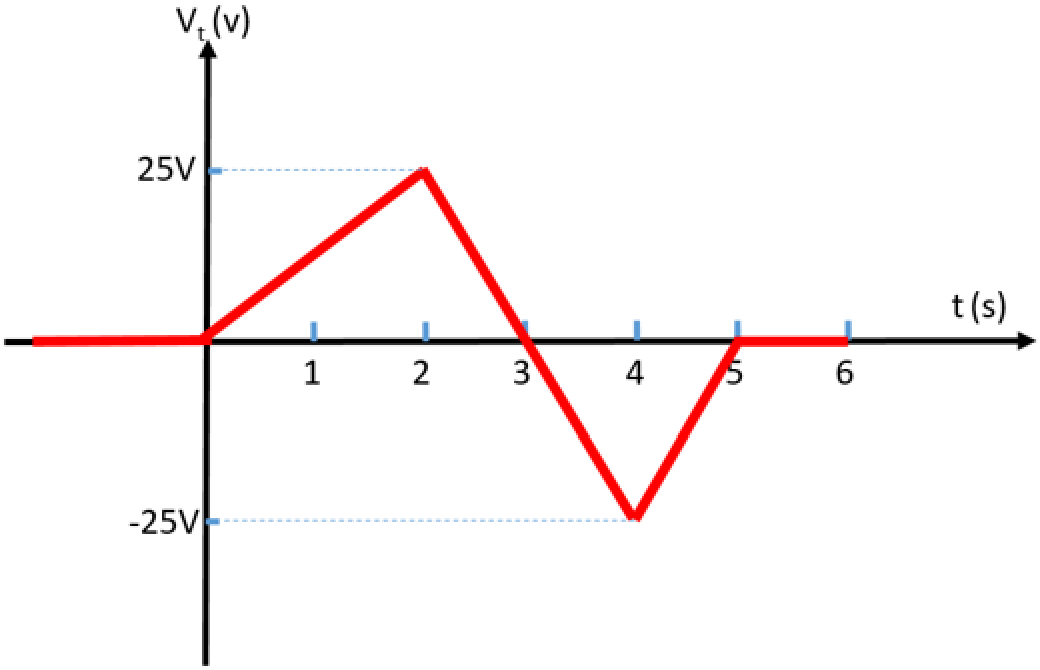
4. Diodes

1. Hlaf-wave Rectifier
2. Limiting circuits
3. Zener regulator circuit

5. Capacitor or Inductor

4. Suppose the voltage V_t shown below is applied to a 100 mF capacitor

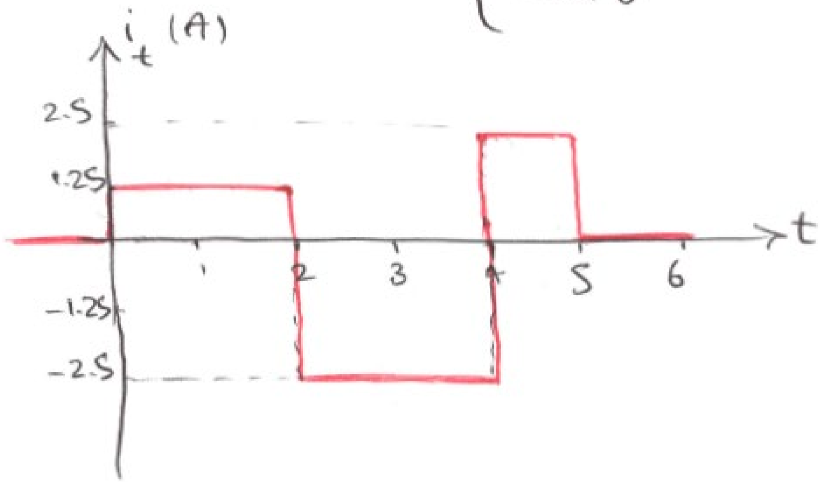
   a. Find the current through the capacitor versus time and plot it
   b. Find the power delivered to the capacitor over time and plot it.



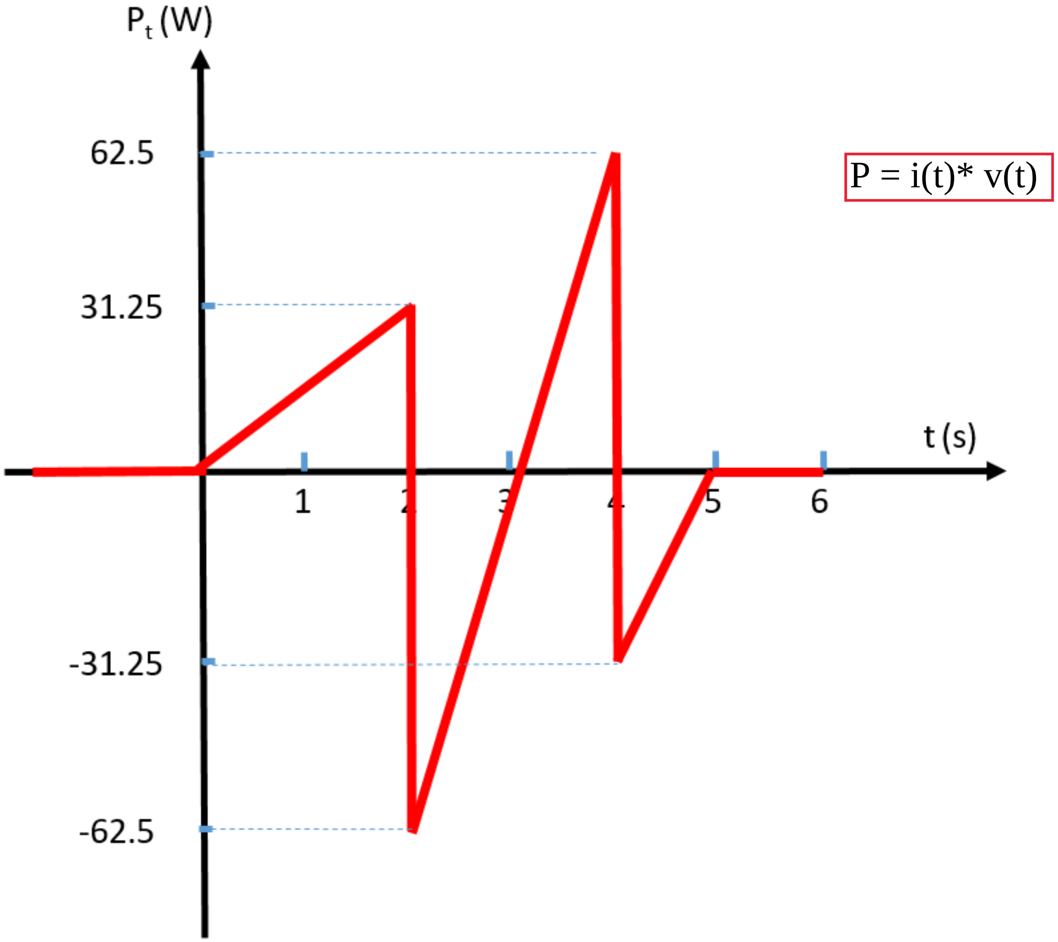
5. Capacitor or Inductor

4. $C = 100\,mF = 10^{2} \times 10^{-3}\,F = 0.1F$

(a) $\boxed{i_t = C\,\frac{dv_t}{dt}} = \begin{cases} 0.1 \times 0 & t<0 \\ 0.1 \times \frac{2.5-0}{2-0} & 0<t<2 \\ 0.1 \times \frac{-2.5-2.5}{4-2} & 2<t<4 \\ 0.1 \times \frac{0-(-2.5)}{5-4} & 4<t<5 \\ 0.1 \times 0 & t>5 \end{cases} = \begin{cases} 0 & t<0 \\ 1.25 & 0<t<2 \\ -2.5 & 2<t<4 \\ 2.5 & 4<t<5 \\ 0 & t>5 \end{cases}$



5. Capacitor or Inductor

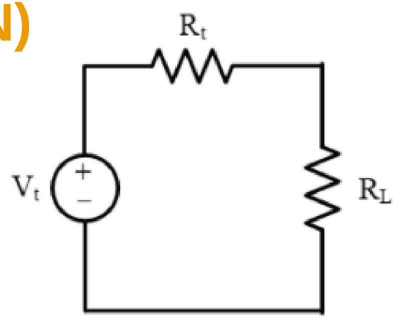


UNIVERSITY OF TECHNOLOGY, SYDNEY

Thank You

Peiyuan.qin@uts.edu.au

11-6 MAXIMUM POWER TRANSFER (COMPLEX VERSION)



In pure resistive circuit, maximum Power Transfer occurs when the resistive value of the load is equal in value to that of the voltage source's internal resistance **(R_L=R_s)** allowing maximum power to be supplied.

The maximum power received by the load is $P_L = \frac{V_t^2}{4R_t}$, which is half of the total power generated by the source.

In comple circuit, how to adjust a load impedance $\boldsymbol{Z_L}$ to extract the maximum average power from a two-terminal circuit? (Note $\boldsymbol{Z_t}$ is the inner impedance or Thevenin equivalent impedance.)

1) **If the load can take on any complex value**, maximum power transfer is attained for a load impedance equal to the complex conjugate of the Thévenin impedance.

$R + jX$

$$Z_{load} = Z_t^*$$

$R - jX$

2) **If the load is required to be a pure resistance**, maximum power transfer is attained for a load resistance equal to the magnitude of the Thévenin impedance.

$R + jX$

$$Z_{load} = |Z_t| = \sqrt{R^2 + X^2}$$

$\sqrt{R^2 + X^2}$

11-6 MAXIMUM POWER TRANSFER (COMPLEX VERSION)

Note that the power should be a pure real number. Only resistance can receive power. Reactance (capacitance and inductance) does not take power. The average AC power on capacitors and inductors are always 0. With the AC source, the capacitors and inductors keep charging and discharging, thus the energy comes and goes away but no real energy is consumed.

The maximum power delivered on the load is only determined by the effective RMS current and the resistance of the impedance:

$$P_{max} = I_{rms}^2 R_{load} \qquad \text{Where } R_{load} = \begin{cases} R_t & \text{if load is a complex impedance} \\ |Z_t| & \text{if load is a pure resistive load} \end{cases}$$