48510 LEC 2

Basic Circuit Laws

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48510 LEC 2 – BASIC CIRCUIT LAWS

Topic 2-1: Series and parallel circuits

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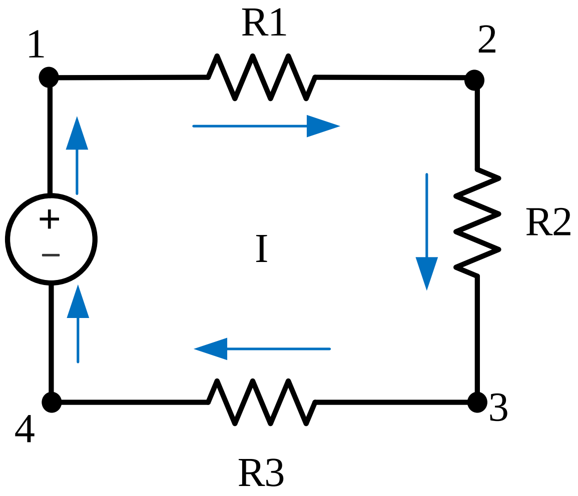
SERIES AND PARALLEL CIRCUITS

Series circuit:

Here, we have three resistors (R1, R2, and R3), connected in a long chain from one terminal of the source to the other.

The defining characteristic of a series circuit is that **there is only one path for current (electrons) to flow.**



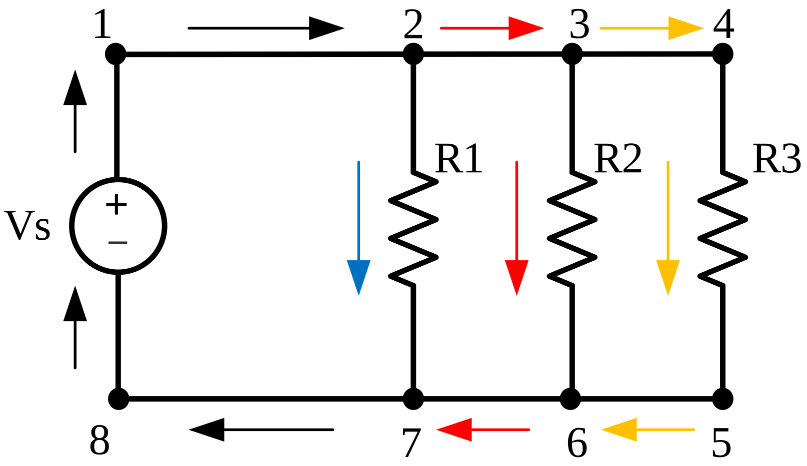
Note that in a series circuit, the current is identical everywhere.

SERIES AND PARALLEL CIRCUITS

Parallel circuit:

The defining characteristic of a parallel circuit is that all components are connected between the same set of electrically common points.

If two or more points are directly connected by lossless wires, they are electrically common points. This is because the length of lossless wires can be neglected when analysing circuit.

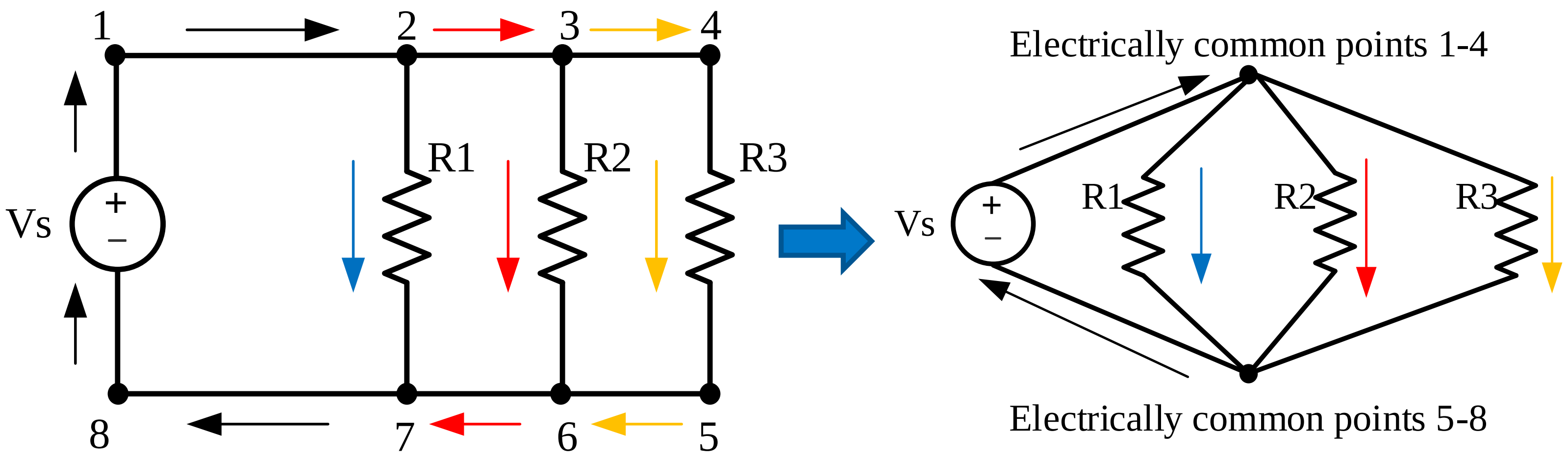


SERIES AND PARALLEL CIRCUITS

Parallel circuit:

The defining characteristic of a parallel circuit is that all components are connected between the same set of electrically common points.

Neglect the length of the wires connecting the electrically common points



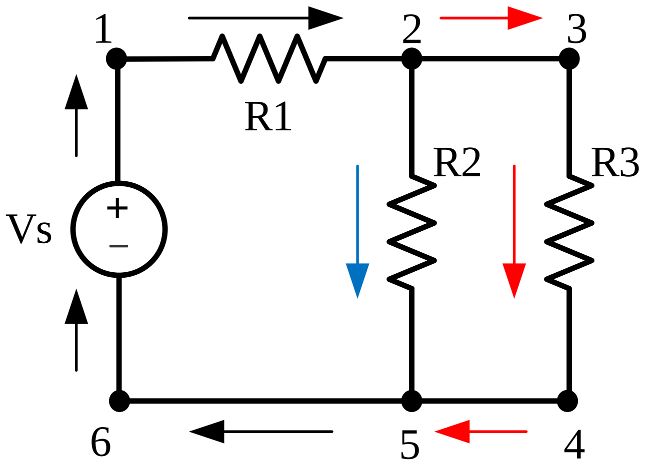
Note that in a parallel circuit, each branch share the same voltage.

SERIES AND PARALLEL CIRCUITS

Combination of series and parallel circuit:

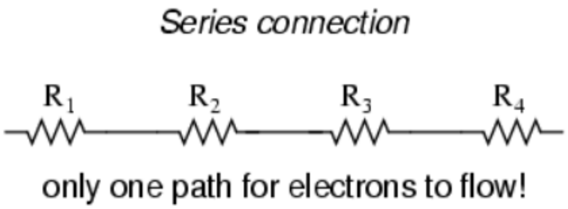
We can have circuits that are a combination of series and parallel.

In this circuit, we have two loops for current to flow through: one from 1-2-5-6, and the other from 1-2-3-4-5-6. Notice how both current paths go through R1. In this configuration, we'd say that R2 and R3 are in parallel with each other, while R1 is in series with the parallel combination of R2 and R3.

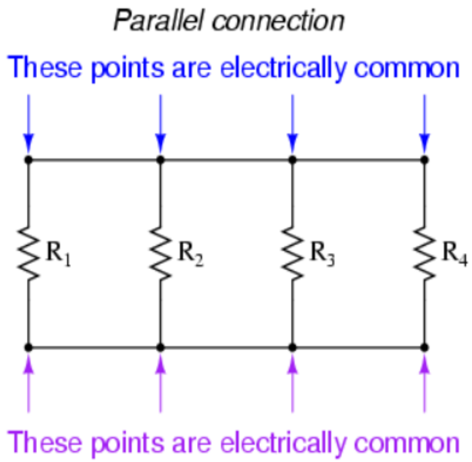


SERIES AND PARALLEL CIRCUITS

The basic idea of a **“series” connection** is that components are **connected end-to-end in a line** to form a single path for electrons to flow.



The basic idea of a **“parallel” connection**, on the other hand, is that all components are **connected across each other’s leads**. In a purely parallel circuit, there are never more than two sets of electrically common points, no matter how many components are connected. There are many paths for electrons to flow, but only one voltage across all components:



48510 LEC 2 – BASIC CIRCUIT LAWS

Topic 2-2: Kirchhoff’s Current Law (KCL)

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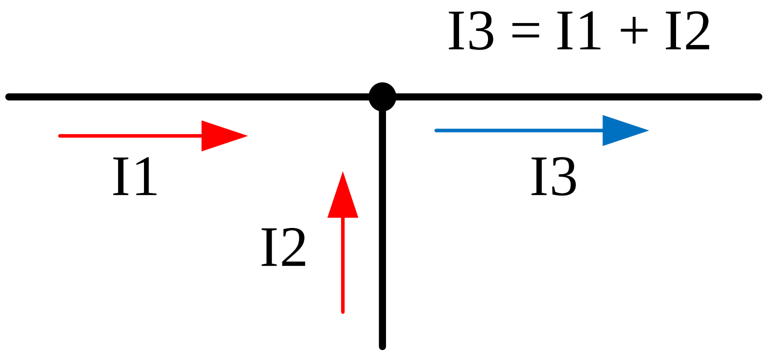
KIRCHHOFF'S CURRENT LAW (KCL)

Kirchhoff's Current Law, often shortened to KCL, states that "**The algebraic sum of all currents entering and exiting a node must equal zero**."

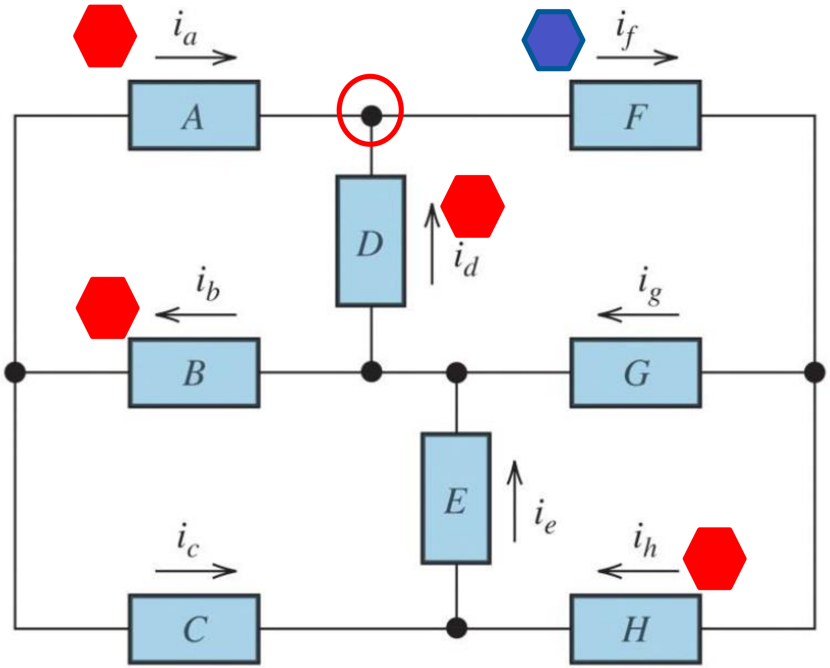
In other words, the algebraic sum of all the currents entering a junction/node must be equal to the sum of all the current leaving the junction: $\Sigma I_{IN} = \Sigma I_{OUT}$.



If we assign a mathematical sign (polarity) to each current, denoting whether they enter or exit a node, we can use the equation $\Sigma I_{IN} = \Sigma I_{OUT}$ to find out the unknown current.

KIRCHHOFF'S CURRENT LAW (KCL)

Example: *Given parameters:* i_a =1A, i_b = -5A, i_d = 3A, and i_h = 2A, find all the other currents.



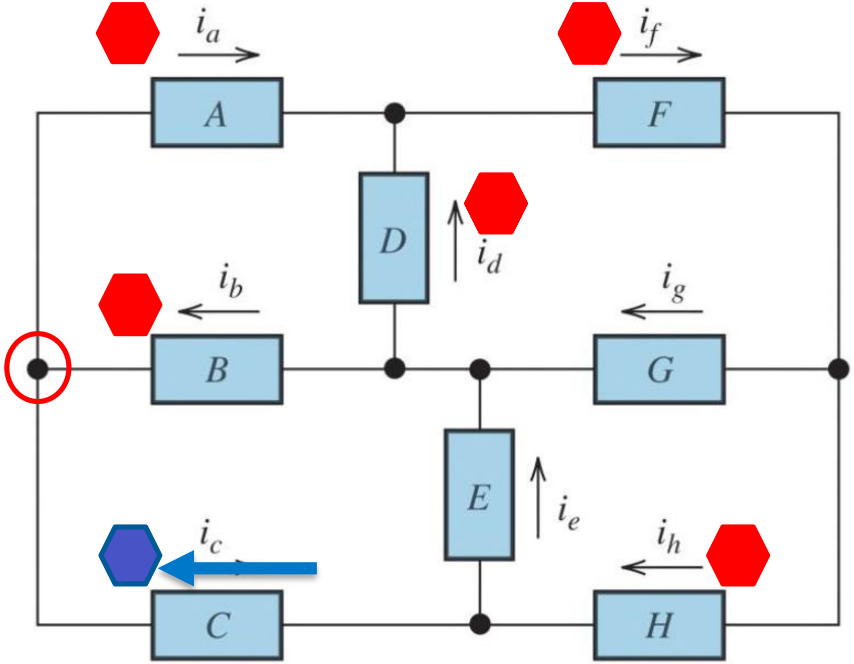
KCL: $\Sigma\ I_{IN} = \Sigma\ I_{OUT}$

$$i_a + i_d = i_f$$

$$i_f = 1 + 3 = 4A$$

KIRCHHOFF'S CURRENT LAW (KCL)

Known parameters: $i_a = 1A$, $i_b = -5A$, $i_d = 3A$, $i_h = 2A$, $i_f = 4A$



$$i_b = i_a + i_c$$

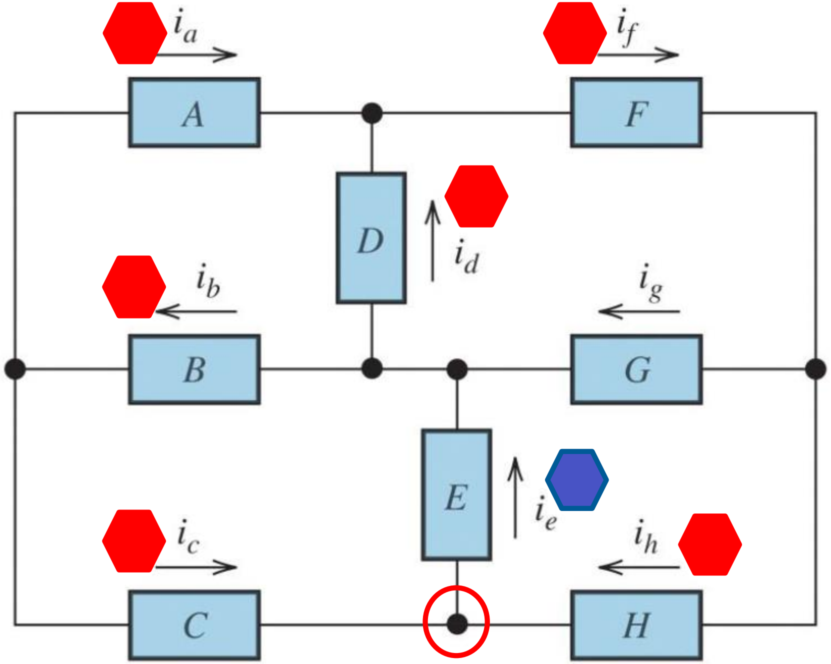
$$i_c = i_b - i_a = -5 - 1 = -6A$$

$$i_b + i_c = i_a$$

$$i_c = i_a - i_b = 1 + 5 = 6A$$

KIRCHHOFF'S CURRENT LAW (KCL)

Known parameters: i_a =1A, i_b = -5A, i_d = 3A, and i_h = 2A, i_f =4A, i_c = -6A

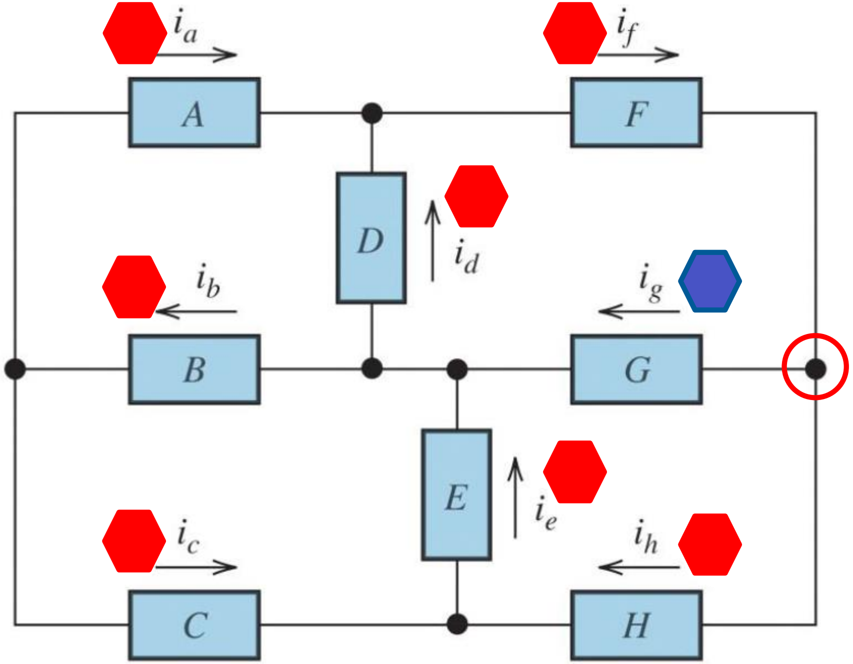


$$i_c + i_h = i_e$$

$$i_e = i_c + i_h = -6 + 2 = -4A$$

KIRCHHOFF'S CURRENT LAW (KCL)

Given parameters: i_a =1A, i_b = -5A, i_d = 3A, and i_h = 2A, i_f =4A, i_c = -6A, i_e = -4A

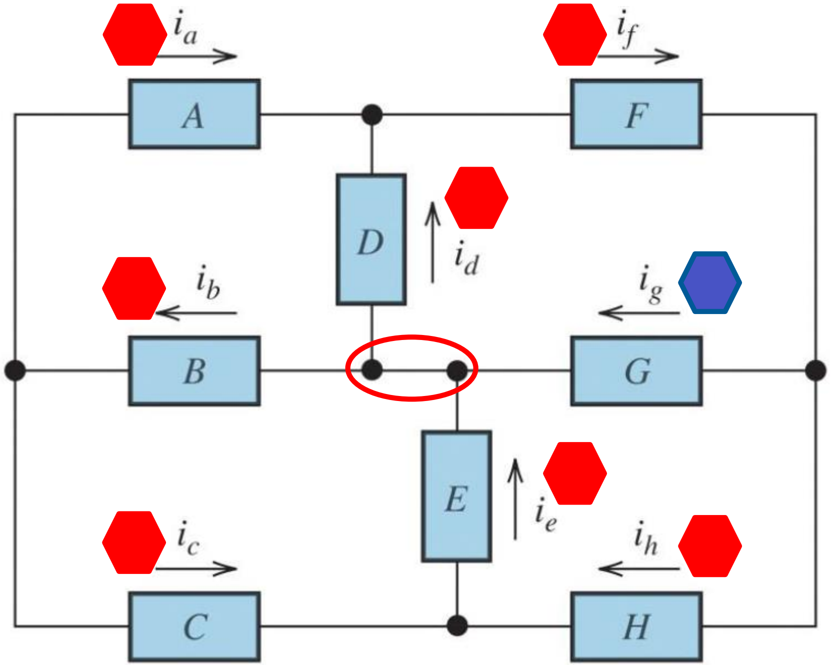


$$i_f = i_g + i_h$$

$$i_g = i_f - i_h = 4 - 2 = 2A$$

KIRCHHOFF'S CURRENT LAW (KCL)

Known parameters: i_a =1A, i_b = -5A, i_d = 3A, and i_h = 2A, i_f =4A, i_c = -6A, i_e = -4A



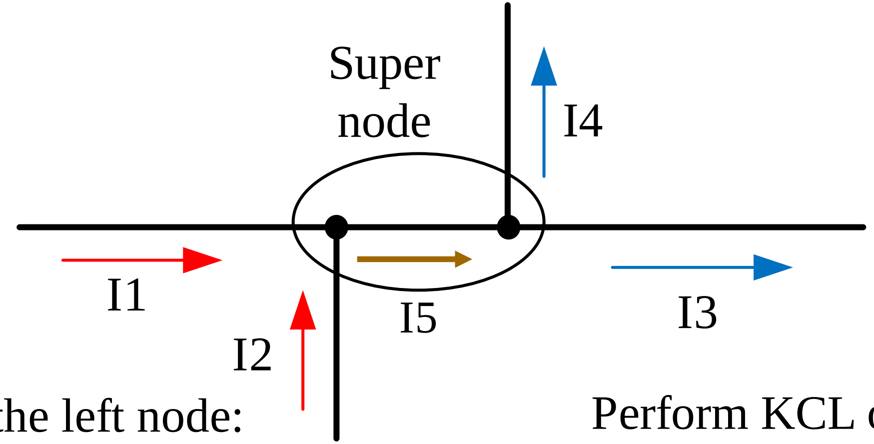
Two electrically common nodes can be seen as a super node:

$$i_g + i_e = i_d + i_b$$

$$i_g = i_d + i_b - i_e$$
$$= 3 – 5 + 4 = 2A$$

KIRCHHOFF'S CURRENT LAW (KCL)

When performing KCL, two electrically common nodes can be seen as a super node.



Perform KCL on the left node:

I1 + I2 = I5

Perform KCL on the right node:

I5 = I3 + I4

The result is the same as performing KCL on the super node

I1 + I2 = I5 = I3 + I4

48510 LEC 2 – BASIC CIRCUIT LAWS

Topic 2-3: Kirchhoff’s Voltage Law (KVL)

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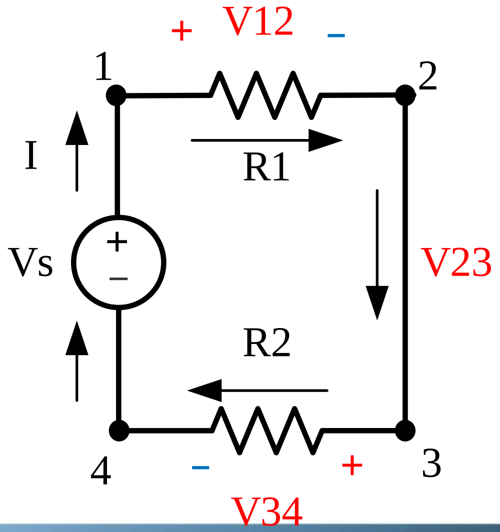
KIRCHHOFF'S VOLTAGE LAW (KVL)

KVL is the Kirchhoff's second law that deals with the conservation of energy around a closed circuit path.

His voltage law states that for a closed loop series path **the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero**. This is because a circuit loop is a closed conducting path so no energy is lost.

Note that the voltage across two points is actually the potential difference between the two points.



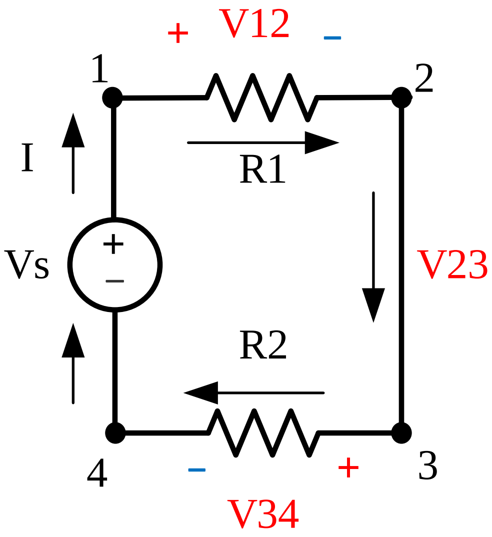
KIRCHHOFF'S VOLTAGE LAW (KVL)

Recommended steps to perform KVL:

Step 1: *Identify the loop on which you want to perform KVL.*

Step 2: *Assign an initial node and assume its voltage is 0. Usually we assign the cathode of the source has a voltage of 0. (Note that even you assign another node to be 0, it won't affect the results.) In this case, node 4 is our initial node and we assume its voltage V4 is 0.*

Step 3: *Add up the voltage change following either clockwise or anticlockwise direction. (Usually I prefer to use the clockwise direction.)*



KIRCHHOFF'S VOLTAGE LAW (KVL)

Recommended steps to perform KVL:

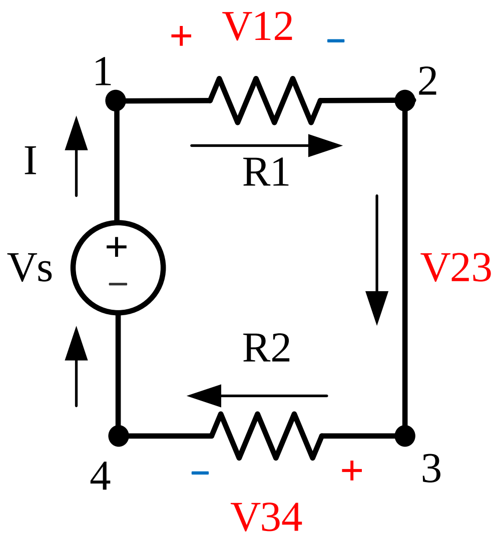
Step 3: *Add up the voltage change following either clockwise or anticlockwise direction.*

From node 4 to node 1	$V4 + Vs = V1$
From node 1 to node 2	$V1 - V_{12} = V2$
From node 2 to node 3	$V2 = V3$
From node 3 to node 4	$V3 - V_{34} = V4$

$$Vs - V_{12} - V_{34} = 0$$

Step 4: *Use Ohm's law to get the value of the voltage drops to complete the equation. In this case, $V_{12} = IR1$, $V_{34} = IR2$.*

Following the current direction, there is a voltage drop across a resistor and the value can be calculated via Ohm's law.



KIRCHHOFF'S VOLTAGE LAW (KVL)

Following the current direction, there is a voltage drop across a resistor and the value can be calculated via Ohm's law.

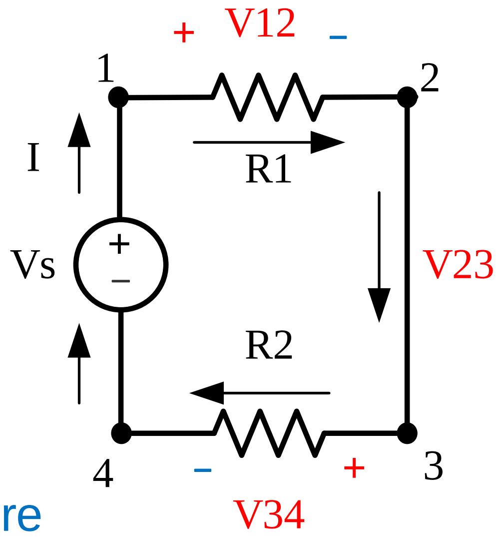
Recommended steps to perform KVL:

Step 1: *Identify the loop on which you want to perform KVL.*

Step 2: *Assign an initial node and assume its voltage is 0.*

Step 3: *Add up the voltage change following either clockwise or anticlockwise direction. Note that whether the voltage is increased or decreased can be determined by the current direction or source polarities.*

Step 4: *Use Ohm's law to get the value of the voltage drops to complete the equation.*



In the future, I encourage you to always follow this procedure to apply KVL as it helps you to avoid mistakes.

KIRCHHOFF'S VOLTAGE LAW (KVL)

Previously we applied KVL on a series circuit, what about a parallel circuit? Let's try to use KVL to determine the relationship between the voltages.

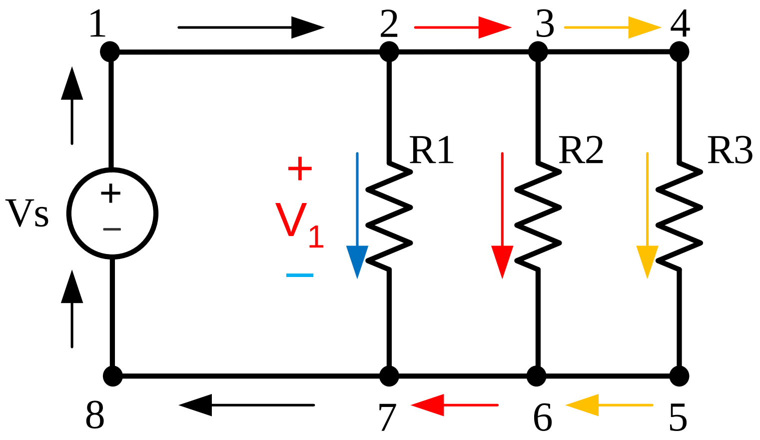
Step 1: *Identify the loop on which you want to perform KVL. (Loop 1-2-7-8-1)*

Step 2: *Assign an initial node and assume its voltage is 0. (Node 8)*

Step 3: *Add up the voltage change following clockwise.*

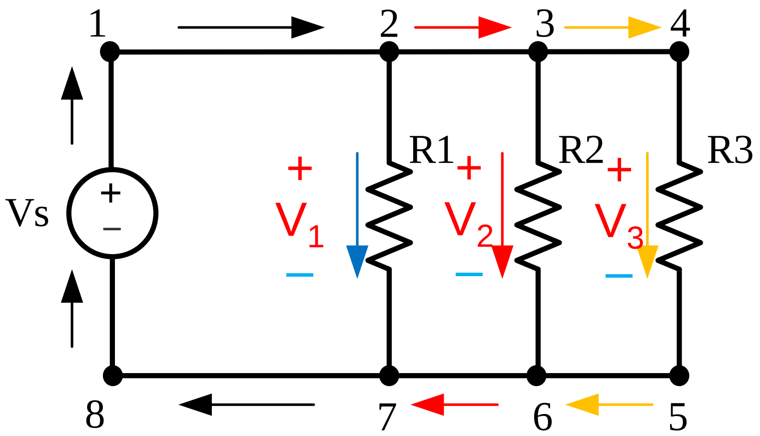
$$Vs - V_1 = 0$$

$$Vs = V_1$$



KIRCHHOFF'S VOLTAGE LAW (KVL)

Previously we applied KVL on a series circuit, what about a parallel circuit? Let's try to use KVL to determine the relationship between the voltages.



Step 1: *Identify the loop on which you want to perform KVL. (Loop 1-2-7-8-1)*

Step 2: *Assign an initial node and assume its voltage is 0. (Node 8)*

Step 3: *Add up the voltage change following clockwise.*

$$Vs = V_1$$

Repeat these steps along loop 1-3-6-8-1 $Vs = V_2$

Repeat these steps along loop 1-4-5-8-1 $Vs = V_3$

Repeat these steps along loop 3-4-5-6-3 $V_2 – V_3 = 0$

48510 LEC 2 – BASIC CIRCUIT LAWS

Topic 2-4: Equivalent Resistance

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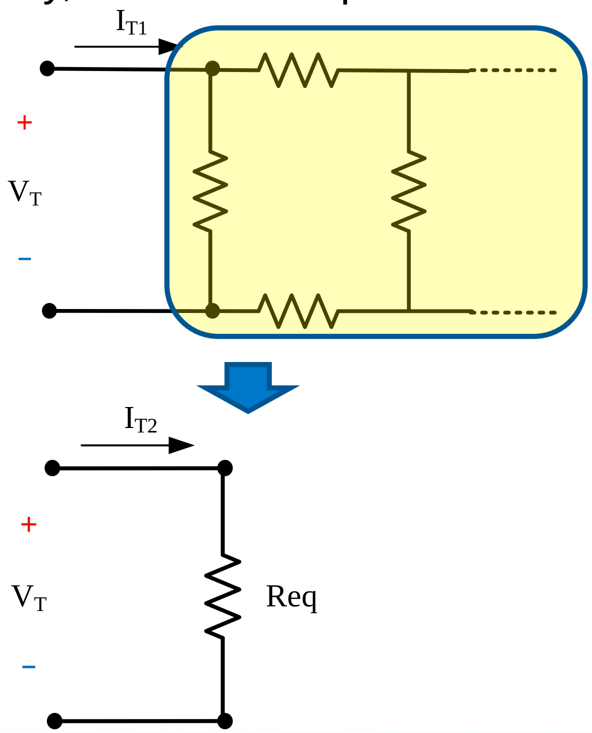
EQUIVALENT RESISTANCE

In electrical and electronic engineering, an **equivalent circuit** refers to a theoretical circuit that retains all of the electrical characteristics of a given circuit. Often, an equivalent circuit is sought that simplifies calculation, and more broadly, that is a simplest form of a more complex circuit in order to aid analysis.

In this topic, we will investigate the equivalent circuit of a combination of resistors only. **A combination of several resistors can be equivalent to one resistor.**

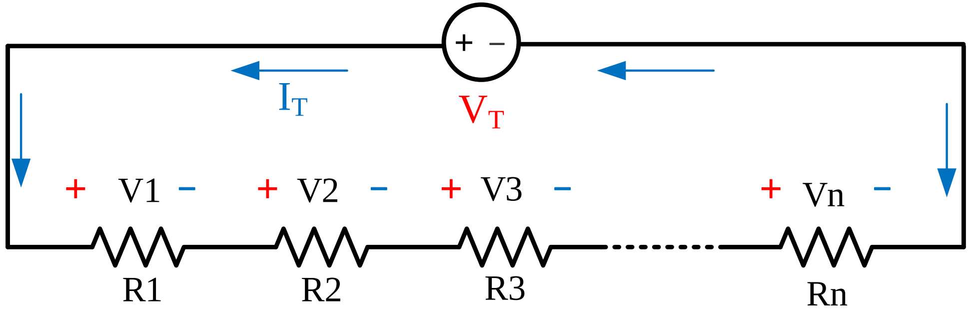
When we say two linear circuits are equivalent to each other, it means that the two circuits have the same response given a same excitation. In other words, if we excite two circuits with the same voltage input (or same current input), the resultant currents (or voltages) for the two circuits are the same. For example, given the same V_T, the current $I_{T1} = I_{T2}$.

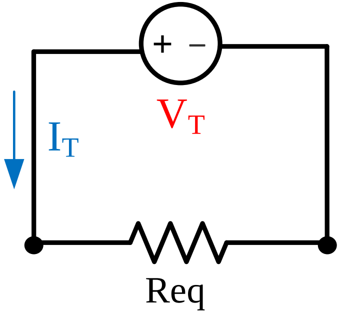


EQUIVALENT RESISTANCE

Equivalent resistance of resistors connected in series

Consider a series circuit consisting of *n* resistors, the current flowing through the circuit is identical everywhere and we assume it is I_T.





According to Ohm' law:

$$V_1 = I_T R_1$$

$$V_2 = I_T R_2$$

$$V_n = I_T R_n$$

According to KVL:

$$V_T = V_1 + V_2 + \cdots V_n$$

$$V_T = I_T (R_1 + R_2 + \cdots R_n)$$

$$V_T = I_T R_{eq}$$

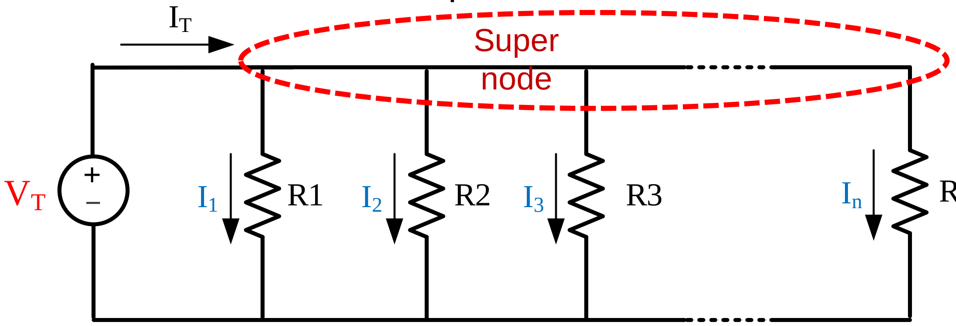
$$R_{eq} = R_1 + R_2 + \cdots R_n$$

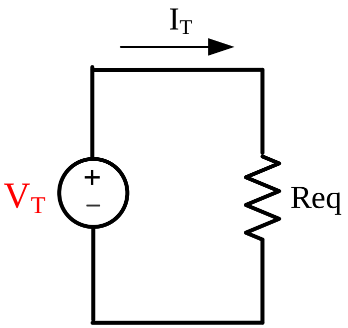
For resistors connected in series, the equivalent resistance equals to the sum of all the individual resistors added together.

EQUIVALENT RESISTANCE

Equivalent resistance of resistors connected in parallel

Consider *n* resistors connected in parallel, all the resistors share the same voltage.





According to Ohm' law:

$$I_1 = \frac{V_T}{R_1} \qquad I_2 = \frac{V_T}{R_2} \quad \cdots\cdots \quad I_n = \frac{V_T}{R_n}$$

$$I_T = V_T \frac{1}{R_{eq}}$$

According to KCL:

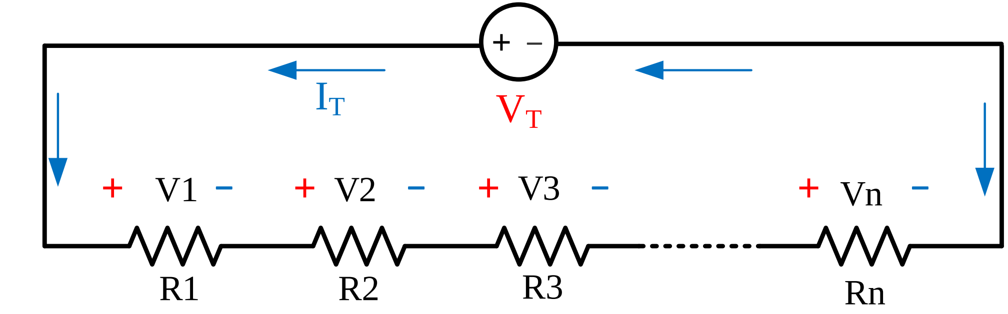
$$I_T = I_1 + I_2 + \cdots I_n$$

$$I_T = \frac{V_T}{R_1} + \frac{V_T}{R_2} + \cdots \frac{V_T}{R_n} = V_T\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}\right)$$

The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistances.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}$$

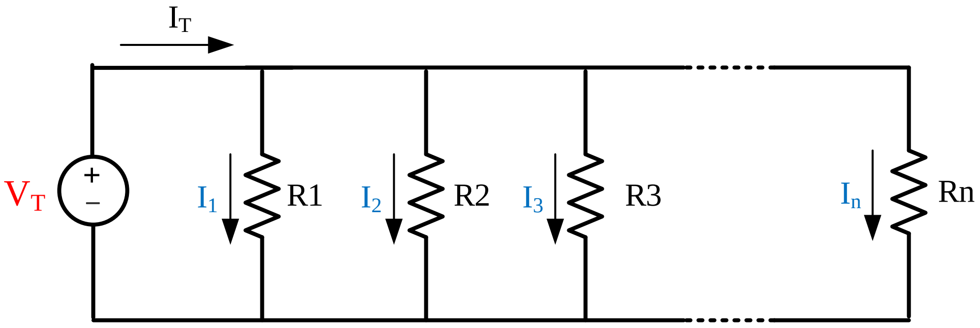
EQUIVALENT RESISTANCE



For resistors connected in series, the equivalent resistance equals to the sum of all the individual resistors added together.

$$R_{eq} = R_1 + R_2 + \cdots R_n$$

The equivalent resistance is larger than any of the individual resistance.



The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistances.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}$$

$$R_{eq} = R_1 // R_2 // \cdots R_n$$

The equivalent resistance is smaller than any of the individual resistance.

Specifically, the equivalent resistance of two resistor connected in parallel is

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

EQUIVALENT RESISTANCE

Equivalent resistance of series and parallel circuits

What if we want to connect various resistors together in both parallel and series combinations, how do we calculate the combined or total circuit resistance?

Tip: The key is to find out how the current flows in the circuit. Based on the current flow, you are able to determine in which way the resistors are combined.

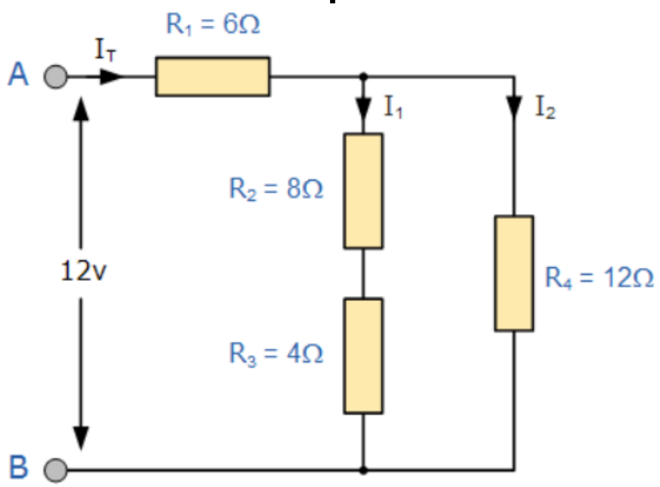
The current passes through R1 first, and then split to two paths and later join together again.

On the left path, the current passes through two resistors in series. The equivalent resistance is R2+R3=12 Ω.

The two branches are connected in parallel, so the total resistance is (R2+R3)//R4 = 12//12 = 6 Ω.

The combination of the two branches are connected in series with R1, so the total resistance is (R2+R3)//R4 + R1 = 6 + 6 = 12 Ω.

Example 1

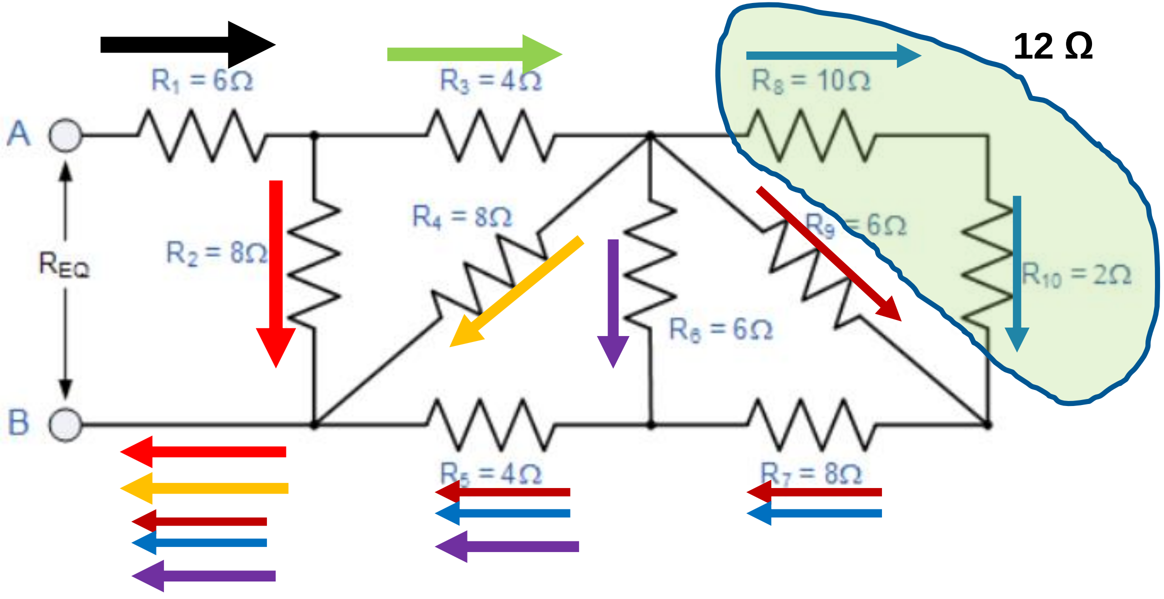


EQUIVALENT RESISTANCE

Equivalent resistance of series and parallel circuits

Tip: The key is to find out how the current flows in the circuit. Based on the current flow, you are able to determine in which way the resistors are combined.

R8 and R10 are connected in series. The resultant resistance is R8+R10 =12 Ω

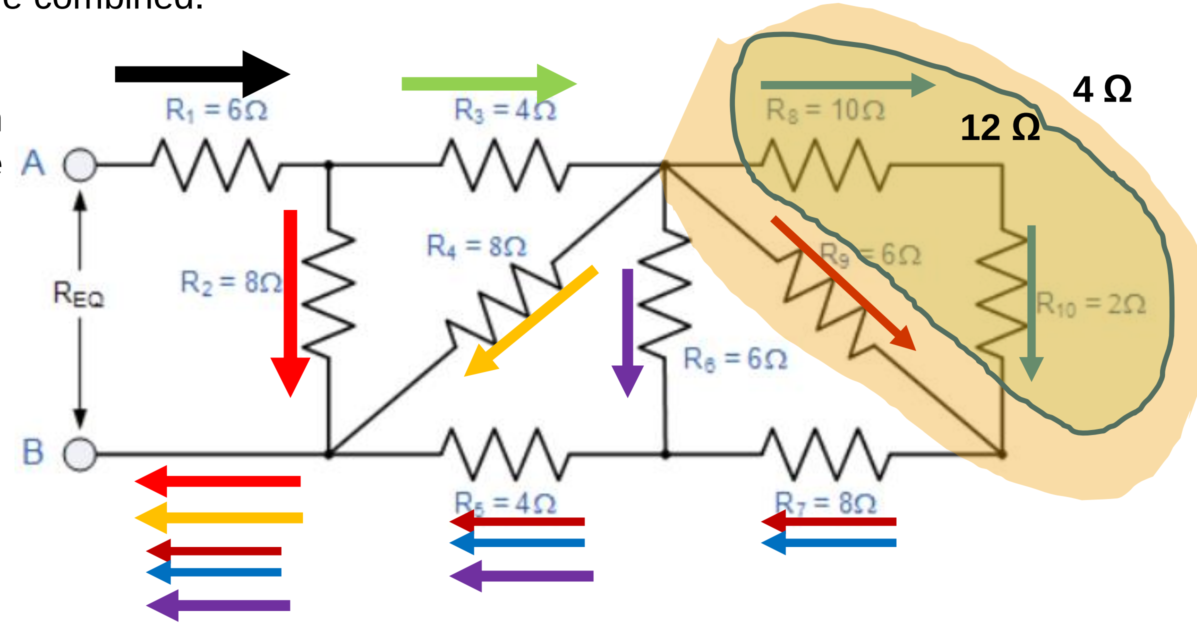


EQUIVALENT RESISTANCE

Equivalent resistance of series and parallel circuits

Tip: The key is to find out how the current flows in the circuit. Based on the current flow, you are able to determine in which way the resistors are combined.

The resultant resistance is then connected with R9 in parallel. The equivalent resistance is 12//6 = 4 Ω.

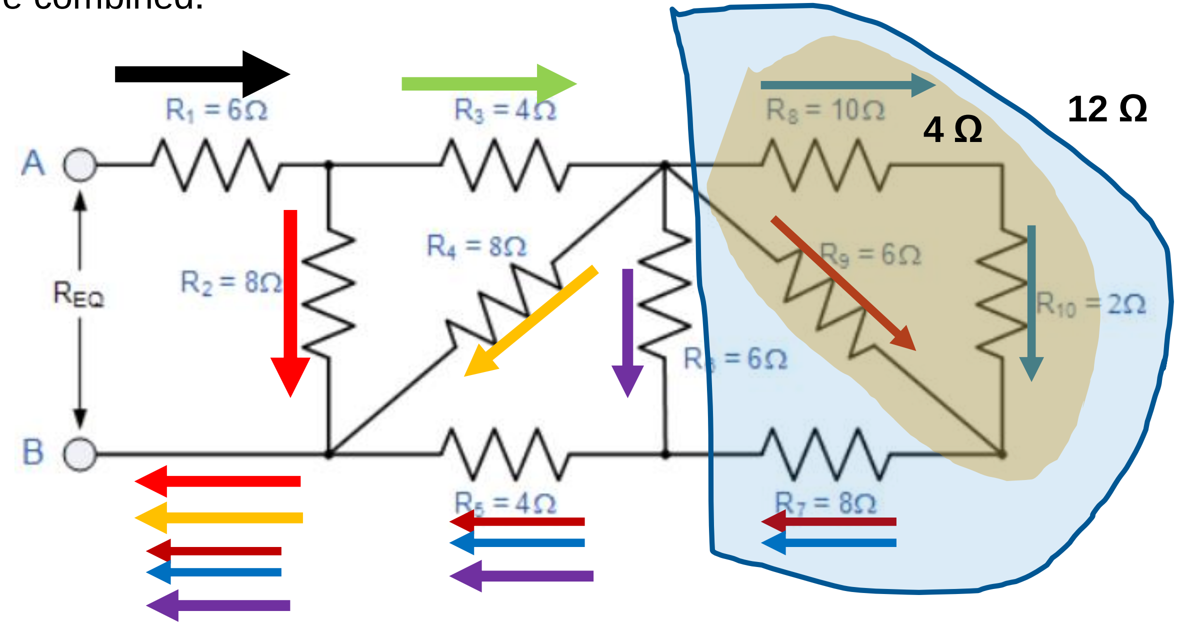


EQUIVALENT RESISTANCE

Equivalent resistance of series and parallel circuits

Tip: The key is to find out how the current flows in the circuit. Based on the current flow, you are able to determine in which way the resistors are combined.

The resultant resistance is then connected with R7 in series. The equivalent resistance is 4 + 8 = 12 Ω.

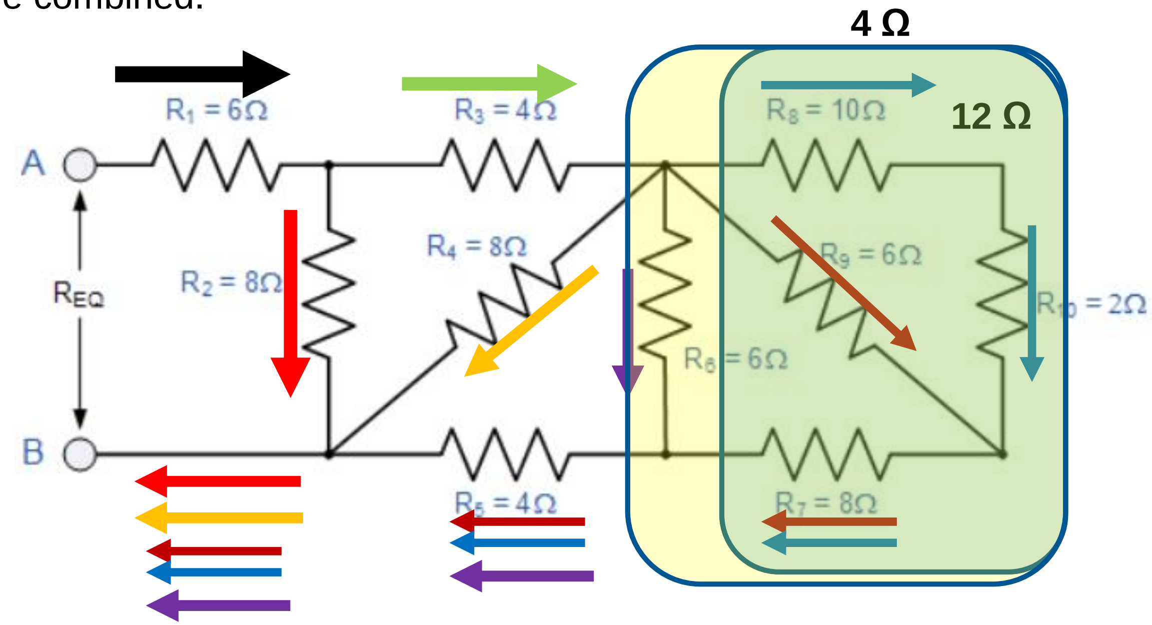


EQUIVALENT RESISTANCE

Equivalent resistance of series and parallel circuits

Tip: The key is to find out how the current flows in the circuit. Based on the current flow, you are able to determine in which way the resistors are combined.

The resultant resistance is then connected with R6 in parallel. The equivalent resistance is 12//6= 4 Ω.

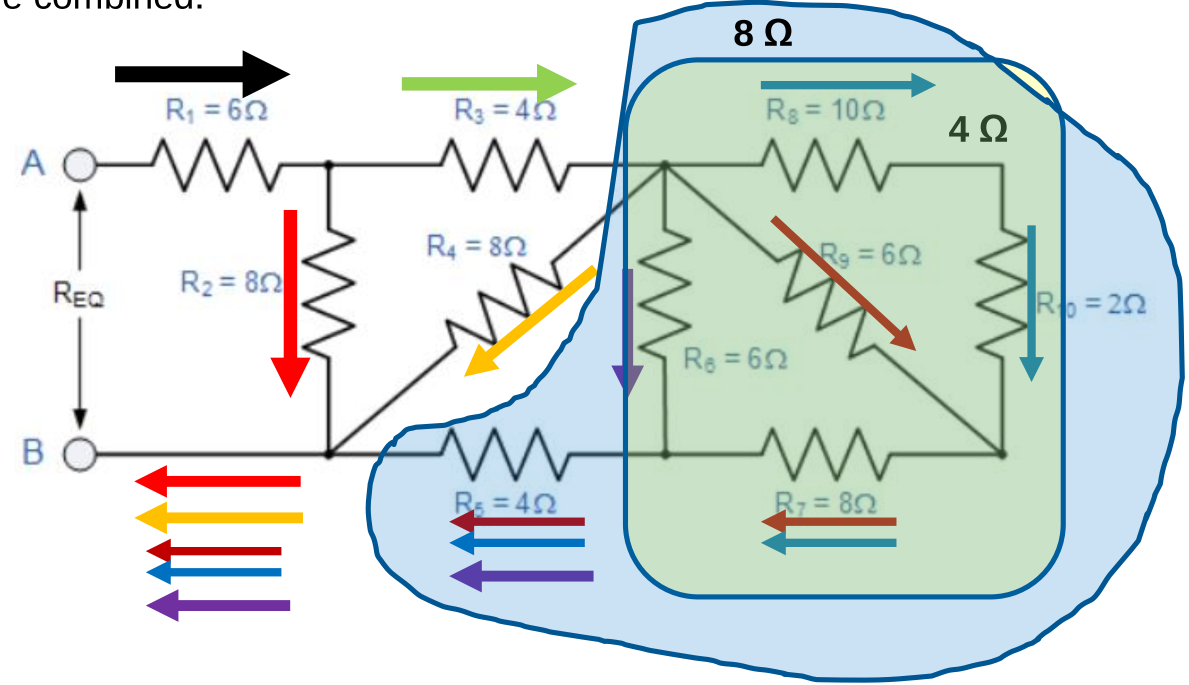


EQUIVALENT RESISTANCE

Equivalent resistance of series and parallel circuits

Tip: The key is to find out how the current flows in the circuit. Based on the current flow, you are able to determine in which way the resistors are combined.

The resultant resistance is then connected with R5 in series. The equivalent resistance is 4+4= 8 Ω.

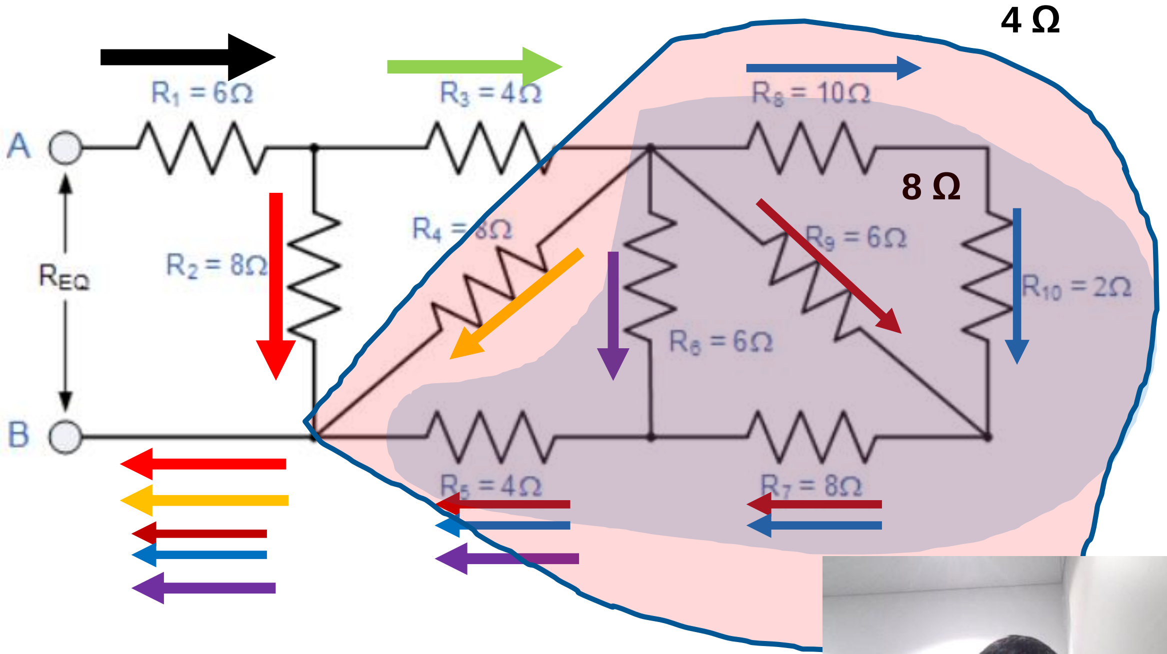


EQUIVALENT RESISTANCE

Equivalent resistance of series and parallel circuits

Tip: The key is to find out how the current flows in the circuit. Based on the current flow, you are able to determine in which way the resistors are combined.

The resultant resistance is then connected with R4 in parallel. The equivalent resistance is 8//8= 4 Ω.

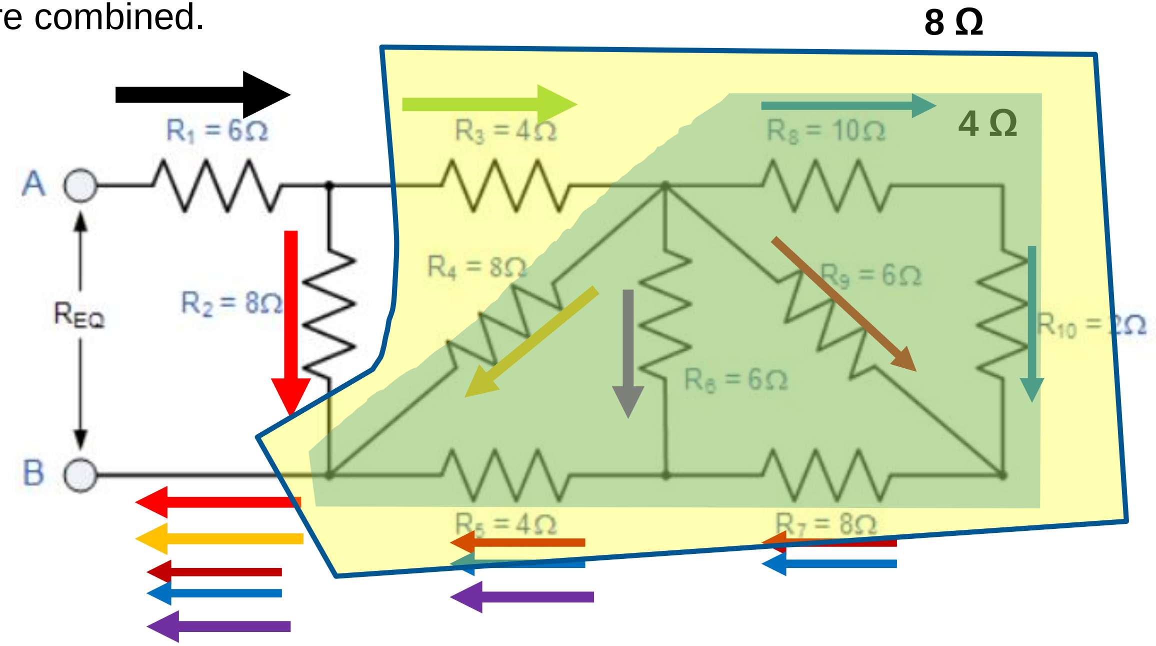


EQUIVALENT RESISTANCE

Equivalent resistance of series and parallel circuits

Tip: The key is to find out how the current flows in the circuit. Based on the current flow, you are able to determine in which way the resistors are combined.

The resultant resistance is then connected with R3 in series. The equivalent resistance is 4+4= 8 Ω.

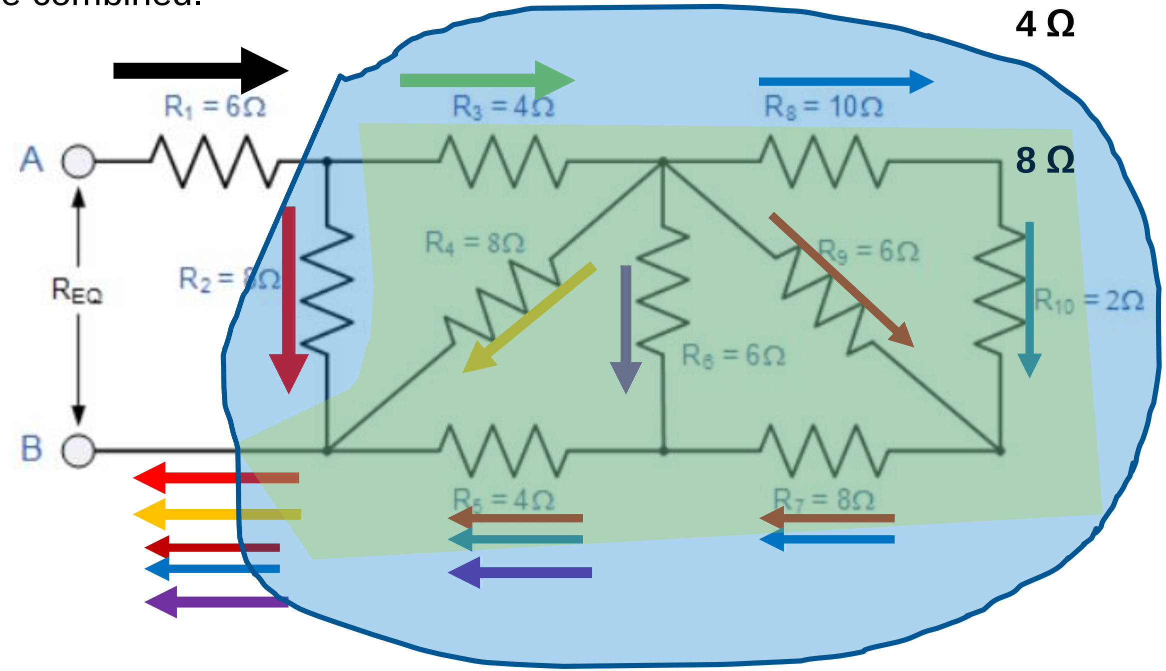


EQUIVALENT RESISTANCE

Equivalent resistance of series and parallel circuits

Tip: The key is to find out how the current flows in the circuit. Based on the current flow, you are able to determine in which way the resistors are combined.

The resultant resistance is then connected with R2 in parallel. The equivalent resistance is 8//8= 4 Ω.

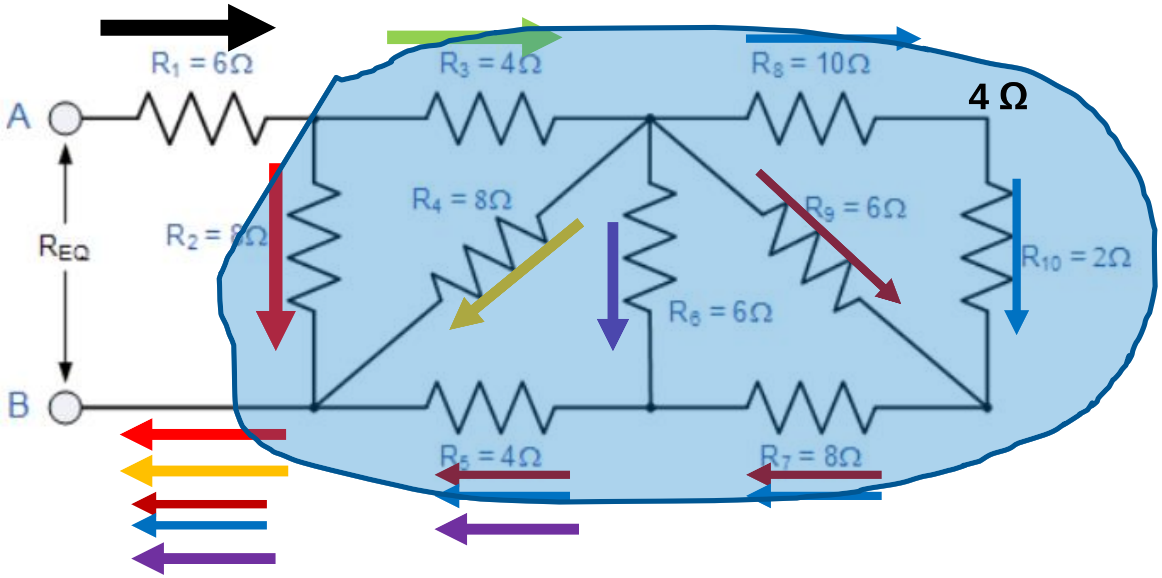


EQUIVALENT RESISTANCE

Equivalent resistance of series and parallel circuits

Tip: The key is to find out how the current flows in the circuit. Based on the current flow, you are able to determine in which way the resistors are combined.

At last, the resultant resistance is then connected with R1 in series. The equivalent resistance is 4+6= 10 Ω.



48510 LEC 2 – BASIC CIRCUIT LAWS

Topic 2-5: Current Division Principle

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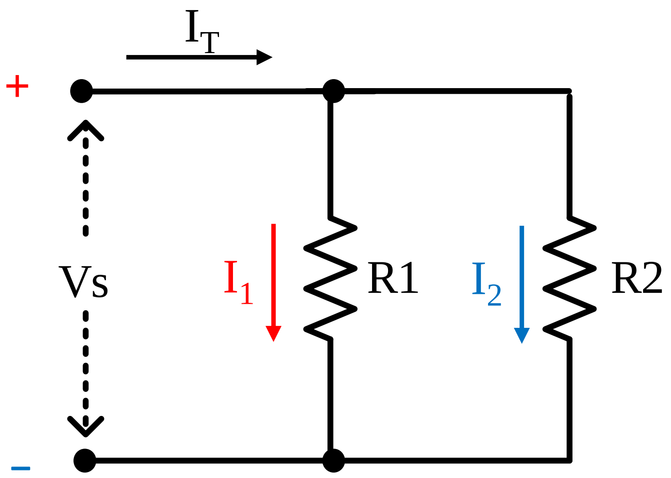
CURRENT DIVISION PRINCIPLE

In a parallel circuit having two or more branches, all the branches share the same voltage. Each branch allows currents to pass along but the currents can have different values through different branches.

Here is a basic current divider circuit consists of two resistors: R_1, and R_2 in parallel which splits the source current I_T between them into two separate currents I_1 and I_2 before joining together again and returning back to the source.

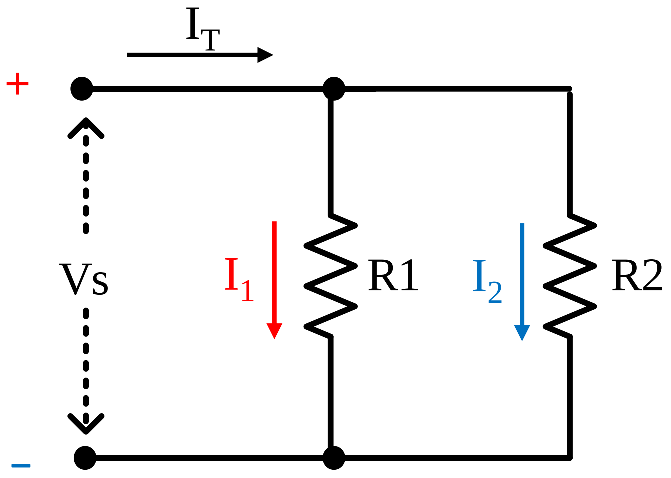


According to KCL, $I_T = I_1 + I_2$

As the same voltage is present across each resistor, according to Ohm's law, we have

$$I_1 = \frac{V_s}{R_1} \qquad I_2 = \frac{V_s}{R_2} \quad \Rightarrow \quad \frac{I_1}{I_2} = \frac{R_2}{R_1}$$

CURRENT DIVISION PRINCIPLE



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Combining these two equations

$$I_T = I_1 + I_2 = \frac{R_2}{R_1} I_2 + I_2 = I_2 \frac{R_1 + R_2}{R_1}$$

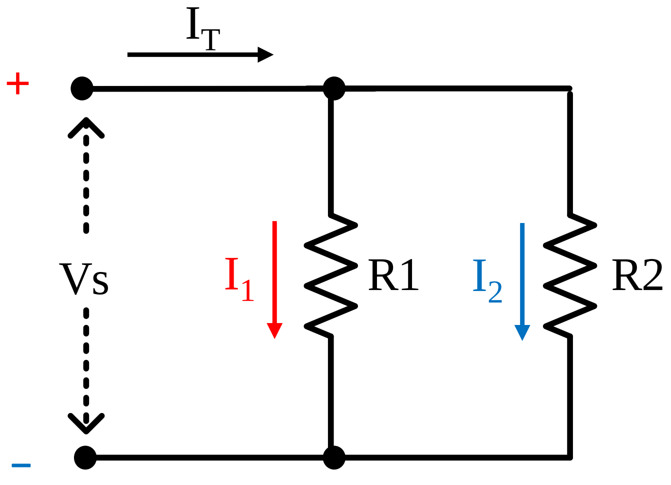
$$\boxed{I_2 = I_T \frac{R_1}{R_1 + R_2}}$$

$$I_T = I_1 + I_2 = I_1 + \frac{R_1}{R_2} I_1 = I_1 \frac{R_1 + R_2}{R_2}$$

$$\boxed{I_1 = I_T \frac{R_2}{R_1 + R_2}}$$

CURRENT DIVISION PRINCIPLE

Notice that the above equations for each branch current has the opposite resistor in its numerator. That is to solve for I1 we use R2, and to solve for I2 we use R1. This is because each branch current is inversely proportional to its resistance resulting in **the smaller resistance having the larger current**.



$$I_1 = I_T \frac{R_2}{R_1 + R_2}$$

$$I_2 = I_T \frac{R_1}{R_1 + R_2}$$

$$\frac{I_1}{I_2} = \frac{R_2}{R_1}$$

What if a short circuit is involved, for example, R1=0?

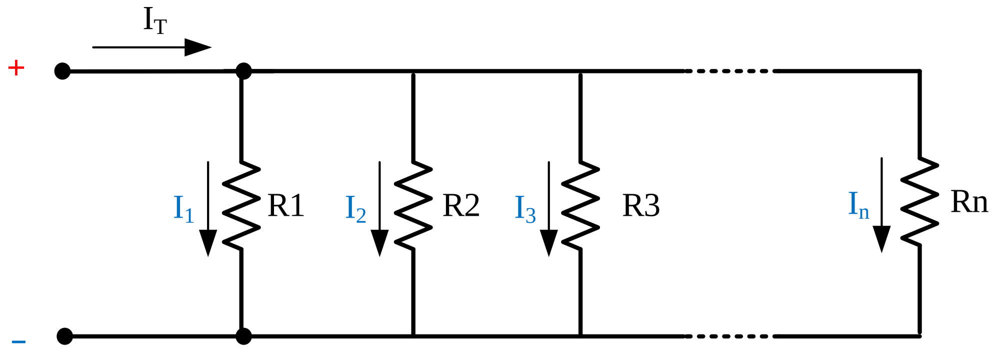
$$I_1 = I_T$$

$$I_2 = 0$$

All the current will pass through the short circuit path and there is no current passing through other paths.

CURRENT DIVISION PRINCIPLE

Question: If there are *n (n>2)* branches in a parallel circuit, given the total current, how to find the current divided on each branch?



Previously, we know the total equivalent resistance of several resistors connected in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}$$

Since we know the total current and the total resistance, we can find out the total voltage using Ohm's law as

$$V = I_T R_{eq}$$

Then the current passing through resistor Rm is

$$I_m = \frac{V}{R_m} = \frac{I_T R_{eq}}{R_m}$$

48510 LEC 2 – BASIC CIRCUIT LAWS

Topic 2-5-2: Voltage Division Principle

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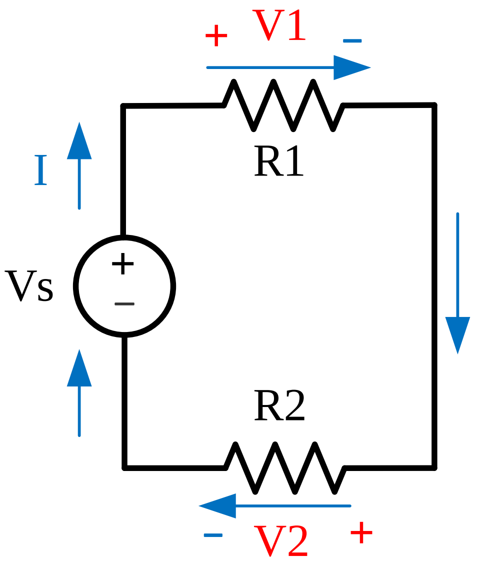
VOLTAGE DIVISION PRINCIPLE

When two or more passive elements are connected in series, the amount of voltage present across each element gets divided (shared) among themselves from the voltage that is available across that entire combination.

This circuit diagram consists of a voltage source, Vs in series with two resistors R1 and R2. The current flowing through these elements is I. The voltage drops across the resistors R1 and R2 are V1 and V2 respectively.



According to KVL, $V_s = V_1 + V_2$

According to Ohm's law $V_1 = IR_1 \qquad V_2 = IR_2$

Substituting V1 and V2 into Vs $V_s = IR_1 + IR_2 = I(R_1 + R_2)$

We can obtain the current as $$I = \frac{V_s}{R_1 + R_2}$$

VOLTAGE DIVISION PRINCIPLE

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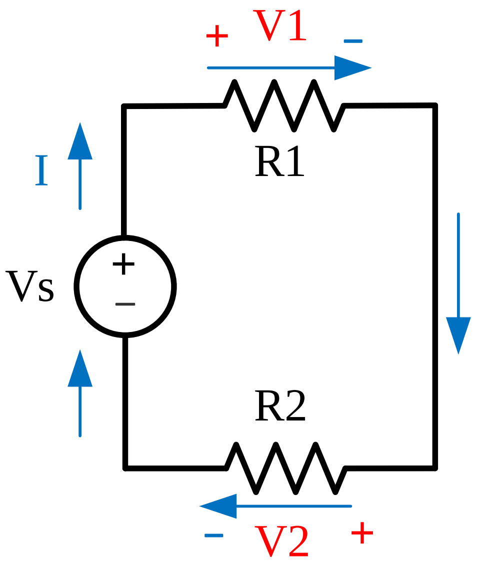
We can obtain the current as $I = \dfrac{V_s}{R_1 + R_2}$

Substituting I into V1 and V2

$$\boxed{V_1 = \frac{V_s R_1}{R_1 + R_2}} \qquad \boxed{V_2 = \frac{V_s R_2}{R_1 + R_2}}$$

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$



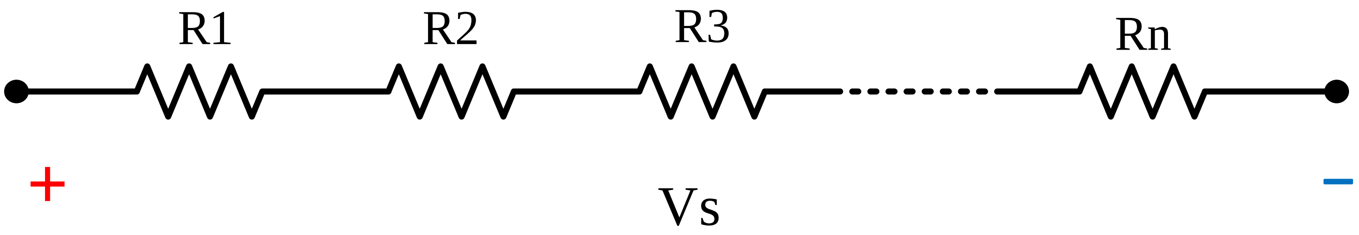
It is noted that, the larger the resistance, the higher the voltage divided on the resistor.

What if an open circuit is involved, for example, R1=∞?

All the voltage is divided on R1 and R2 got 0 voltage (current is 0).

VOLTAGE DIVISION PRINCIPLE

Question: If there are *n (n*>2*)* resistors connected in series, given the total voltage is Vs, how to find the voltage divided on each resistor?



$$V_m = \frac{V_s R_m}{R_1 + R_2 + .. R_n}$$