48510 LEC 3 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-1

Topic 3-1: Mesh Current Analysis

Topic 3-1-1: Planar circuit, loop, mesh and mesh current

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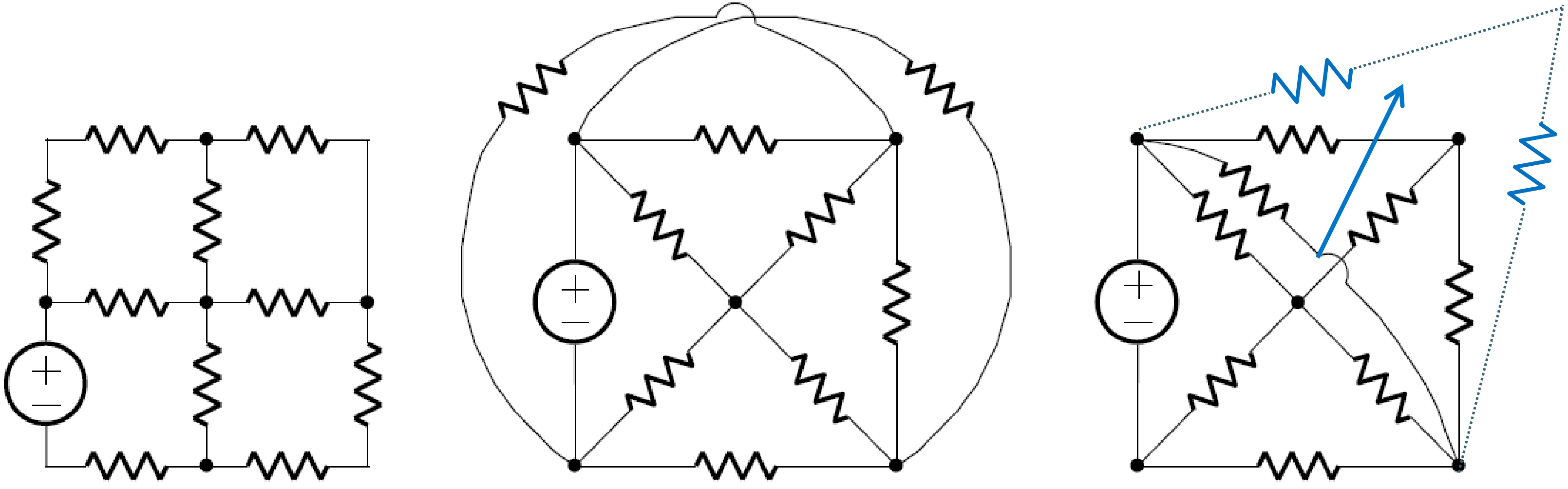
3-1-1 PLANAR CIRCUIT

A **planar circuit** is one where it is possible to draw the circuit on a plane surface in such a way that **no branch passes over or under any other branch**.

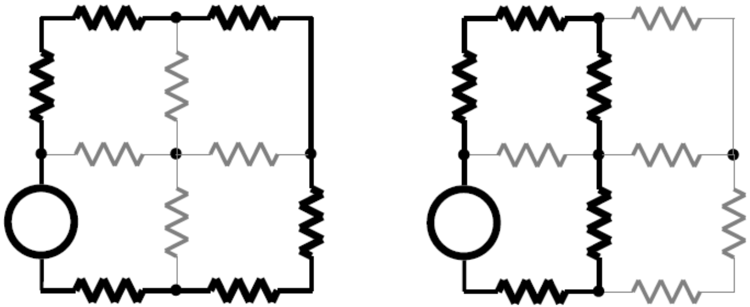


(a) (b) (c)

In the figure above, circuit (a) is planar, circuit (b) is nonplanar and circuit (c) is planar, but drawn so that it appears nonplanar.

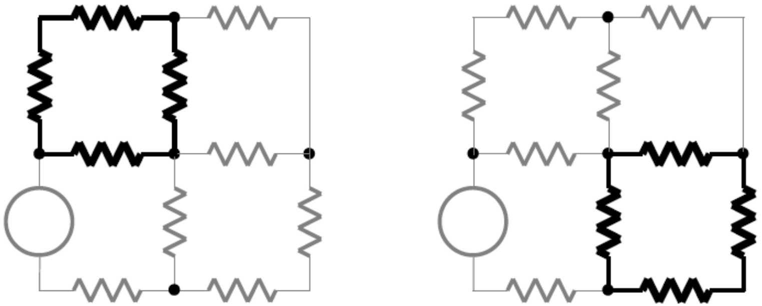
3-1-1 LOOPS AND MESHES

A ***loop*** is any closed path, i.e., the last node visited is the same as the starting node.



A ***mesh*** is a loop which does not contain any other loops within it. A mesh is a property of a planar circuit and is not defined for a nonplanar circuit.

A mesh must be a loop, but a loop is not necessarily a mesh.



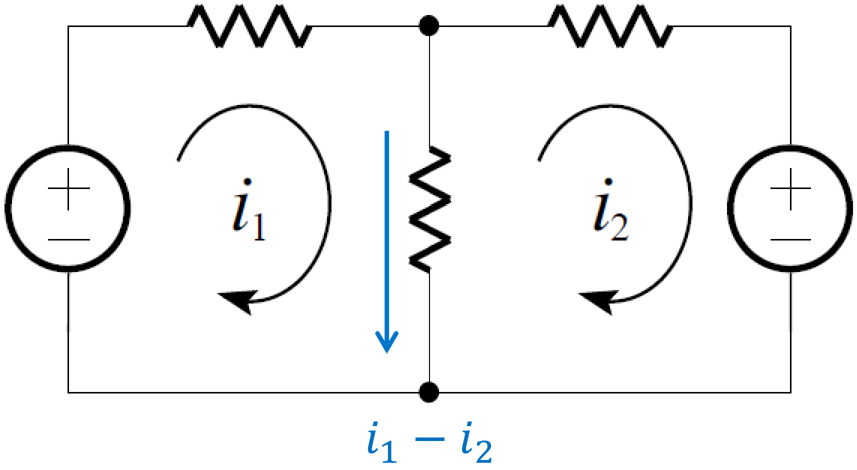
3-1-1 MESH CURRENTS

We define a **mesh current** as a “mathematical” (or imaginary) current in which charge flows only around the perimeter of a mesh. A mesh current is indicated by a curved arrow that almost closes on itself and is drawn inside the appropriate mesh.

Although the direction of mesh currents is arbitrary, we draw the mesh currents in a **clockwise direction** so that a symmetry in the equations results when performing mesh analysis.

One of the great advantages of mesh currents is that *KCL is automatically satisfied*, and no branch can appear in more than two meshes.

We no longer have a current or current arrow shown on each branch in the circuit. **The current through any branch may be determined by superimposing each mesh current that exists in it.**



48510 LEC 3 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-1

Topic 3-1: Mesh Current Analysis

Topic 3-1-2: Mesh Current Analysis Methodology

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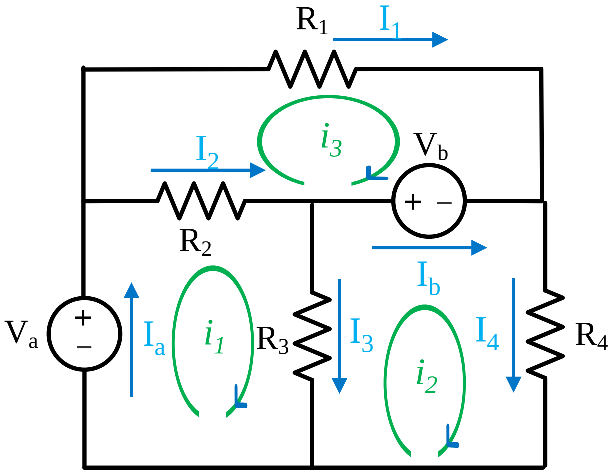
3-1-2 MESH CURRENT ANALYSIS METHODOLOGY

Mesh current analysis is a method that is used to solve planar circuits for the currents (and indirectly the voltages) at any place in the electrical circuit. It works by arbitrarily assigning mesh currents in the essential meshes. Note that the currents in essential meshes are independent to each other.

Solving for mesh currents instead of directly applying KCL and KVL can greatly reduce the amount of calculation required.



$$I_1 = i_3$$
$$I_2 = i_1 - i_3$$
$$I_3 = i_1 - i_2$$
$$I_4 = i_2$$
$$I_a = i_1$$
$$I_b = i_2 - i_3$$

3-1-2 MESH CURRENT ANALYSIS METHODOLOGY

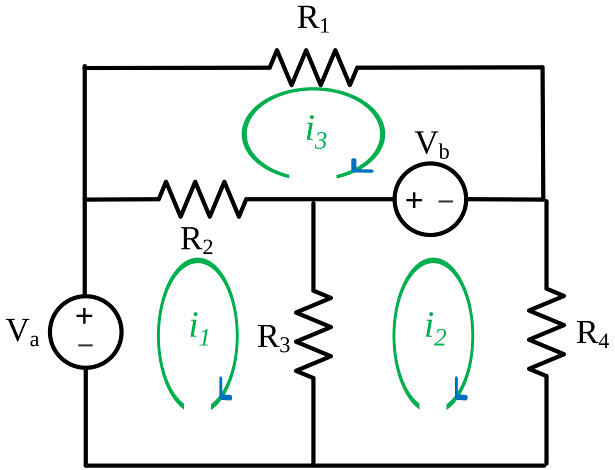
A recommended procedure to conduct mesh current analysis:

Step 1: Identify the mesh and define the mesh currents

Note: Clockwise direction is recommended

Step 2: Construct equations in each mesh, usually using KVL

Note1: This is because every mesh is a loop. KVL states that for a closed loop series path the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero.



3-1-2 MESH CURRENT ANALYSIS METHODOLOGY

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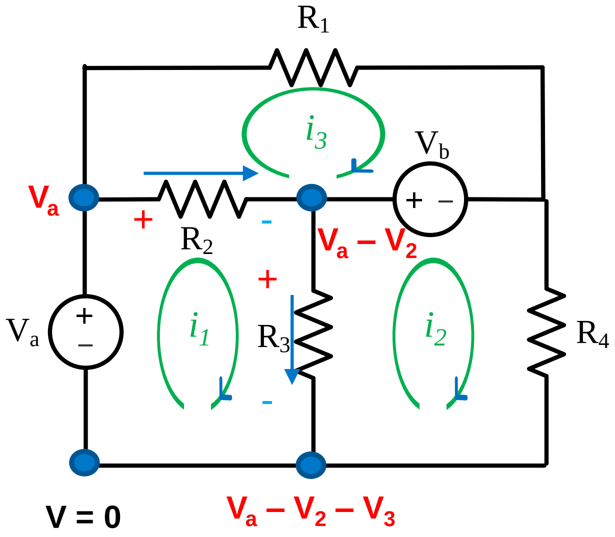

Mesh 1:

$$V_a - V_2 - V_3 = 0 \qquad \begin{aligned} V_2 &= R_2(i_1 - i_3) \\ V_3 &= R_3(i_1 - i_2) \end{aligned}$$

$$V_a - R_2(i_1 - i_3) - R_3(i_1 - i_2) = 0$$

$$(R_2 + R_3)i_1 - R_3 i_2 - R_2 i_3 = V_a$$

3-1-2 MESH CURRENT ANALYSIS METHODOLOGY

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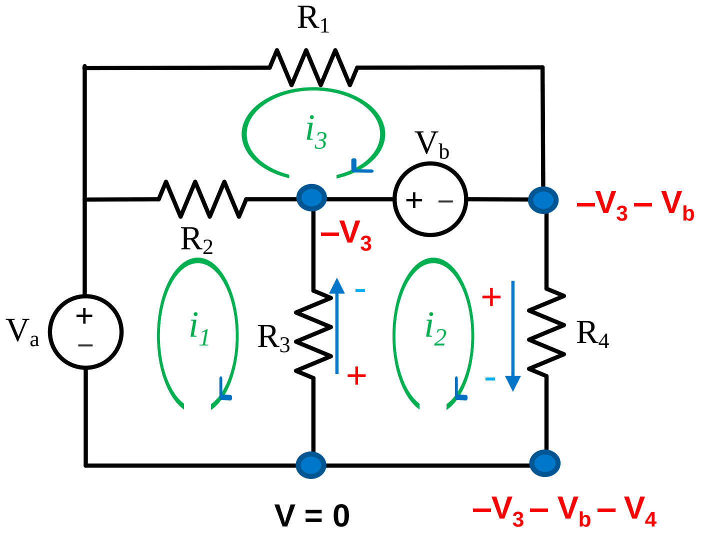

Mesh 1:

$$(R_2 + R_3)i_1 - R_3 i_2 - R_2 i_3 = V_a$$

Mesh 2:

$$V_3 + V_4 + V_b = 0$$

$$V_3 = R_3(i_2 - i_1)$$

$$V_4 = R_4 i_2$$

$$R_3(i_2 - i_1) + R_4 i_2 + V_b = 0$$

$$R_3 i_1 - (R_3 + R_4) i_2 = V_b$$

3-1-2 MESH CURRENT ANALYSIS METHODOLOGY

A recommended procedure to conduct mesh current analysis:

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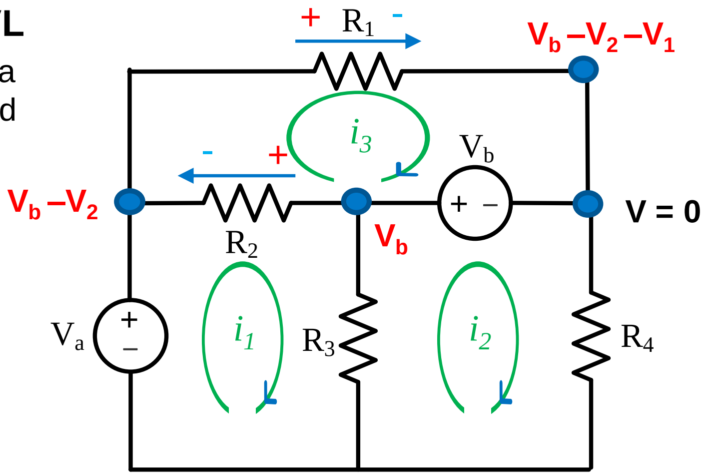

Mesh 1:

$$(R_2 + R_3)i_1 - R_3 i_2 - R_2 i_3 = V_a$$

Mesh 2:

$$R_3 i_1 - (R_3 + R_4)i_2 = V_b$$

Mesh 3:

$$V_b - V_2 - V_1 = 0 \qquad V_1 = R_1 i_3$$

$$V_2 = R_2(i_3 - i_1)$$

$$V_b + R_2(i_1 - i_3) - R_1 i_3 = 0$$

$$R_2 i_1 - (R_1 + R_2)i_3 = -V_b$$

3-1-2 MESH CURRENT ANALYSIS METHODOLOGY

A recommended procedure to conduct mesh current analysis:

Step 1: Identify the mesh and define the mesh currents

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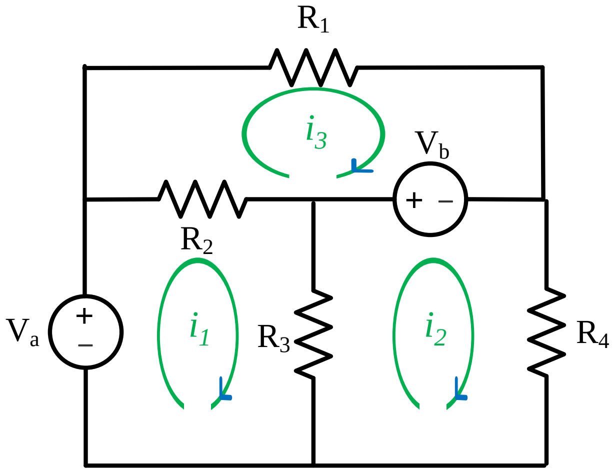

Mesh 1: $(R_2 + R_3)i_1 - R_3 i_2 - R_2 i_3 = V_a$

Mesh 2: $R_3 i_1 - (R_3 + R_4)i_2 = V_b$

Mesh 3: $R_2 i_1 - (R_1 + R_2)i_3 = -V_b$

3 meshes, 3 unknown currents, 3 equations

Step 3: Write the equations in standard form

$$r_{11}i_1 + r_{12}i_2 + r_{13}i_3 = v_1$$
$$r_{12}i_1 + r_{22}i_2 + r_{23}i_3 = v_2$$
$$r_{31}i_1 + r_{32}i_2 + r_{33}i_3 = v_3$$

Step 4: Solve the equation set or transform to matrix

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

48510 LEC 3 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-1

Topic 3-1: Mesh Current Analysis

Topic 3-1-3: Mesh Current Analysis – Special cases

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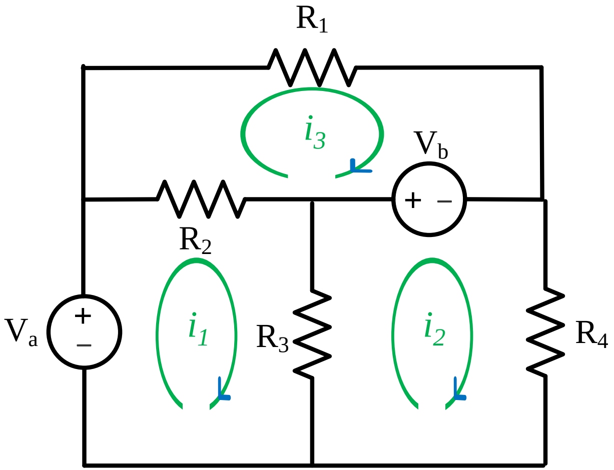
3-1-3 SPECIAL CASES

A recommended procedure to conduct mesh current analysis:

Step 1: Identify the mesh and define the mesh currents Note: Clockwise direction is recommended

Step 2: Construct equations in each mesh, usually using KVL

Note that *n* meshes correspond to *n* unknown mesh currents, thus we need *n* equations.



Step 3: Write the equations in standard form

$$r_{11}i_1 + r_{12}i_2 + r_{13}i_3 = v_1$$
$$r_{12}i_1 + r_{22}i_2 + r_{23}i_3 = v_2$$
$$r_{31}i_1 + r_{32}i_2 + r_{33}i_3 = v_3$$

Step 4: Solve the equation set or transform to matrix

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

3-1-3 SPECIAL CASES

A recommended procedure to conduct mesh current analysis:

Step 1: Identify the mesh and define the mesh currents Note: Clockwise direction is recommended

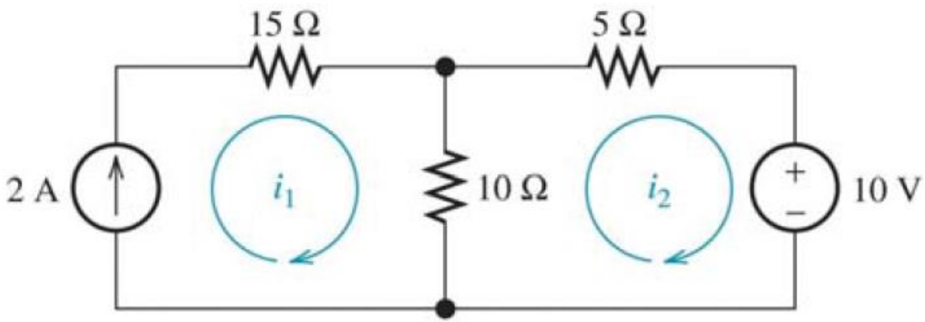
Step 2: Construct equations in each mesh, usually using KVL

Note that n meshes correspond to *n* unknown mesh currents, thus we need *n* equations.

Usually we can get the required equations by conducting KVL analysis, however, there are some special cases that need different considerations.

Special Cases – When **current sources** are involved, we are not able to conduct KVL because the voltage across a current source is unknown.

***Special case 1**: Current source in one mesh only*



Equation 1: conduct KVL on mesh 2

Equation 2: by comparing the current source with the mesh current, we can obtain $i_1 = 2\ A$

3-1-3 SPECIAL CASES

A recommended procedure to conduct mesh current analysis:

Step 1: Identify the mesh and define the mesh currents Note: Clockwise direction is recommended

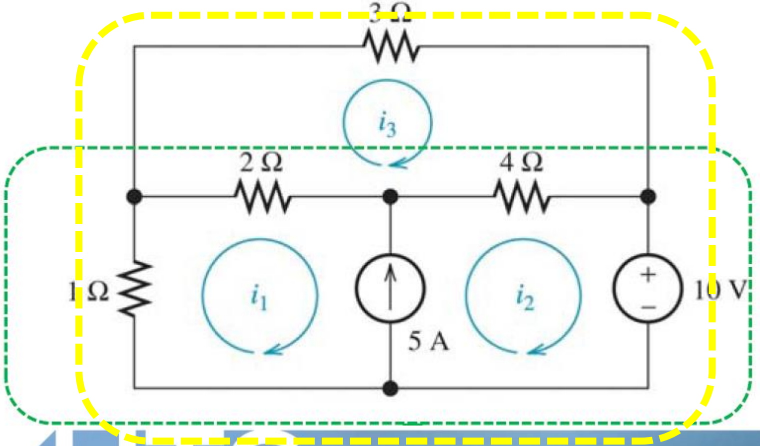
Step 2: Construct equations in each mesh, usually using KVL

Note that n meshes correspond to *n* unknown mesh currents, thus we need *n* equations.

Usually we can get the required equation by conducting KVL analysis, however, there are some special cases that need different considerations.

Special Cases – When **current sources** are involved, we are not able to conduct KVL because the voltage across a current source is unknown.

***Special case 2**:* Current source belongs to two meshes



Equation 1: conduct KVL on mesh 3

Equation 2: by comparing the current source with the mesh current, we can obtain $i_2 - i_1 = 5$ A

Equation 3: conduct KVL on a larger loop containing the current source, i.e., the green or yellow dashed loops.

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Topic 3-1: Mesh Current Analysis

Topic 3-1-4: Mesh Current Analysis – Example

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3-1-4 EXAMPLE

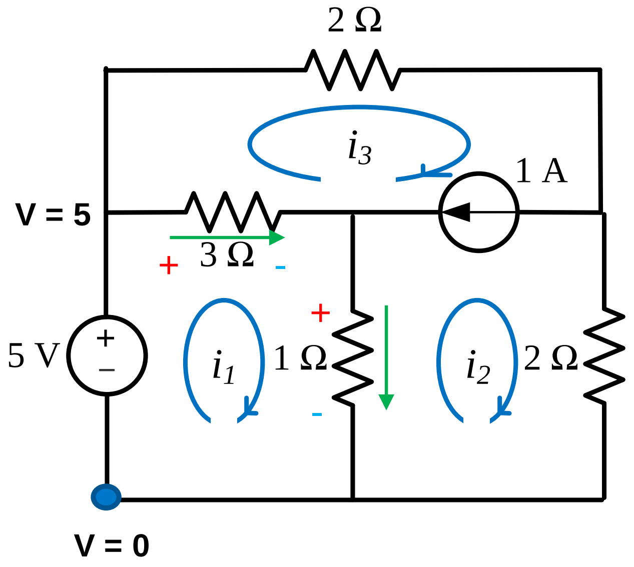
Let's try to solve an example problem following the recommended procedure

Step 1: Identify the mesh and define the mesh currents

Step 2: Construct equations in each mesh, usually using KVL

Note that *n* meshes correspond to *n* unknown mesh currents, thus we need *n* equations.

If current sources present, find relationship between mesh currents according to the current source and conduct KVL around larger loop.



Mesh 1:

$$5 - 3 \times (i_1 - i_3) - 1 \times (i_1 - i_2) = 0$$

$$5 - 3i_1 + 3i_3 - i_1 + i_2 = 0$$

$$-4i_1 + i_2 + 3i_3 = -5$$

$$4i_1 - i_2 - 3i_3 = 5 \qquad \text{Eq (1)}$$

3-1-4 EXAMPLE

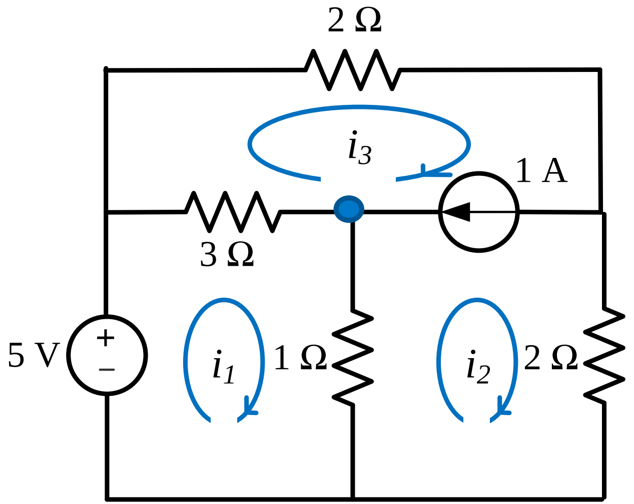
Let's try to solve an example problem following the recommended procedure

Step 1: Identify the mesh and define the mesh currents Note: Clockwise direction is recommended

Step 2: Construct equations in each mesh, usually using KVL

Note that *n* meshes correspond to *n* unknown mesh currents, thus we need *n* equations.

If current sources present, find relationship between mesh currents according to the current source and conduct KVL around larger loop.



Mesh 1: $4i_1 - i_2 - 3i_3 = 5$ Eq (1)

Mesh 2: not able to conduct KVL

Mesh 3: not able to conduct KVL

Comparing current source with mesh currents

$$i_3 - i_2 = 1$$

$$i_2 - i_3 = -1 \quad \text{Eq (2)}$$

3-1-4 EXAMPLE

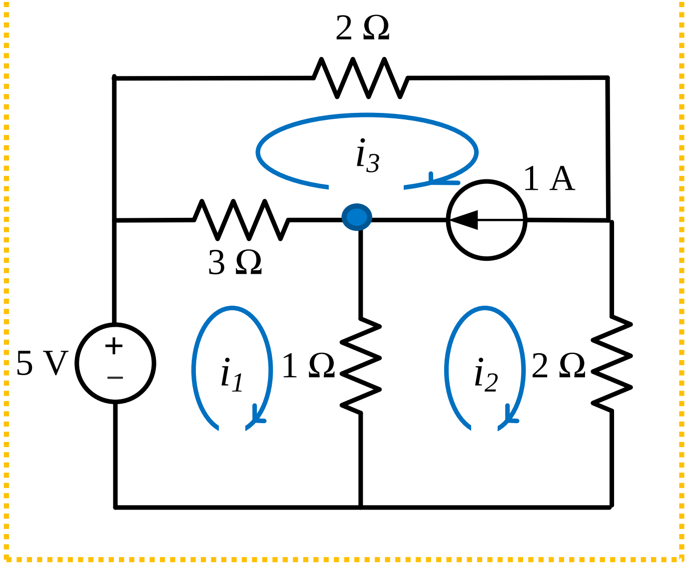
Let's try to solve an example problem following the recommended procedure

Step 1: Identify the mesh and define the mesh currents Note: Clockwise direction is recommended

Step 2: Construct equations in each mesh, usually using KVL

Note that *n* meshes correspond to *n* unknown mesh currents, thus we need *n* equations.

If current sources present, find relationship between mesh currents according to the current source and conduct KVL around larger loop.



Mesh 1: $4i_1 - i_2 - 3i_3 = 5$ Eq (1)

Comparing current source with mesh currents

$i_2 - i_3 = -1$ Eq (2)

Conducting KVL around a larger loop enclosing the current source

$$5 - 2i_3 - 2i_2 = 0$$

$$2i_2 + 2i_3 = 5$$ Eq (3)

3-1-4 EXAMPLE

Let's try to solve an example problem following the recommended procedure

Step 3: Write the equations in standard form

$$4i_1 - i_2 - 3i_3 = 5 \qquad \text{Eq (1)}$$

$$i_2 - i_3 = -1 \qquad \text{Eq (2)}$$

$$2i_2 + 2i_3 = 5 \qquad \text{Eq (3)}$$

Step 4: Solve the equation set

Method 1: substitution method

According to Eq (2), $i_2 = i_3 - 1$ Eq (4)

Substitute Eq(4) into (1) and (3), we have

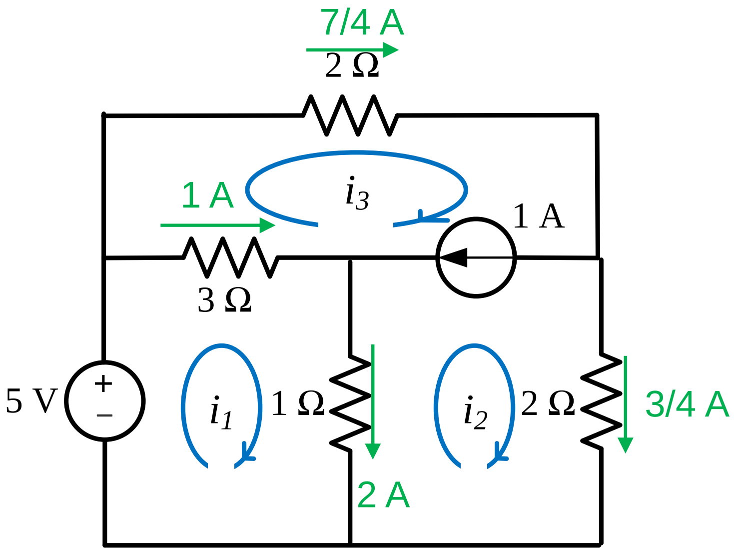
$$4i_1 - 4i_3 = 4 \qquad \text{Eq (5)}$$

$$4i_3 = 7 \qquad i_3 = 7/4 \qquad \text{Eq (6)}$$

Substitute Eq (6) into Eq (5), we have

$$4i_1 = 4i_3 + 4 = 11 \qquad i_1 = 11/4$$

Substitute i3 into Eq (4), we have $i_2 = 3/4$



3-1-4 EXAMPLE

Let's try to solve an example problem following the recommended procedure

Step 3: Write the equations in standard form

$4i_1 - i_2 - 3i_3 = 5$ Eq (1)

$i_2 - i_3 = -1$ Eq (2)

$2i_2 + 2i_3 = 5$ Eq (3)

Step 4: Solve the equation set

Method 2: matrix

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 & -3 \\ 0 & 1 & -1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \mathbf{5} \\ \mathbf{-1} \\ \mathbf{5} \end{bmatrix}$$

$$i_1 = \frac{\begin{vmatrix} \mathbf{5} & -1 & -3 \\ \mathbf{-1} & 1 & -1 \\ \mathbf{5} & 2 & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -1 & -3 \\ 0 & 1 & -1 \\ 0 & 2 & 2 \end{vmatrix}} \qquad i_2 = \frac{\begin{vmatrix} 4 & \mathbf{5} & -3 \\ 0 & \mathbf{-1} & -1 \\ 0 & \mathbf{5} & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -1 & -3 \\ 0 & 1 & -1 \\ 0 & 2 & 2 \end{vmatrix}}$$

$$i_3 = \frac{\begin{vmatrix} 4 & -1 & \mathbf{5} \\ 0 & 1 & \mathbf{-1} \\ 0 & 2 & \mathbf{5} \end{vmatrix}}{\begin{vmatrix} 4 & -1 & -3 \\ 0 & 1 & -1 \\ 0 & 2 & 2 \end{vmatrix}}$$

Find the determinate of matrix

3-1-4 EXAMPLE

Let's try to solve an example problem following the recommended procedure

Step 3: Write the equations in standard form

Step 4: Solve the matrix equation

$$\begin{bmatrix} 4 & -1 & -3 \\ 0 & 1 & -1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \mathbf{5} \\ \mathbf{-1} \\ \mathbf{5} \end{bmatrix}$$

$$i_1 = \frac{\begin{vmatrix} \mathbf{5} & -1 & -3 \\ \mathbf{-1} & 1 & -1 \\ \mathbf{5} & 2 & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -1 & -3 \\ 0 & 1 & -1 \\ 0 & 2 & 2 \end{vmatrix}}$$

Find the determinate of matrix

$$\begin{vmatrix} 4 & -1 & -3 \\ 0 & 1 & -1 \\ 0 & 2 & 2 \end{vmatrix} = 4\begin{vmatrix} 1 & -1 \\ 2 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ 0 & 2 \end{vmatrix} + (-3)\begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 4[1 \times 2 - (-1) \times 2] - (-1)\,[0 \times 2 - 0 \times (-1)] + (-3)\,(0 \times 2 - 0 \times 1)$$

$$= 16$$

48510 LEC 3 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-1

Topic 3-2: Node Voltage Analysis

Topic 3-2-1: Node Voltage Analysis Methodology

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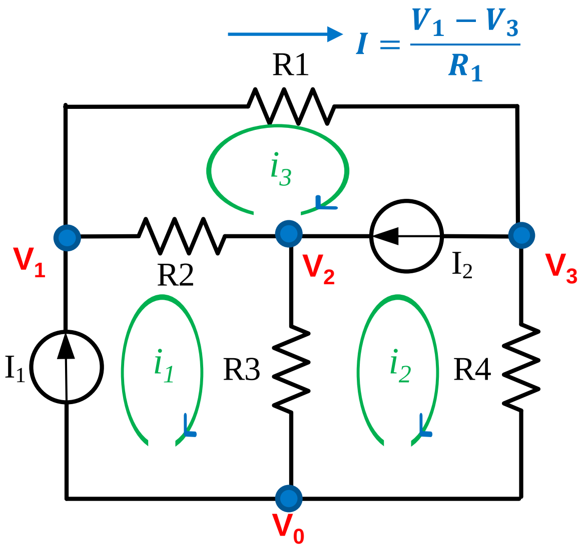
3-2-1 NODE VOLTAGE ANALYSIS METHODOLOGY

Like **mesh current analysis** is based on **mesh currents + KLV** around meshes, **node voltage analysis** utilizes **node voltages + KCL** at nodes.

Node voltage analysis complements the previous mesh analysis in that it is equally powerful and based on the same concepts of matrix analysis.

By using mesh current analysis, you can find the currents flowing around each mesh. By superimposing the mesh currents on each branch, the currents at all the paths can be determined. Then you are able to obtained the voltage using Ohm's law.

By using node voltage analysis, you can find the voltage at each node. Then you are able to use Ohm's law and KCL to find all the currents.



3-2-1 NODE VOLTAGE ANALYSIS METHODOLOGY

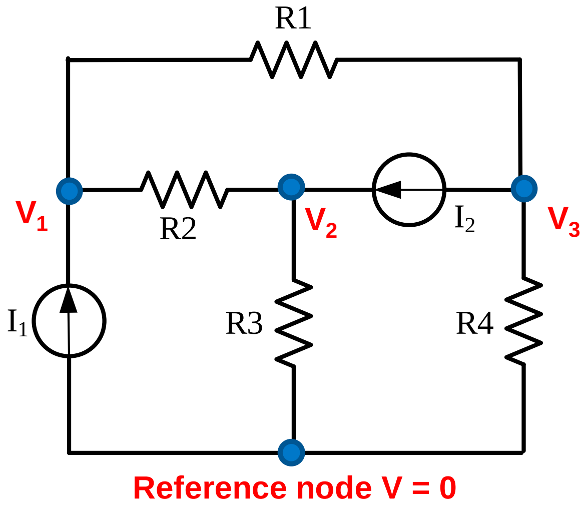
A recommended procedure to conduct node voltage analysis:

Step 1: Identify the nodes and define a reference node

Note1: Electrically common nodes can be seen as one node.

Note2: You can assume any node to be the reference node and the voltage at the reference node is 0 V.

Note3: If you have "*n*" nodes in a circuit, there are only "*n-1*" unknown node voltages.



3-2-1 NODE VOLTAGE ANALYSIS METHODOLOGY

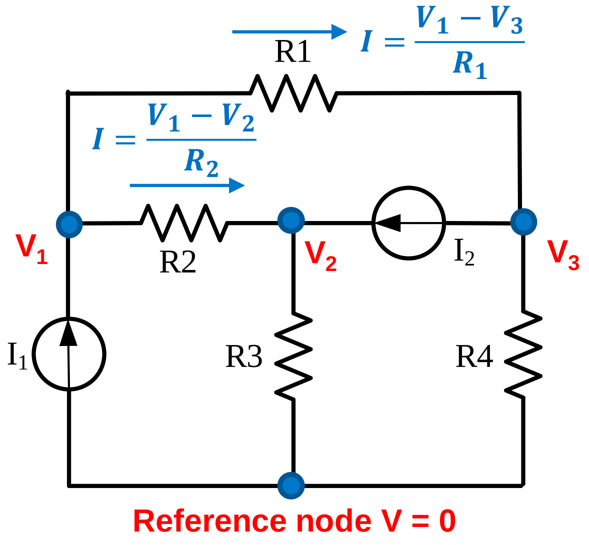
A recommended procedure to conduct node voltage analysis:

Step 1: Identify the nodes and define a reference node

Step 2: Construct equations at each node, usually using KCL

Note1: Current_in = Current_out

Note2: Always assume currents are leaving the node when you see resistors



Node 1: $I_1 = I_{R1} + I_{R2}$ $\quad I_{R1} = \frac{V_1 - V_3}{R_1}$ $\quad I_{R2} = \frac{V_1 - V_2}{R_2}$

$$I_1 = \frac{V_1 - V_3}{R_1} + \frac{V_1 - V_2}{R_2}$$

$$I_1 R_1 R_2 = R_2(V_1 - V_3) + R_1(V_1 - V_2)$$

$$(R_1 + R_2)V_1 - R_1 V_2 - R_2 V_3 = R_1 R_2 I_1 \quad \textbf{Eq (1)}$$

3-2-1 NODE VOLTAGE ANALYSIS METHODOLOGY

A recommended procedure to conduct node voltage analysis:

Step 1: Identify the nodes and define a reference node

Step 2: Construct equations at each node, usually using KCL

Note1: Current_in = Current_out

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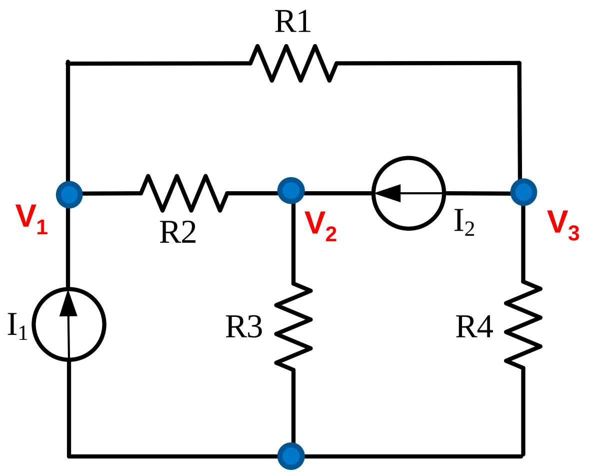


Reference node V = 0

Node 1: $(R_1 + R_2)V_1 - R_1V_2 - R_2V_3 = R_1R_2I_1$ **Eq (1)**

Node 2:

$$I_2 = I_{R2} + I_{R3} \qquad I_{R2} = \frac{V_2 - V_1}{R_2} \qquad I_{R3} = \frac{V_2 - 0}{R_3}$$

$$I_2R_2R_3 = R_3(V_2 - V_1) + R_2V_2$$

$$-R_3V_1 + (R_2 + R_3)V_2 = R_2R_3I_2$$

$$R_3V_1 - (R_2 + R_3)V_2 = -R_2R_3I_2 \qquad \textbf{Eq (2)}$$

3-2-1 NODE VOLTAGE ANALYSIS METHODOLOGY

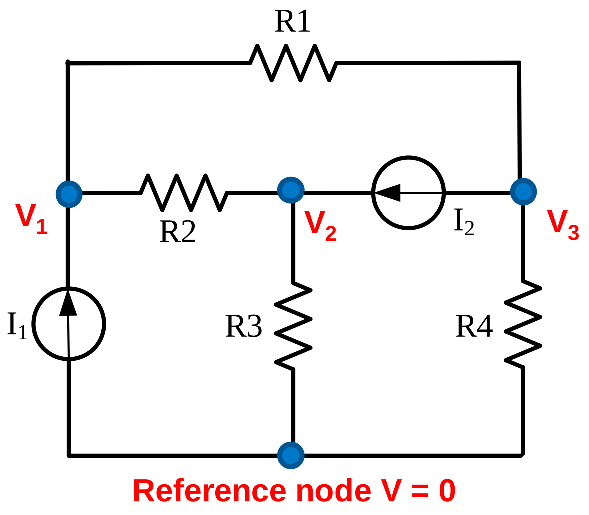
A recommended procedure to conduct node voltage analysis:

Step 1: Identify the nodes and define a reference node

Step 2: Construct equations at each node, usually using KCL

Note1: Current_in = Current_out

Note2: Always assume currents are leaving the node when you see resistors



Node 1: $(R_1 + R_2)V_1 - R_1V_2 - R_2V_3 = R_1R_2I_1$ **Eq (1)**

Node 2: $R_3V_1 - (R_2 + R_3)V_2 = -R_2R_3I_2$ **Eq (2)**

Node 3:

$$0 = I_2 + I_{R1} + I_{R4} \qquad I_{R1} = \frac{V_3 - V_1}{R_1} \qquad I_{R4} = \frac{V_3 - 0}{R_4}$$

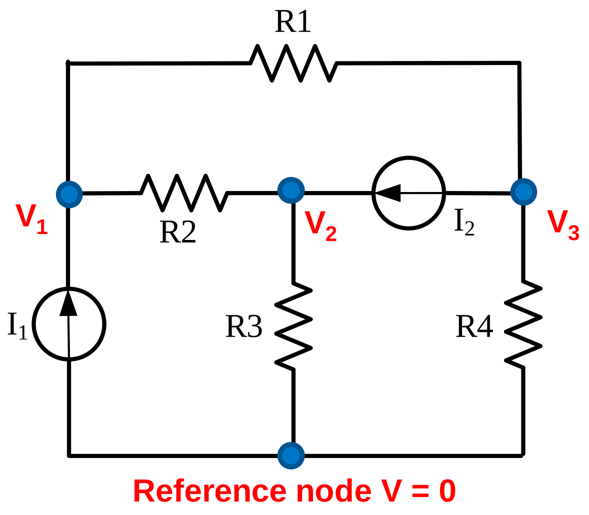
$$I_2R_1R_4 + R_4(V_3 - V_1) + R_1V_3 = 0$$

$$-R_4V_1 + (R_1 + R_4)\,V_3 = -R_1R_4I_2$$

$$R_4V_1 - (R_1 + R_4)\,V_3 = R_1R_4I_2$$ **Eq (3)**

3-2-1 NODE VOLTAGE ANALYSIS METHODOLOGY

A recommended procedure to conduct node voltage analysis:



Step 1: Identify the nodes and define a reference node

Step 2: Construct equations at each node, usually using KCL

Note1: Current_in = Current_out

Note2: Always assume currents are leaving the node when you see resistors

Node 1: $(R_1 + R_2)V_1 - R_1 V_2 - R_2 V_3 = R_1 R_2 I_1$ **Eq (1)**

Node 2: $R_3 V_1 - (R_2 + R_3)V_2 = -R_2 R_3 I_2$ **Eq (2)**

Node 3: $R_4 V_1 - (R_1 + R_4)\, V_3 = R_1 R_4 I_2$ **Eq (3)**

Step 3: Write the equations in standard form

$$g_{11} V_1 + g_{12} V_2 + g_{13} V_3 = i_1$$
$$g_{21} V_1 + g_{22} V_2 + g_{23} V_3 = i_2$$
$$g_{31} V_1 + g_{32} V_2 + g_{33} V_3 = i_3$$

Step 4: Solve the equations using substitution method or via matrix

48510 LEC 3 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-1

Topic 3-2: Node Voltage Analysis

Topic 3-2-2: Node Voltage Analysis Methodology – Special cases

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3-2-1 NODE VOLTAGE ANALYSIS SPECIAL CASE

A recommended procedure to conduct node voltage analysis:

Step 1: Identify the nodes and define a reference node

Note1: Electrically common nodes can be seen as one node.

Note2: You can assume any node to be the reference node and the voltage at the reference node is 0 V.

Note3: If you have "*n*" nodes in a circuit, there are only "*n-1*" unknown node voltages.

Step 2: Construct equations at each node, usually using KCL

Note1: Current_in = Current_out

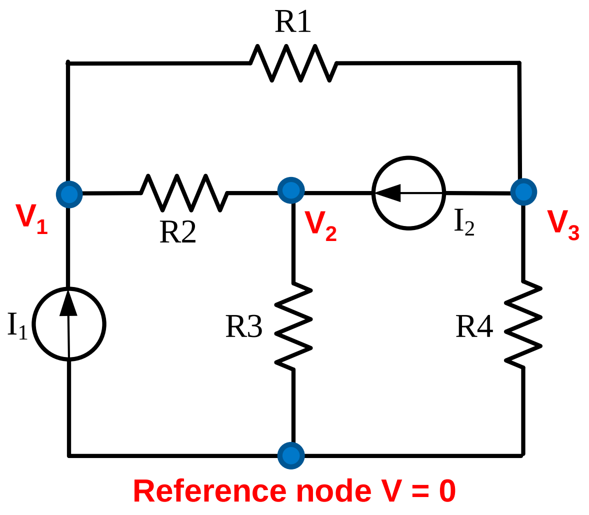
Note2: Always assume currents are leaving the node when you see resistors

Usually we can get the required equations by conducting KCL analysis, however, there are some special cases that need different considerations.

Step 3: Write the equations in standard form

$$g_{11}V_1 + g_{12}V_2 + g_{13}V_3 = i_1$$
$$g_{21}V_1 + g_{22}V_2 + g_{23}V_3 = i_2$$
$$g_{31}V_1 + g_{32}V_2 + g_{33}V_3 = i_3$$

Step 4: Solve the equations using substitution method or via matrix



3-2-1 NODE VOLTAGE ANALYSIS SPECIAL CASE

A recommended procedure to conduct node voltage analysis:

Step 1: Identify the nodes and define a reference node

Step 2: Construct equations at each node, usually using KCL

Note1: Current_in = Current_out

Note2: Always assume currents are leaving the node when you see resistors

Special Cases – When **voltage sources** are involved, we are not able to conduct KCL because the voltage across a current source is unknown.

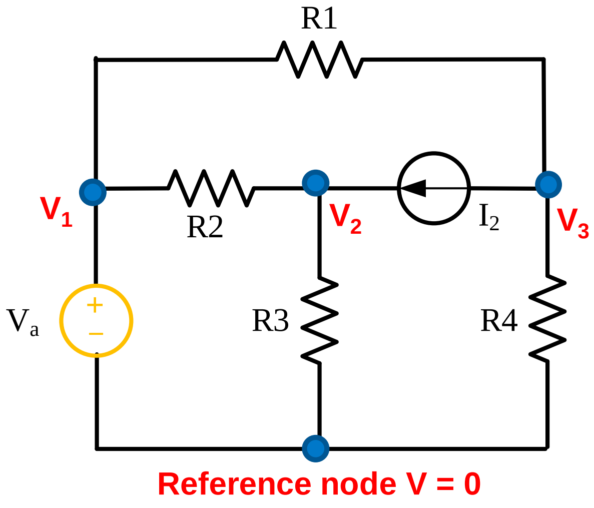
Special case 1*: Voltage source connecting to one unknown node only*

Note that if a voltage source sits between one unknown node and the reference node, we believe the source is connected to one node only.

In this case, you are not able to get an equation from this node. Note the reference node is a known node.

But the equation can be obtained by observing the voltage difference between the voltage source.

$$Equation\ 1\colon V_a = V_1 - 0$$



3-2-1 NODE VOLTAGE ANALYSIS SPECIAL CASE

A recommended procedure to conduct node voltage analysis:

Step 1: Identify the nodes and define a reference node

Step 2: Construct equations at each node, usually using KCL

Note1: Current_in = Current_out

Note2: Always assume currents are leaving the node when you see resistors

Special Cases – When **voltage sources** are involved, we are not able to conduct KCL because the voltage across a current source is unknown.

Special case 2: *Voltage source sits between two unknown node*

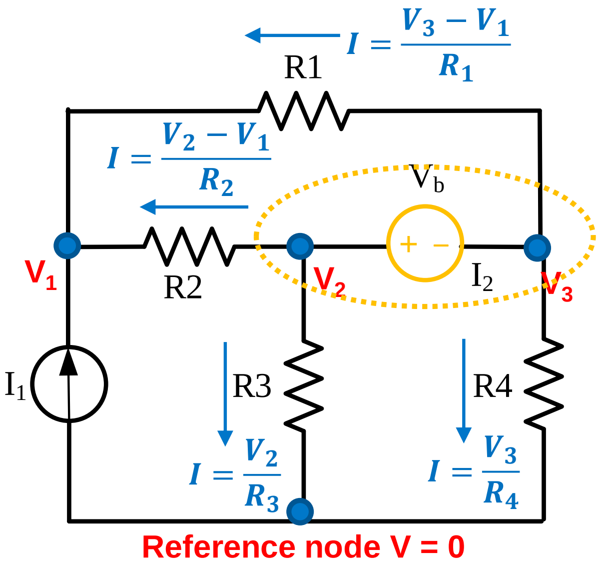
In this case, you are not able to get two equations from the nodes connected to the voltage source.

Eq. 1: *by observing the voltage difference between the voltage source.*

$$Equation\ 1{:}\ V_b = V_2 - V_3$$

Eq. 2: Taking the two nodes as a super node and conduct KCL

$$Equation\ 2{:}\ 0 = I_{R1} + I_{R2} + I_{R3} + I_{R4}$$



48510 LEC 3 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-1

Topic 3-2: Node Voltage Analysis

Topic 3-2-3: Node Voltage Analysis -- Example

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3-2-3 NODE VOLTAGE ANALYSIS EXAMPLE

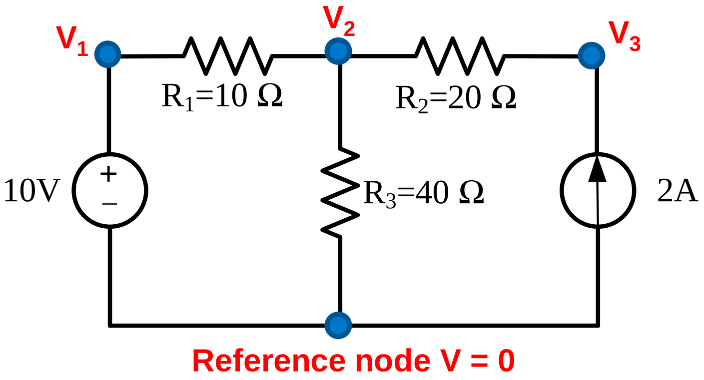
Let's try to solve an example problem following the recommended procedure

Step 1: Identify the nodes and define a reference node

Note1: Electrically common nodes can be seen as one node.

Note2: You can assume any node to be the reference node and the voltage at the reference node is 0 V.

Note3: If you have "*n*" nodes in a circuit, there are only "*n-1*" unknown node voltages.



3-2-3 NODE VOLTAGE ANALYSIS EXAMPLE

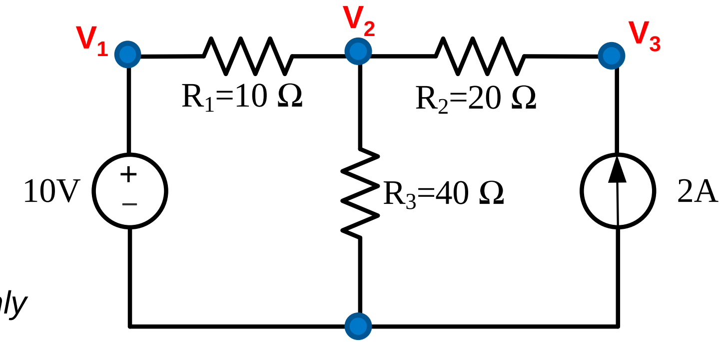
Let's try to solve an example problem following the recommended procedure

Step 1: Identify the nodes and define a reference node

Step 2: Construct equations at each node, usually using KCL

Note1: Current_in = Current_out

Note2: Always assume currents are leaving the node when you see resistors



Reference node V = 0

Node 1:

***Special case 1**: Voltage source connecting to one unknown node only*

The equation can be obtained by observing the voltage difference between the voltage source.

$V_1 = 10$ **Eq (1)**

Node 2:

$0 = I_{R1} + I_{R2} + I_{R3}$ $\quad I_{R1} = \frac{V_2 - 10}{10}$ $\quad I_{R2} = \frac{V_2 - V_3}{20}$ $\quad I_{R3} = \frac{V_2 - 0}{40}$

$4(V_2 - 10) + 2(V_2 - V_3) + V_2 = 0$

$7V_2 - 2V_3 = 40$ **Eq (2)**

Node 3:

$2 = \frac{V_3 - V_2}{20}$ $\quad V_3 - V_2 = 40$ **Eq (3)**

$5V_2 - 80 = 40$

$V_2 = 24$

$V_3 = 64$

3-2-3 NODE VOLTAGE ANALYSIS EXAMPLE

Let’s try to solve an example problem following the recommended procedure

Step 1: Identify the nodes and define a reference node

Step 2: Construct equations at each node, usually using KCL

Step 3: Write the equations in standard form

Step 4: Solve the equation set

$$V_1 = 10$$
$$V_2 = 24$$
$$V_3 = 64$$

Now lets validate our results by finding the current on each branch and check whether it agrees with KCL