48510 LEC 4 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-2

Topic 4-2-1: Superposition principle

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Lecturer

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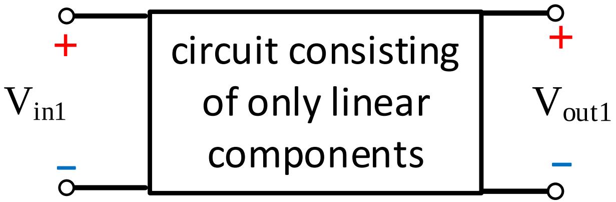
4-2-1 SUPERPOSITION PRINCIPLE

Considering a circuit consisting of only linear components and no independent sources are involved, we pick two nodes in arbitrary and put in between a voltage source providing a voltage input of $V_{in1.}$ We measure the voltage across another two arbitrary nodes and define the voltage as V_{out1}.

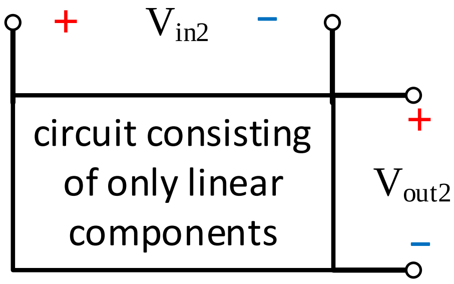


Linear circuits possess the property that "outputs are proportional to inputs".

If V_{in1}=0, V_{out1}=0

If V_{in1} is doubled, V_{out1} is doubled.

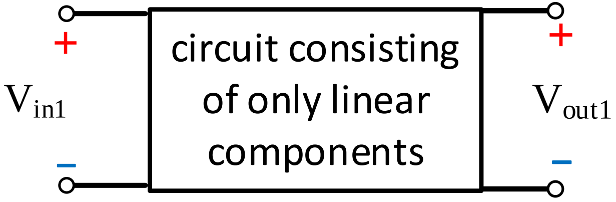
Then we disconnect the voltage source V_{in1}. We feed the circuit at another two arbitrary nodes with a voltage source $V_{in2.}$ In this case, the output voltage we measured is $V_{out2.}$ The output voltage is also in proportional to the input.

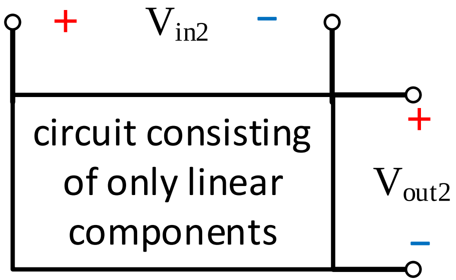


If V_{in2}=0, V_{out2}=0

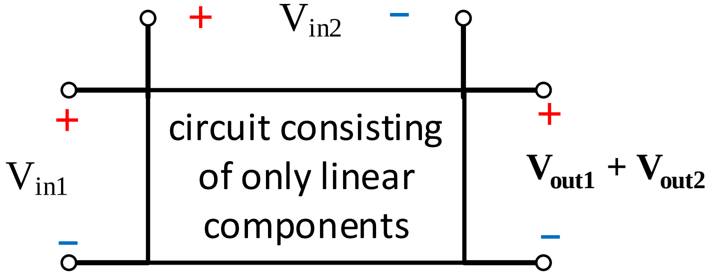
If V_{in2} is doubled, V_{out2} is doubled.

4-2-1 SUPERPOSITION PRINCIPLE





Now if we feed the circuit with two voltage sources at the same time, what is the output voltage?



The output voltage is $V_{out1} + V_{out2}$.

This example is a manifestation of the superposition principle.

4-2-1 SUPERPOSITION PRINCIPLE

Definition: The **superposition principle** states that a response in a linear circuit is equal to the sum of the responses for each independent source acting alone with the other independence source zeroed.

When zeroed, current source became open circuits and voltage source became short circuit.

Case 1: When both the two sources present

Node voltage analysis

Conduct KCL at node 2:

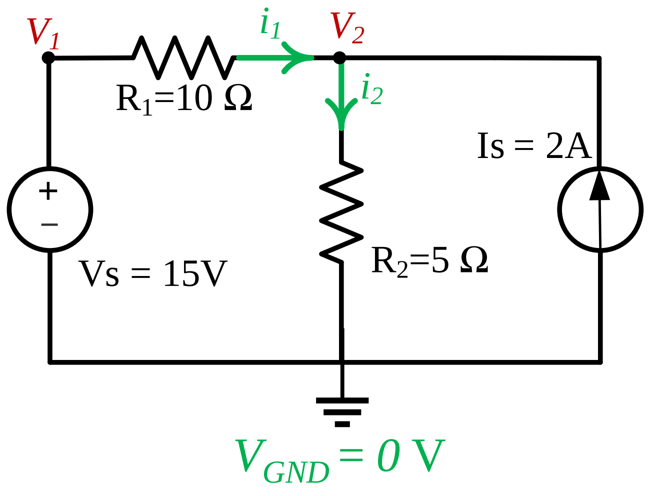
$$i_1 + I_s = i_2 \Rightarrow \frac{V_1 - V_2}{R_1} + I_s = \frac{V_2 - 0}{R_2}$$

Special case at node 1:

$$V_1 = 15\text{ V}$$

$$V_1 = 15\text{ V} \qquad i_1 = \frac{1}{3}\text{ A}$$

$$V_2 = \frac{35}{3}\text{ V} \qquad i_2 = \frac{7}{3}\text{ A}$$



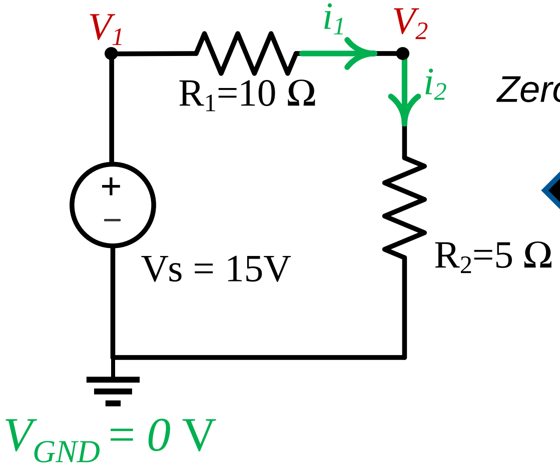
4-2-1 SUPERPOSITION PRINCIPLE

Definition: The **superposition principle** states that a response in a linear circuit is equal to the sum of the responses for each independent source acting alone with the other independence source zeroed.

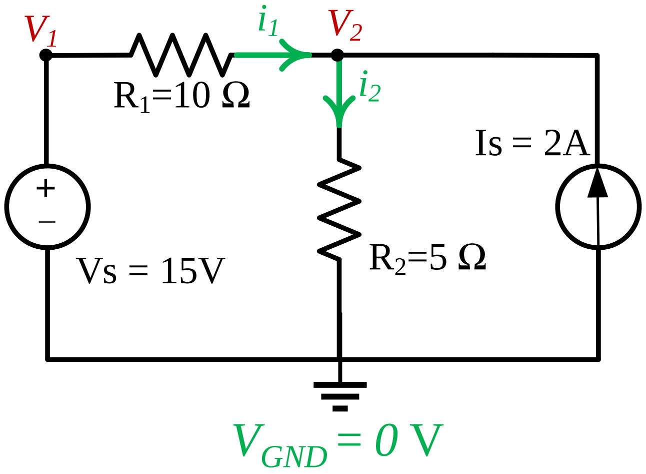
When zeroed, current source became open circuits and voltage source became short circuit.

Case 2: Only the voltage source presents

$$V_1 = 15\text{ V}$$
$$V_2 = 5\text{ V}$$
$$i_1 = 1\text{ A}$$
$$i_2 = 1\text{ A}$$



Zero the current source:



4-2-1 SUPERPOSITION PRINCIPLE

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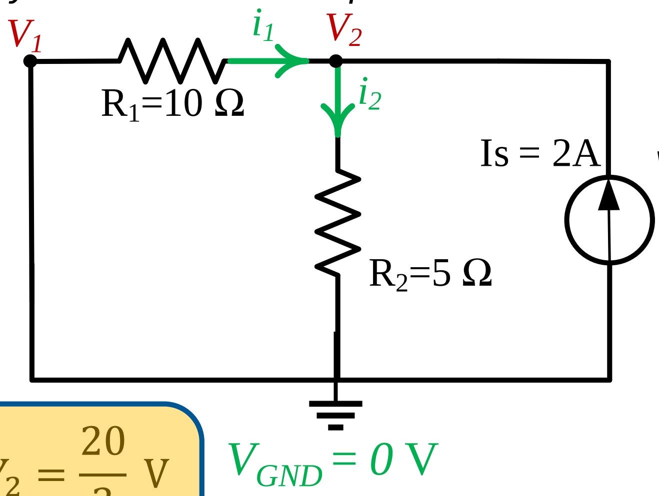
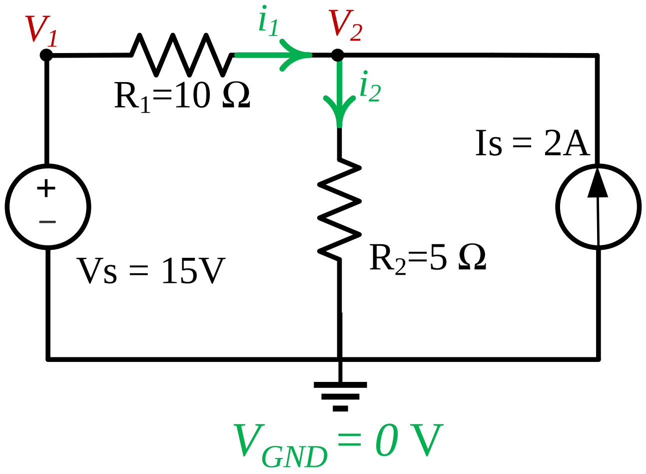
Case 3: Only the current source presents



Zero the voltage source:



$$V_1 = 0\ \text{V}, V_2 = \frac{20}{3}\ \text{V}$$

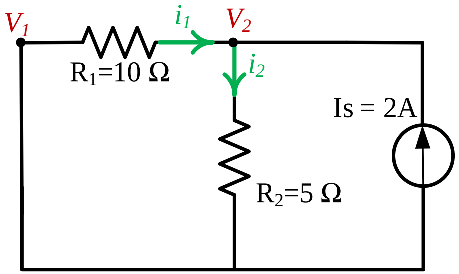
$$i_1 = -\frac{2}{3}\text{A}, i_2 = \frac{4}{3}\text{A}$$

4-2-1 SUPERPOSITION PRINCIPLE

Definition: The **superposition principle** states that a response in a linear circuit is equal to the sum of the responses for each independent source acting alone with the other independence source zeroed.

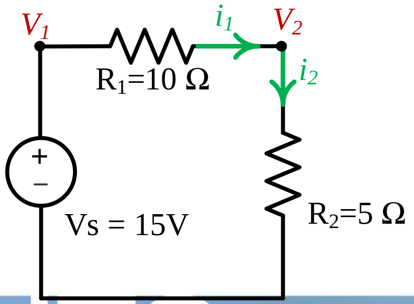
When zeroed, current source became open circuits and voltage source became short circuit.

Only the current source presents



$$V_1 = 0\ \text{V}, V_2 = \frac{20}{3}\ \text{V}$$
$$i_1 = -\frac{2}{3}\ \text{A}, i_2 = \frac{4}{3}\ \text{A}$$

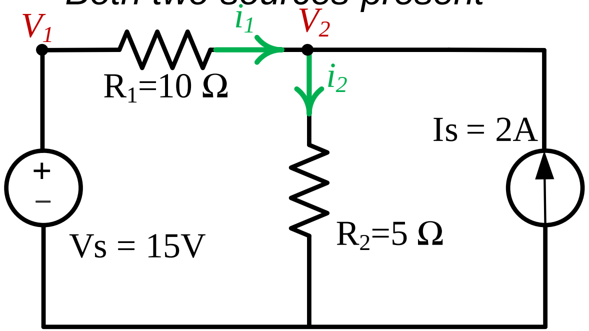
Only the voltage source presents



$$V_1 = 15\ \text{V}$$
$$V_2 = 5\ \text{V}$$
$$i_1 = 1\ \text{A}$$
$$i_2 = 1\ \text{A}$$

+

Both two sources present



=

$$V_1 = 15\ \text{V} \quad i_1 = \frac{1}{3}\ \text{A}$$
$$V_2 = \frac{35}{3}\ \text{V} \quad i_2 = \frac{7}{3}\ \text{A}$$

48510 LEC 4 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-2

Topic 4-2-2: Superposition principle example

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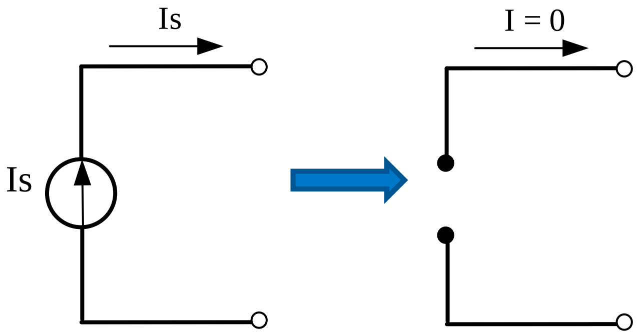
4-2-2 SUPERPOSITION PRINCIPLE

Definition: The **superposition principle** states that a response in a linear circuit is equal to the sum of the responses for each independent source acting alone with the other independence source zeroed.

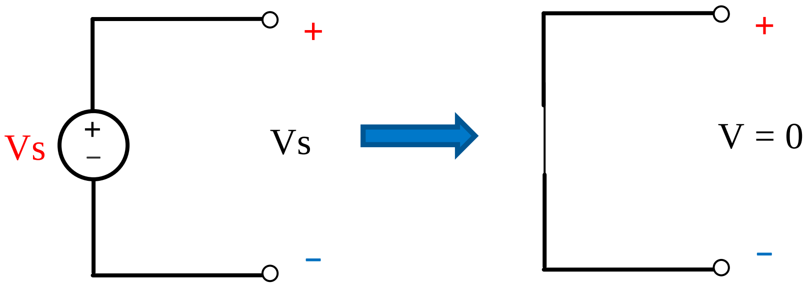
When zeroed, current source became open circuits and voltage source became short circuit.

Zero/suppress/deactivate



Current sources became open circuits because the current flow across an open circuit is 0.

Zero/suppress/deactivate

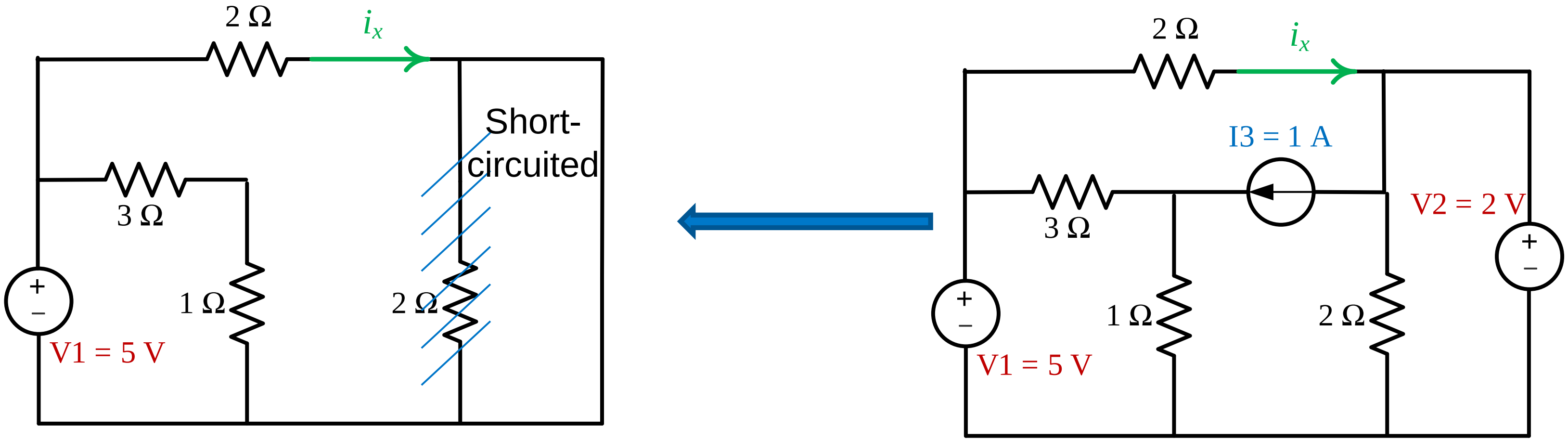


Voltage sources became short circuits because the voltage across a short circuit is 0.

4-2-2 SUPERPOSITION PRINCIPLE

When zeroed, current source became open circuits and voltage source became short circuit.

Keep source 1, zero source 2 and 3, conduct the circuit analysis;

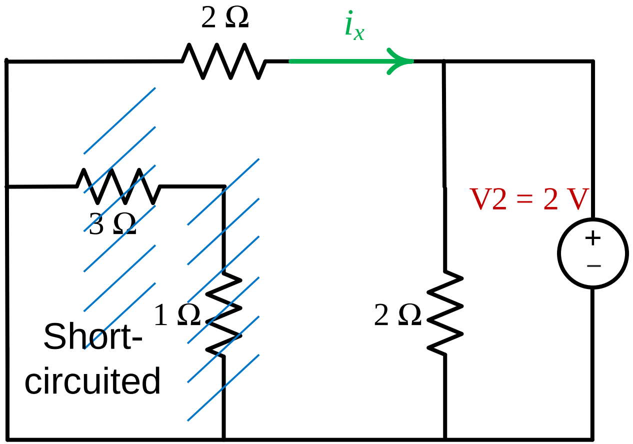


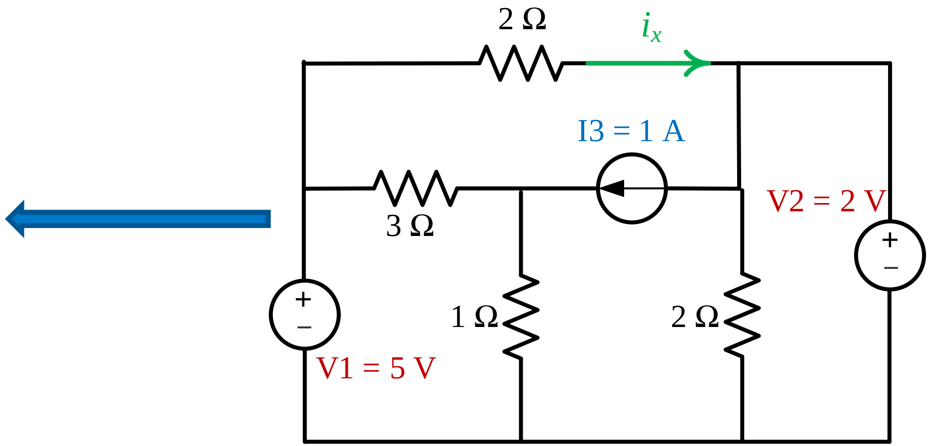
$$i_{x1} = 2.5 \text{ A}$$

4-2-2 SUPERPOSITION PRINCIPLE

When zeroed, current source became open circuits and voltage source became short circuit.

Keep source 2, zero source 1 and 3, conduct the circuit analysis;



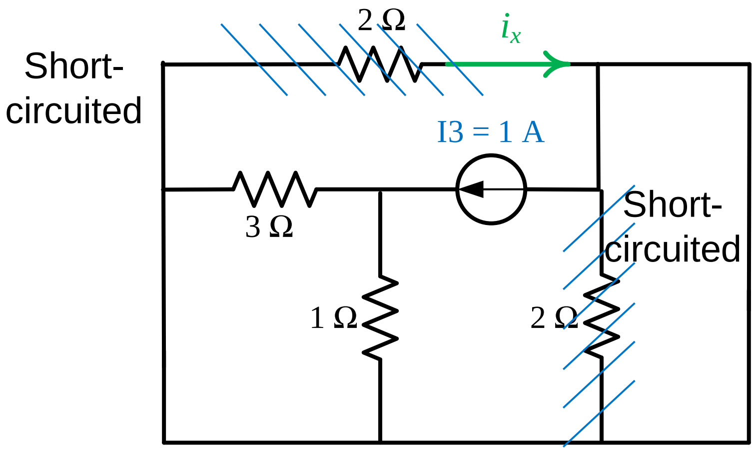


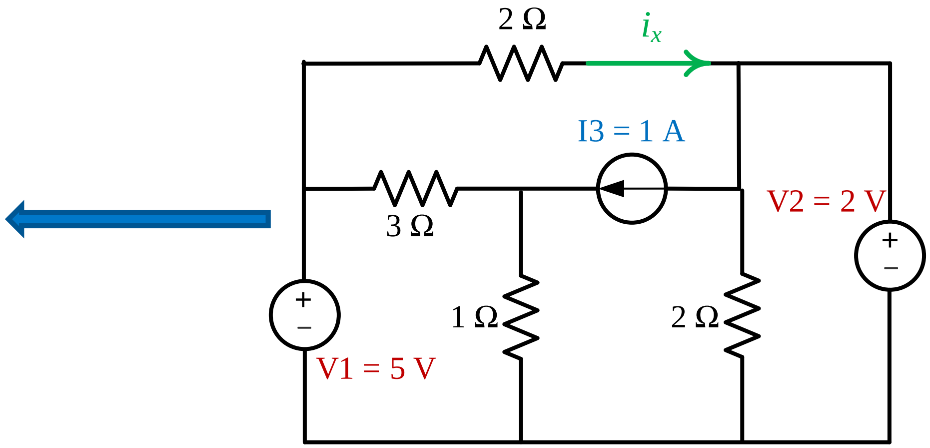
$$i_{x2} = -1\ \text{A}$$

4-2-2 SUPERPOSITION PRINCIPLE

When zeroed, current source became open circuits and voltage source became short circuit.

Keep source 3, zero source 1 and 2, conduct the circuit analysis;



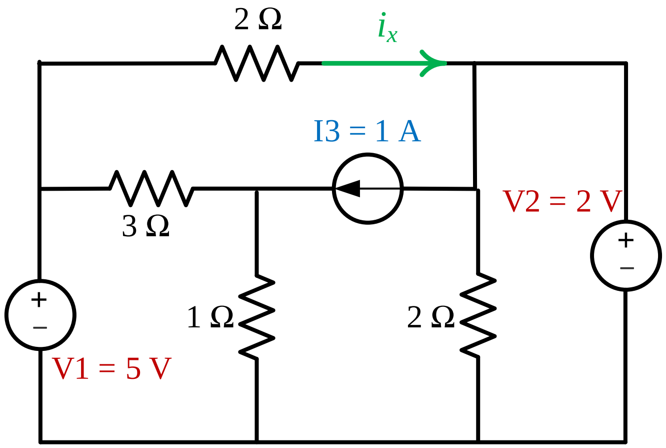


$$i_{x3} = 0 \text{ A}$$

4-2-2 SUPERPOSITION PRINCIPLE

When zeroed, current source became open circuits and voltage source became short circuit.

Keep source 1, zero source 2 and 3, conduct the circuit analysis;
Keep source 2, zero source 1 and 3, conduct the circuit analysis;
Keep source 3, zero source 1 and 2, conduct the circuit analysis;



$$i_{x1} = 2.5\ \text{A}$$
$$i_{x2} = -1\ \text{A}$$
$$i_{x3} = 0\ \text{A}$$

$$i_{x1} = i_{x1} + i_{x2} + i_{x3} = 1.5\ \text{A}$$

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Topic 4-3-1: Practical sources

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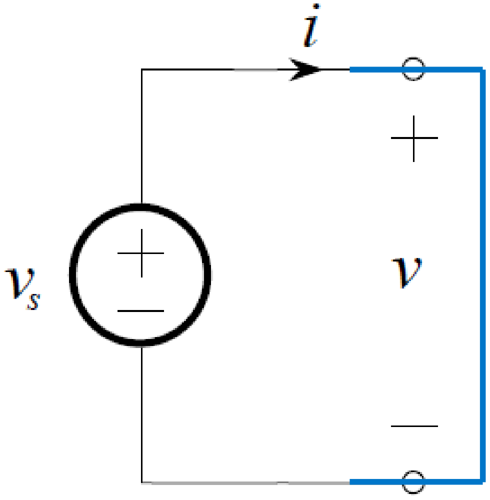
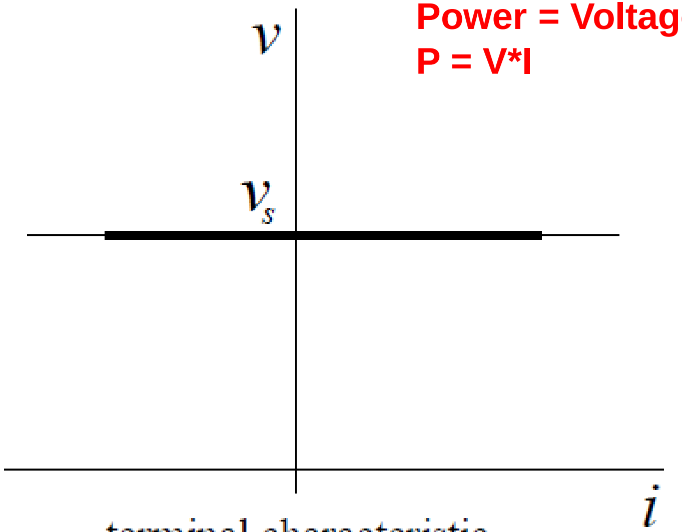
4-3-1 PRACTICAL SOURCE

The **ideal voltage source** is defined as a device whose terminal voltage is independent of the current through it.



ideal source



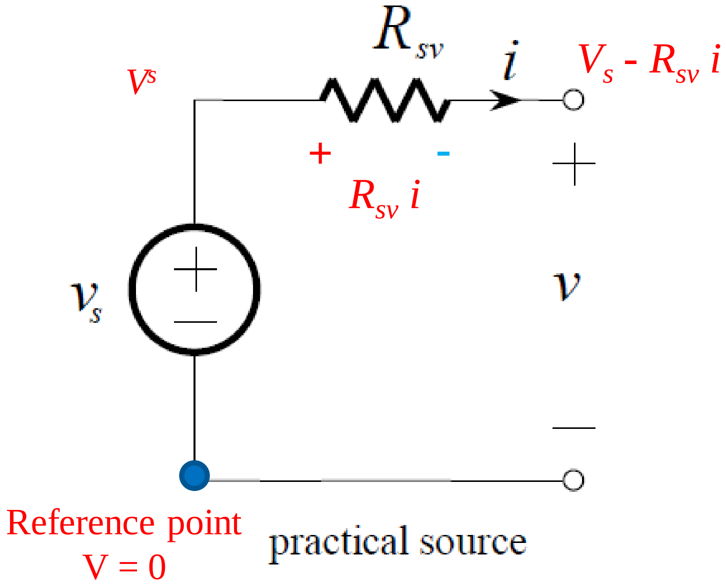
terminal characteristic

Power = Voltage * Current

P = V*I

4-3-1 PRACTICAL SOURCE

All **practical voltage sources suffer from a voltage drop when they deliver current** – the larger the current, the larger the voltage drop. Such behaviour can be modelled by the inclusion of a resistor in series with an ideal voltage source.



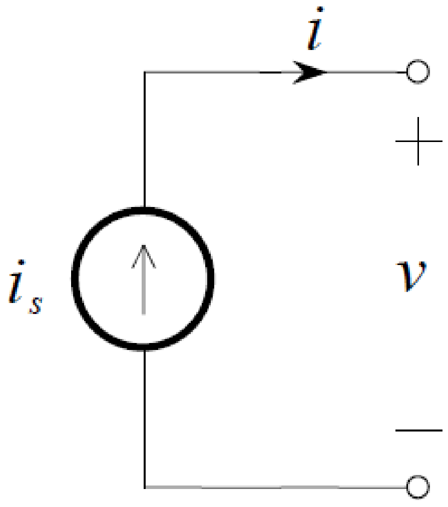
R_{sv} is known as the internal resistance or output resistance.

This resistor (in most cases) is not a real physical resistor! It is more like an equivalent resistor that represents the characteristic of the source.

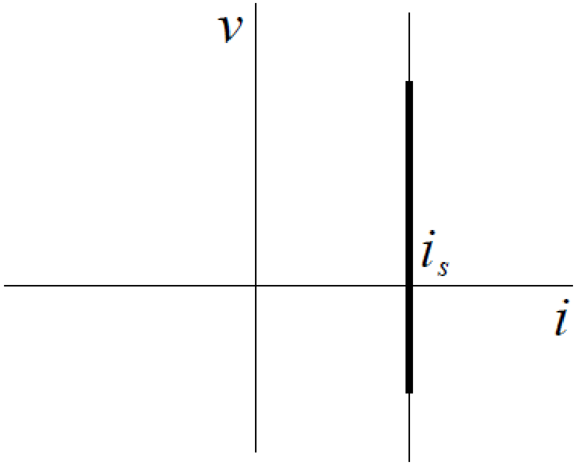
$$v = v_s - R_{sv} i$$

4-3-1 PRACTICAL SOURCE

The **ideal current source** is defined as a device whose current is independent of the voltage across it.



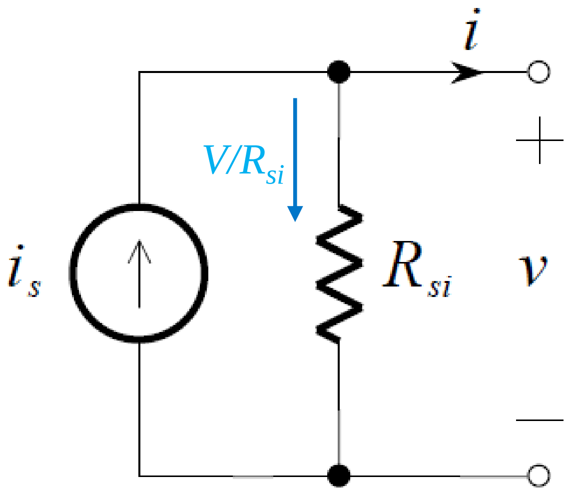
ideal source



terminal characteristic

4-3-1 PRACTICAL SOURCE

A **practical current source** can be modelled by the **inclusion of an inner resistor in parallel** with an ideal current source:



practical source

$$i = i_s - \frac{v}{R_{si}}$$

THINK.
CHANGE.
DO

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48510 LEC 4 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-2

Topic 4-3-2: Equivalent practical source and source transformation

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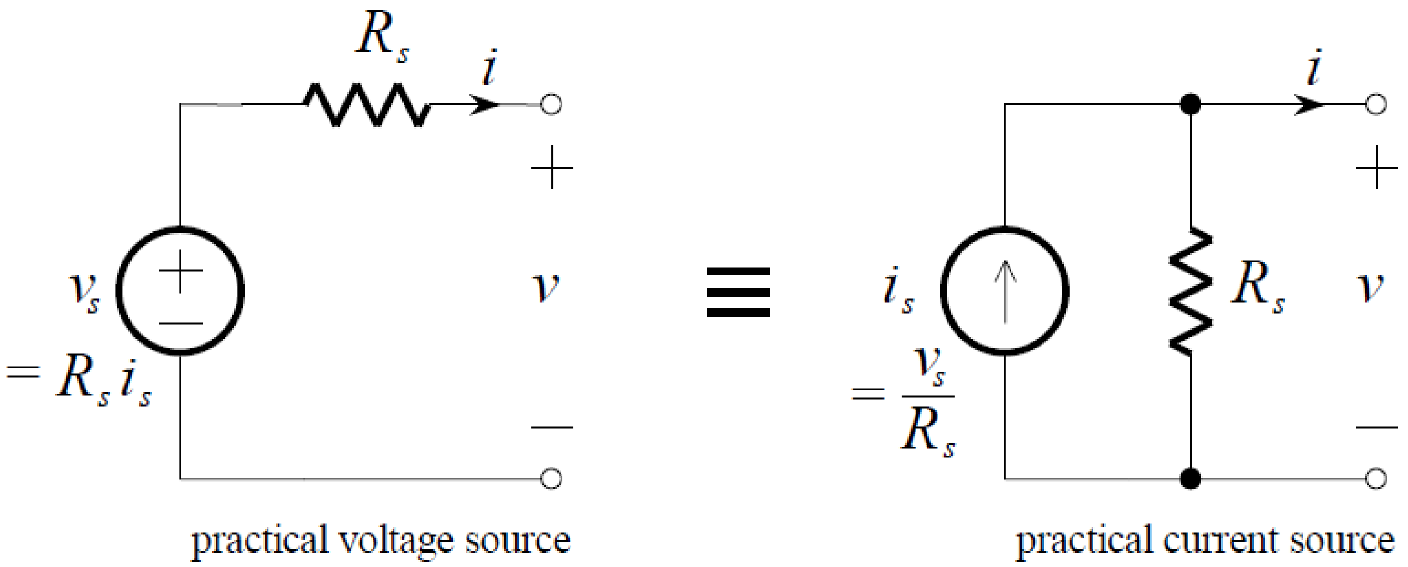
4-3-2 EQUIVALENT PRACTICAL SOURCE

A practical voltage source (an ideal voltage source in series with a resistor) is equivalent to a practical current source (an ideal current source in parallel with a resistor) if

$$R_{sv} = R_{si} \qquad v_s = i_s R_{si}$$



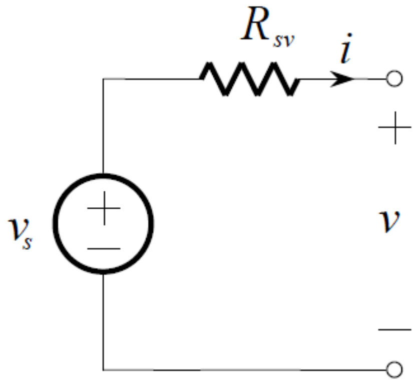
4-3-2 EQUIVALENT PRACTICAL SOURCE

A practical voltage source (an ideal voltage source in series with a resistor) is equivalent to a practical current source (an ideal current source in parallel with a resistor) if

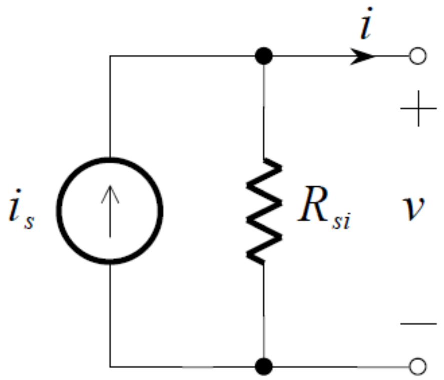
$$R_{sv} = R_{si} \qquad v_s = i_s R_{si}$$

The two sources are considered to be equivalent to each other because they provide the same voltage and current outputs when connected to a same load.

Case 1: Open circuit



practical source



practical source

$i = 0,\ V = V_s$ $\equiv$ $i = 0,\ V = i_s R_{si}$

4-3-2 EQUIVALENT PRACTICAL SOURCE

A practical voltage source (an ideal voltage source in series with a resistor) is equivalent to a practical current source (an ideal current source in parallel with a resistor) if

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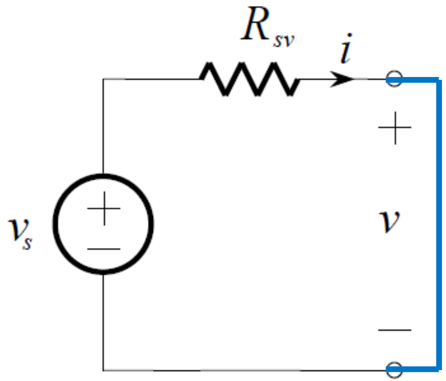
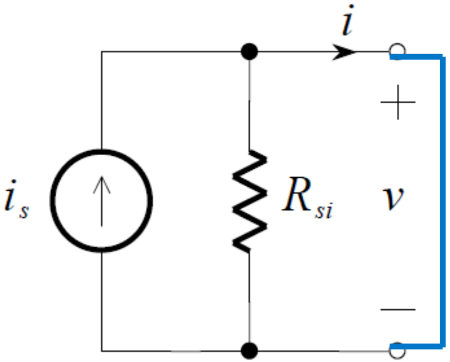
The two sources are considered to be equivalent to each other because they provide the same voltage and current outputs when connected to a same load.

Case 2: Short circuit



practical source



practical source

$i = V_s/R_{sv}$, $V = 0$ $\equiv$ $i = i_s$, $V = 0$

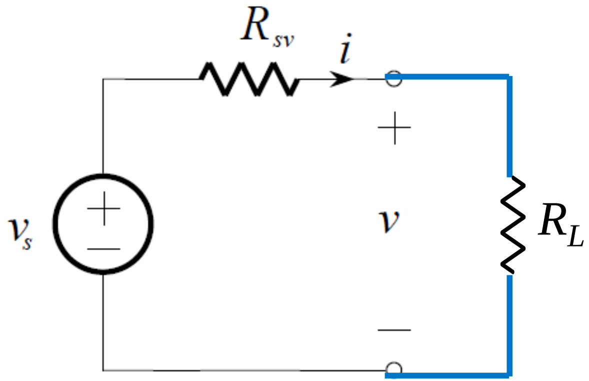
4-3-2 EQUIVALENT PRACTICAL SOURCE

A practical voltage source (an ideal voltage source in series with a resistor) is equivalent to a practical current source (an ideal current source in parallel with a resistor) if

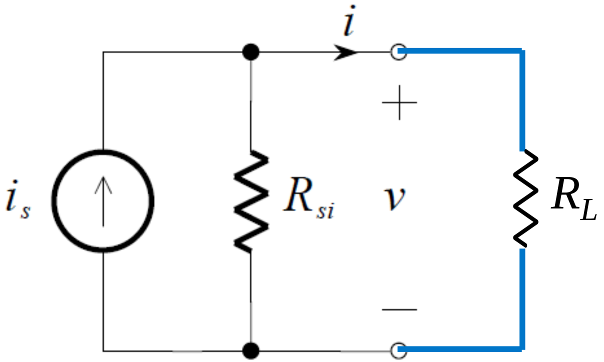
$$R_{sv} = R_{si} \qquad v_s = i_s R_{si}$$

The two sources are considered to be equivalent to each other because they provide the same voltage and current outputs when connected to a same load.

Case 3: Connect with a load resistor R_L



practical source

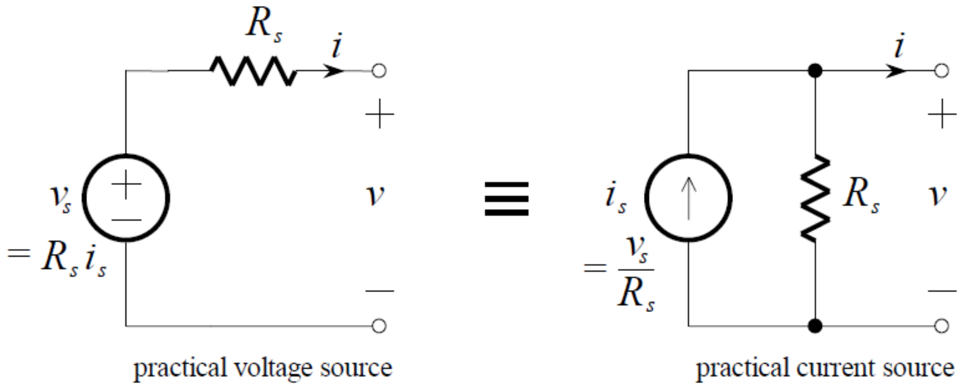


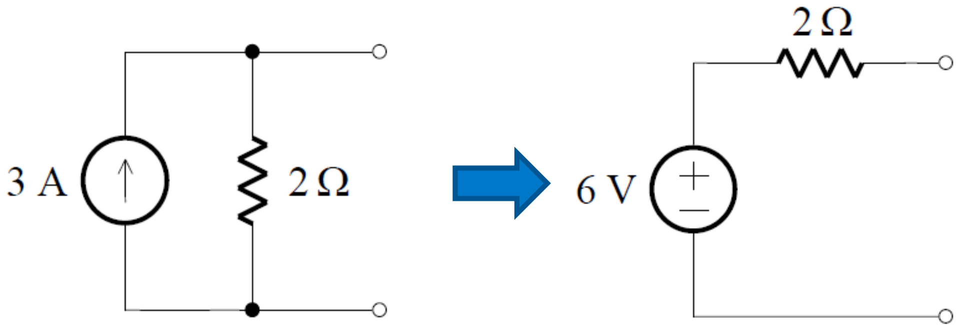
practical source

$$i = \frac{V_s}{R_s + R_L} \qquad V = \frac{R_L V_s}{R_s + R_L} \qquad \equiv \qquad i = \frac{R_s i_s}{R_s + R_L} \qquad V = \frac{R_s i_s R_L}{R_s + R_L}$$

4-3-2 SOURCE TRANSFORMATION

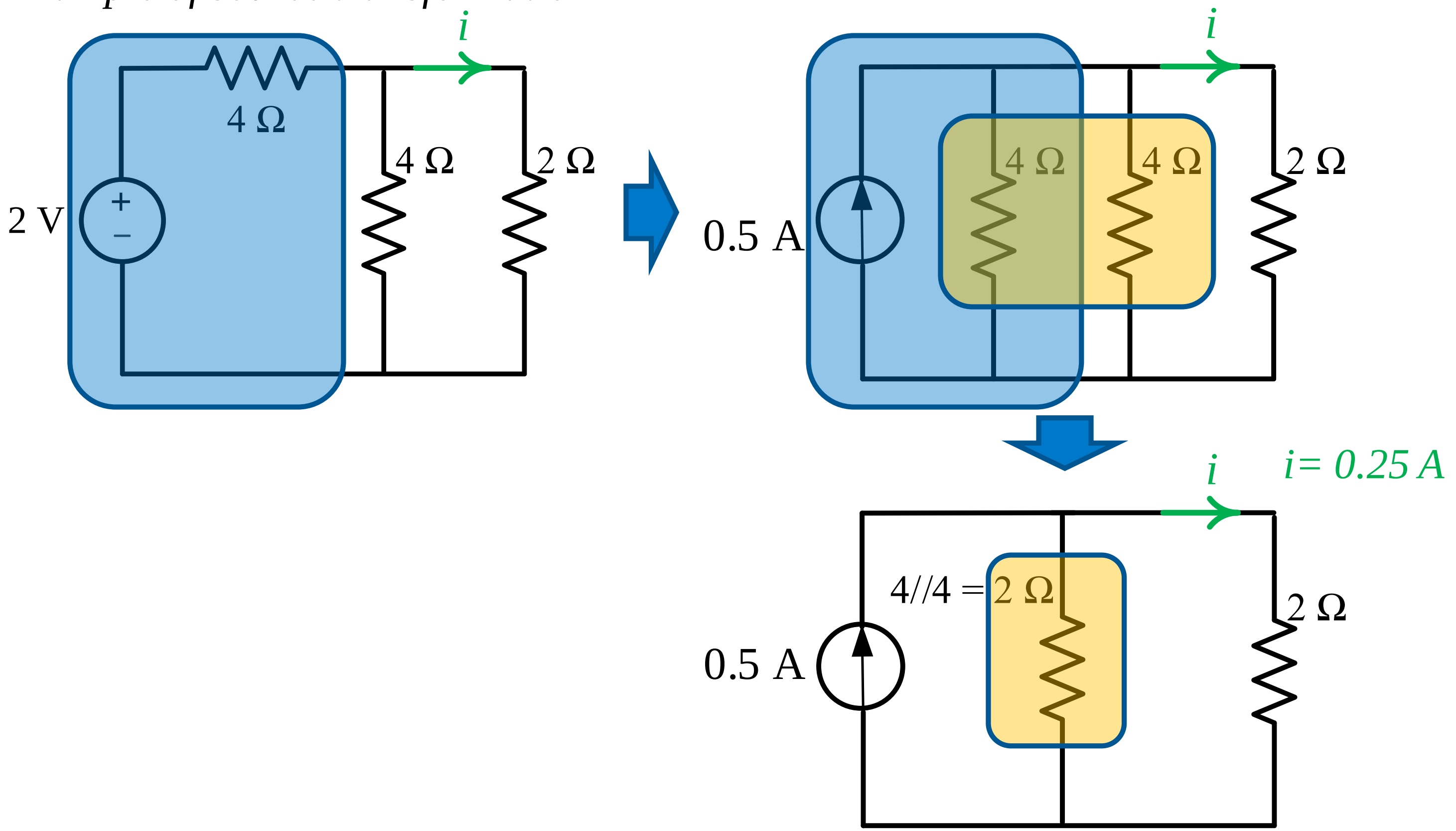
Source transformation is the process of simplifying a circuit solution, especially with mixed sources, by transforming voltage sources into current sources, and vice versa.





4-3-2 SOURCE TRANSFORMATION

Example of source transformation



48510 LEC 4 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-2

Topic 4-4-1: Thevenin Equivalent Circuit

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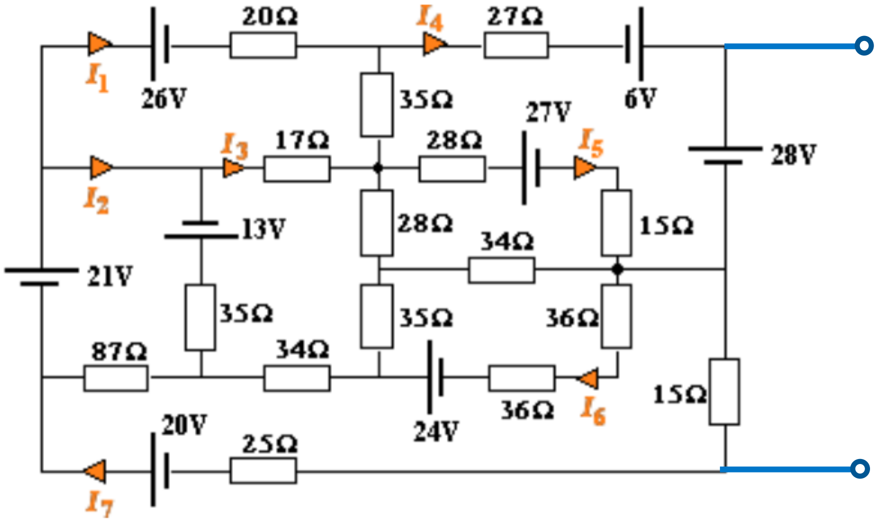
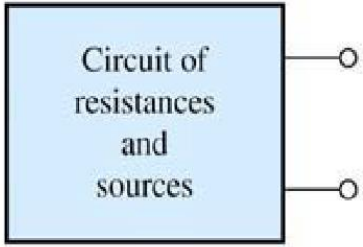
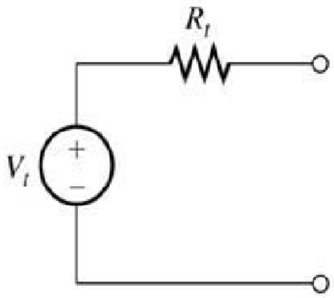
4-4-1 THEVENIN & NORTON EQUIVALENT CIRCUIT

Thévenin's and Norton's equivalent circuits are circuit simplification techniques that focus on terminal behaviour.

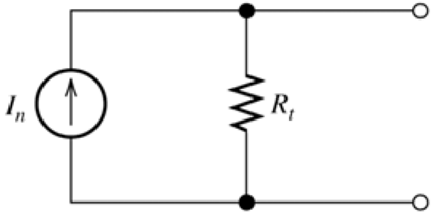
They state that any circuit consisting of independent sources and a network of resistors can be transformed into **one voltage source in series with one resistor** (Thévenin thereom) or **one current source in parallel with one resistor** (Norton thereom).







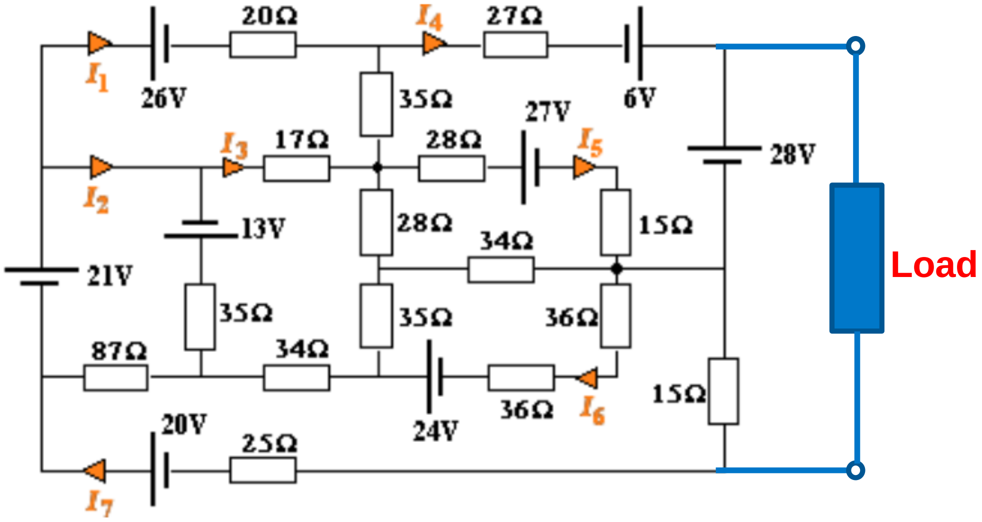
Thévenin equivalent circuit

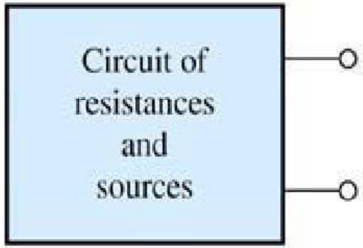


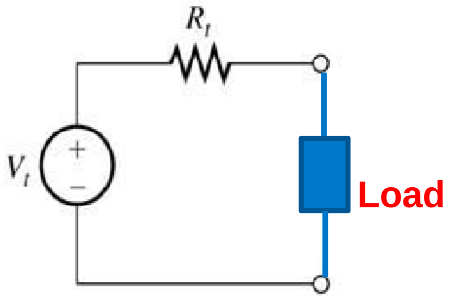
Norton's equivalent circuit

4-4-1 THEVENIN & NORTON EQUIVALENT CIRCUIT

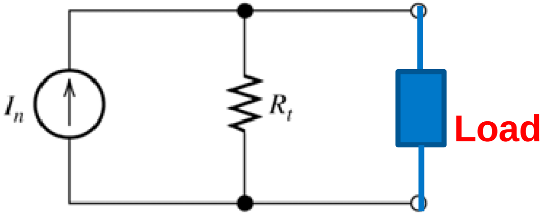
When two circuits have the same load, the voltage on and the current flow across load are exactly the same, we can call the circuits are equivalent to each other.







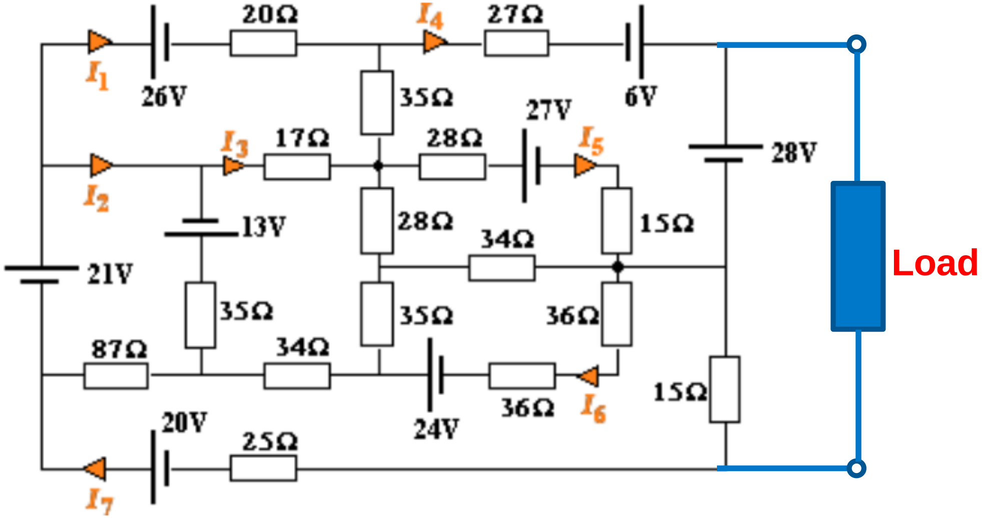
Thévenin equivalent circuit

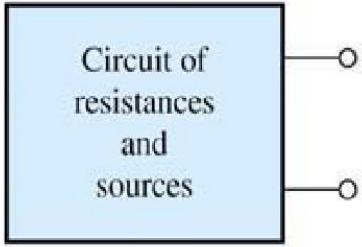


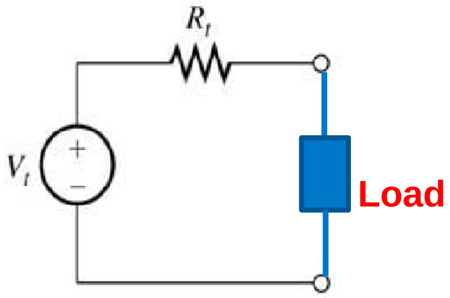
Norton's equivalent circuit

4-4-1 THEVENIN & NORTON EQUIVALENT CIRCUIT

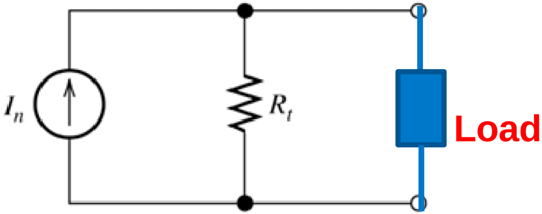
Through the use of Thévenin's theorem and Norton's theorem, we will see that we can replace a large portion of a complex circuit (often a complicated and uninteresting part) with a very simple equivalent circuit, thus enabling analysis and focus on one particular element of the circuit.







Thévenin equivalent circuit



Norton's equivalent circuit

How to find out V_t, I_n, and R_t?

4-4-1 THEVENIN & NORTON EQUIVALENT CIRCUIT

How to find out Vt, In, and Rt?

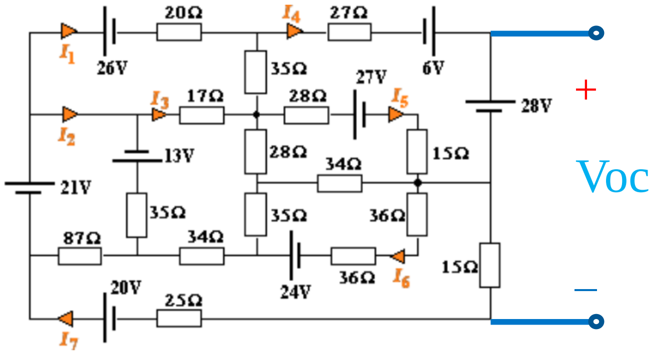
What are known information?

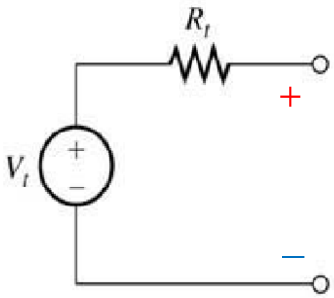
According to the source transformation theorem,

Rt * In = Vt

If we left the circuit open

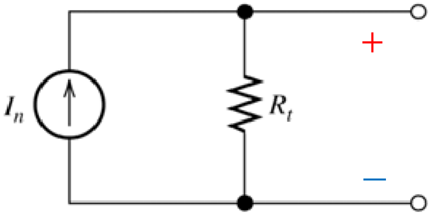
Vt = Voc





Thévenin equivalent circuit

Voc = Vt



Norton's equivalent circuit

4-4-1 THEVENIN & NORTON EQUIVALENT CIRCUIT

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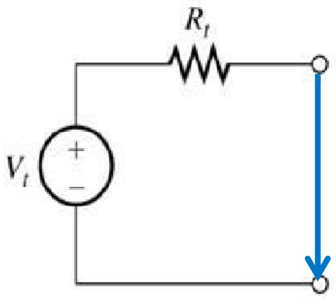
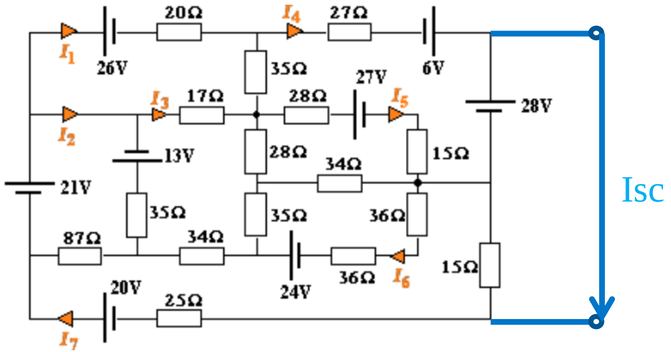


Thévenin equivalent circuit

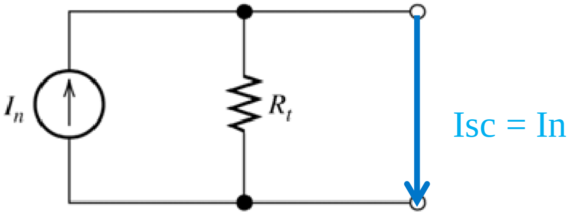
If we left the circuit open

Vt = Voc



If we short-circuit the two terminals

In = Isc



Norton's equivalent circuit

We can obtain **Vt** and **In** directly from the circuit

Can we obtain **Rt** directly from circuit? Yes!

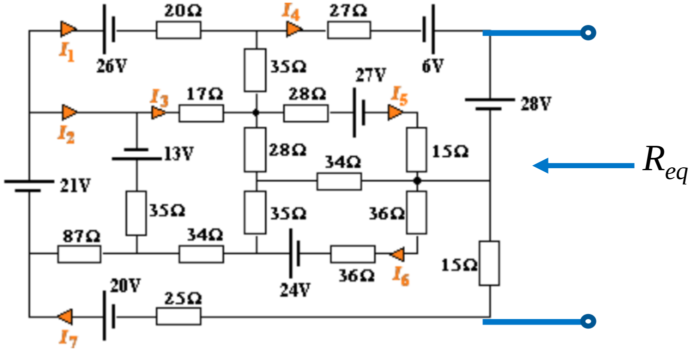
4-4-1 THEVENIN & NORTON EQUIVALENT CIRCUIT

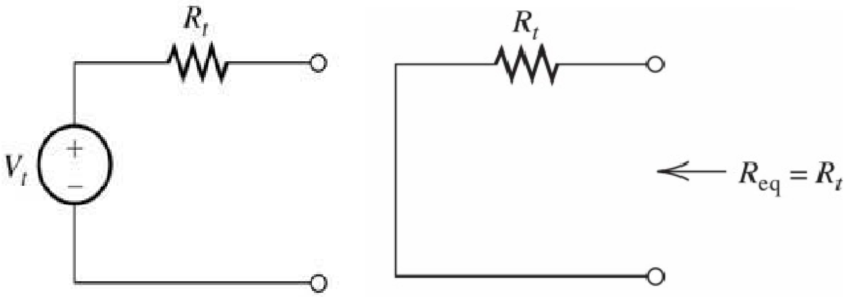
How to find out Vt, In, and Rt?

Can we obtain **Rt** directly from circuit?

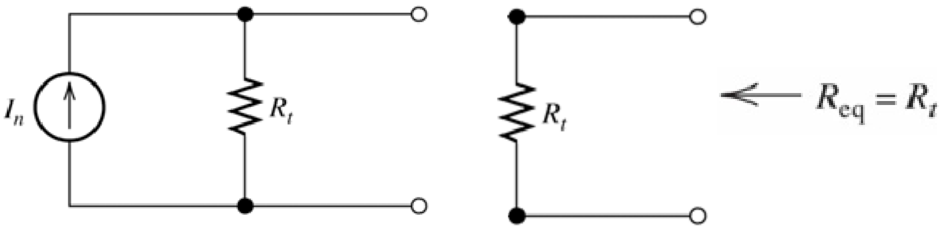
Yes, by zeroing the source

Thevenin equivalent resistance is the equivalent resistant you observed across the two terminals when all the sources are zeroed.





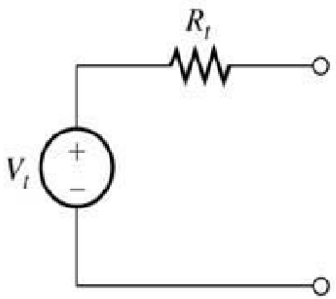
Thévenin equivalent circuit



Norton's equivalent circuit

4-4-1 THEVENIN & NORTON EQUIVALENT CIRCUIT

They state that any circuit consisting of independent sources and a network of resistors can be transformed into **one voltage source in series with one resistor** (Thévenin thereom) or **one current source in parallel with one resistor** (Norton thereom).



Thévenin equivalent circuit

We can get Vt, In, and Rt directly from circuit or from source transition principle.

$$V_t = V_{oc}$$

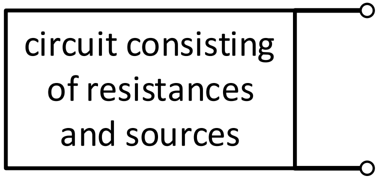
$$I_n = I_{sc}$$

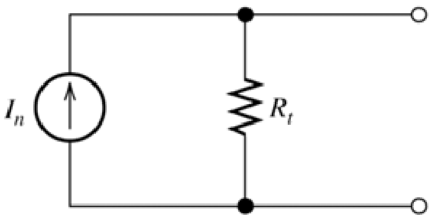
$$R_t = R_{eq}$$ by zeroing the sources

Voltage source = Short circuit

Current source = Open circuit

$$R_t \times I_n = V_t$$





Norton's equivalent circuit

48510 LEC 4 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-2

Topic 4-4-2: Thevenin Equivalent Circuit example

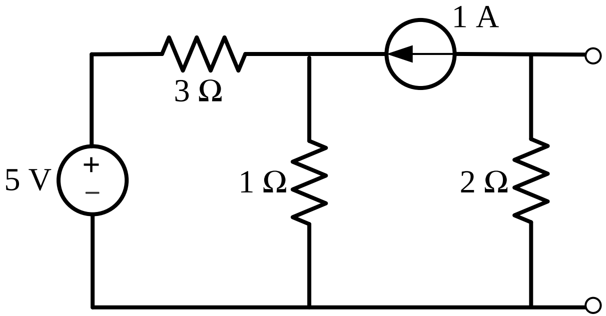
DR CAN DING

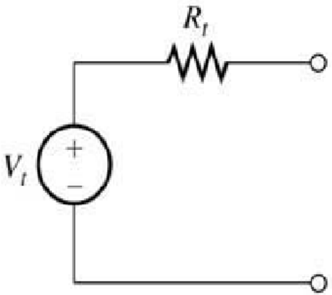
Lecturer

can.ding@uts.edu.au

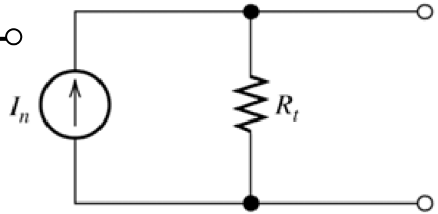
4-4-2 EQUIVALENT CIRCUIT EXAMPLE

Finding the Thevenin and Norton equivalent circuit of the circuit below.





Thévenin equivalent circuit



Norton's equivalent circuit

We can get Vt, In, and Rt directly from circuit or from source transition principle.

$V_t = V_{oc}$

$I_n = I_{sc}$

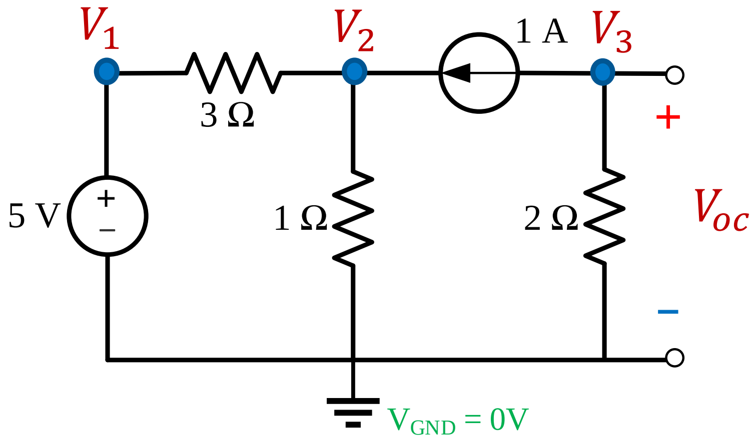
$R_t = R_{eq}$ by zeroing the sources

Voltage source = Short circuit
Current source = Open circuit

$R_t \times I_n = V_t$

4-4-2 EQUIVALENT CIRCUIT EXAMPLE

Finding the Thevenin and Norton equivalent circuit of the circuit below.



1) Find Vt

We can get Vt, In, and Rt directly from circuit or from source transition principle.

$V_t = V_{oc}$

$I_n = I_{sc}$

$R_t = R_{eq}$ by zeroing the sources

Voltage source = Short circuit

Current source = Open circuit

$R_t \times I_n = V_t$

Node voltage analysis

$V_1 = 5$

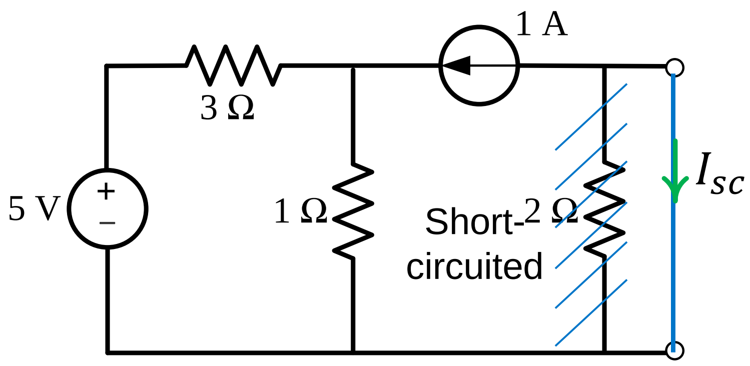
$1 = \frac{V_2 - V_1}{3} + \frac{V_2 - 0}{1}$ ⇒ $V_3 = -2\text{ V} = V_{oc}$

$\frac{V_3 - 0}{2} + 1 = 0$

$V_t = -2\text{ V}$

4-4-2 EQUIVALENT CIRCUIT EXAMPLE

Finding the Thevenin and Norton equivalent circuit of the circuit below.



2) Find In

$I_{sc} = -1\text{ A}$

$I_n = -1\text{ A}$

We can get Vt, In, and Rt directly from circuit or from source transition principle.

$V_t = V_{oc}$

$I_n = I_{sc}$

$R_t = R_{eq}$ by zeroing the sources

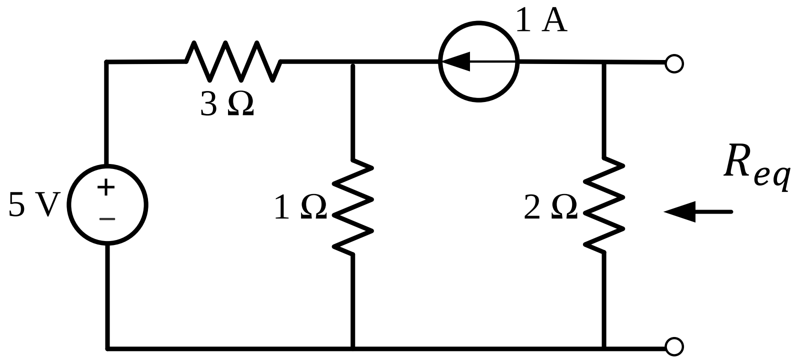
Voltage source = Short circuit

Current source = Open circuit

$R_t \times I_n = V_t$

4-4-2 EQUIVALENT CIRCUIT EXAMPLE

Finding the Thevenin and Norton equivalent circuit of the circuit below.



$V_t = -2$ V
$I_n = -1$ A
$R_t = 2\ \Omega$

We can get Vt, In, and Rt directly from circuit or from source transition principle.

$V_t = V_{oc}$

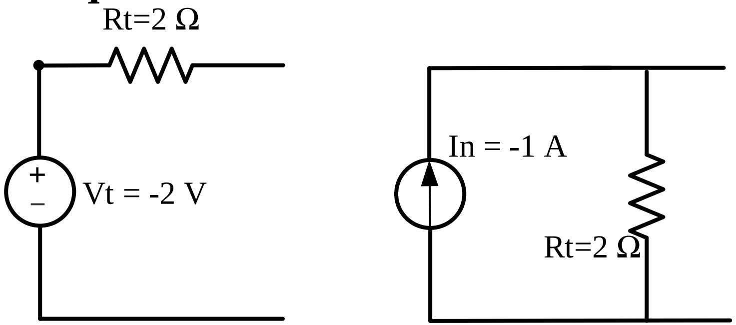
$I_n = I_{sc}$

$R_t = R_{eq}$ by zeroing the sources

Voltage source = Short circuit
Current source = Open circuit

$R_t \times I_n = V_t$

4) Draw equivalent circuits



48510 LEC 4 – HIGHER LEVEL CIRCUIT ANALYSIS TECHNIQUES-2

Topic 4-5: Maximum power transfer

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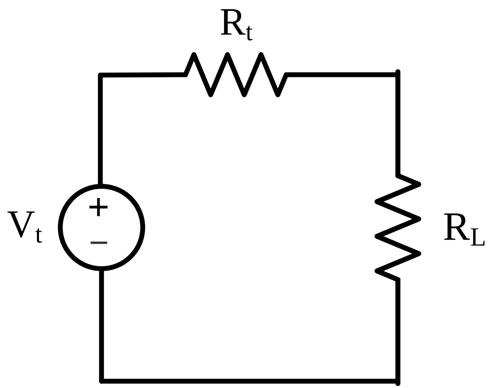
4-5 MAXIMUM POWER TRANSFER

Maximum power transfer theorem states that a practical DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.

Consider a practical DC source shown as below, if we connect a resistive load with this source, **only when $R_L = R_t$, the maximum power is delivered to the load.**



According to the definition of power:

$$P_L = V_L I_L = (I_L R_L) I_L = I_L^2 R_L$$

The current can be obtained as

$$I_L = \frac{V_t}{R_t + R_L}$$

Substitute I_L into P_L, we have

$$P_L = V_t^2 \frac{R_L}{(R_t + R_L)^2}$$

Once the source is given, V_t and R_t are unchangeable. Now we need to find the appropriate value of R_L to achieve maximum power transfer.

4-5 MAXIMUM POWER TRANSFER

The problem turns into finding the maximum value of $\frac{R_L}{(R_t + R_L)^2}$

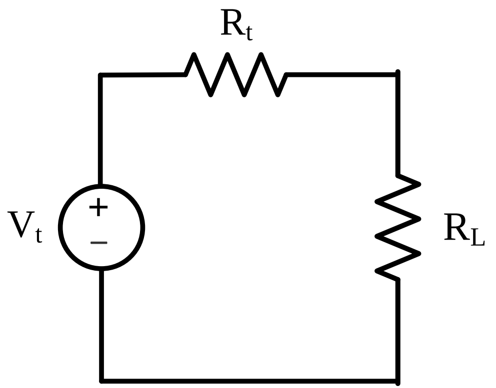
where R_L *is the variable and* R_t *is a constant.*

The result is that only when $R_L = R_t$ *, it has the maximum value.*

The maximum power is

$$P_L = V_t^2 \frac{R_L}{(R_t + R_L)^2} = V_t^2 \frac{R_t}{(R_t + R_t)^2} = V_t^2 \frac{R_t}{4{R_t}^2}$$

$$= \frac{V_t^2}{4R_t}$$



The total power generated by the source is

$$P_T = V_T I_T = \frac{V_t^2}{R_t + R_t} = \frac{V_t^2}{2R_t}$$

It is noticed that, in this case, ***only the half of power generated by the source is received by the load, i.e., ($P_L = 0.5P_T$).***

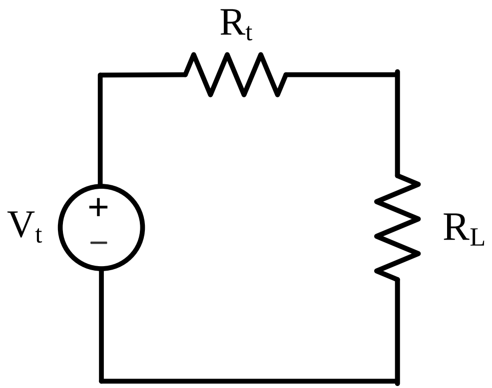
4-5 MAXIMUM POWER TRANSFER

How to find the maximum value of $\frac{R_L}{(R_t+R_L)^2}$ *where* R_L *is the variable and* R_t *is a constant.*

(Knowing this derivation is not compulsory.)

The problem is to find the peak value of a function $f(R_L)=\frac{R_L}{(R_t+R_L)^2}$

It has a maximum or minimum value when $f'(R_L)=0$



$$f'(R_L) = \frac{d[\frac{R_L}{(R_t+R_L)^2}]}{dR_L}$$

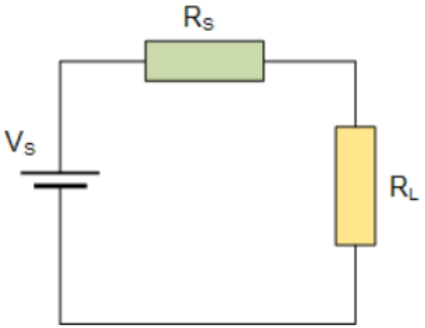
$$= \frac{(R_t+R_L)^2 \times 1 - R_L \times 2(R_t+R_L)}{(R_t+R_L)^4} = 0$$

$\Rightarrow$ $(R_t+R_L)^2 - 2R_L(R_t+R_L) = 0$

$\Rightarrow$ $R_t + R_L - 2R_L = 0$

$\Rightarrow$ $R_t = R_L$ *One can easily find this is the maximum value rather than the minimum value.*

4-5 MAXIMUM POWER TRANSFER



Where:

$R_S = 25\Omega$

R_L is variable between 0 – 100Ω

$V_S = 100v$

If you give specific values to the source, the inner resistance and the load resistance, you can plot the power received by the load.

R_L (Ω)	I (amps)	P (watts)
0	4.0	0
5	3.3	55
10	2.8	78
15	2.5	93
20	2.2	97

R_L (Ω)	I (amps)	P (watts)
25	2.0	**100**
30	1.8	97
40	1.5	94
60	1.2	83
100	0.8	64