48510 LEC 6 – INDUCTOR

Topic 6-1: Introduction to Inductor

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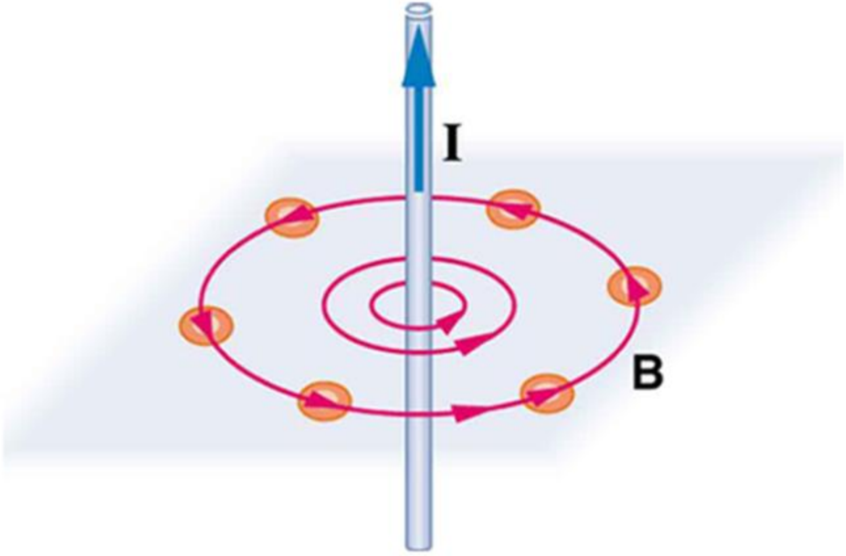
6-1 INTRODUCTION TO INDUCTOR

Inductors can be made by winding any metallic wire commonly around an iron core. An ideal inductor produces a **magnetic field** which is wholly confined within it.

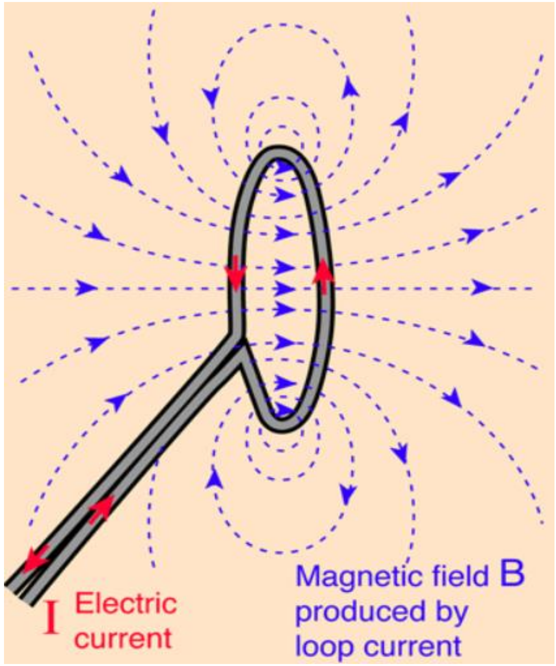
Examples of inductors from Wikipedia

6-1 INTRODUCTION TO INDUCTOR

An electric current will produce a magnetic field, which can be visualized as a series of circular field lines around a wire segment.



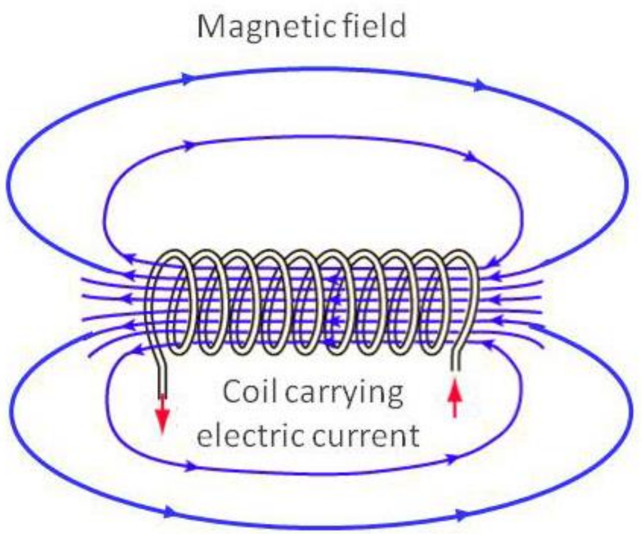
Magnetic field of a straight current



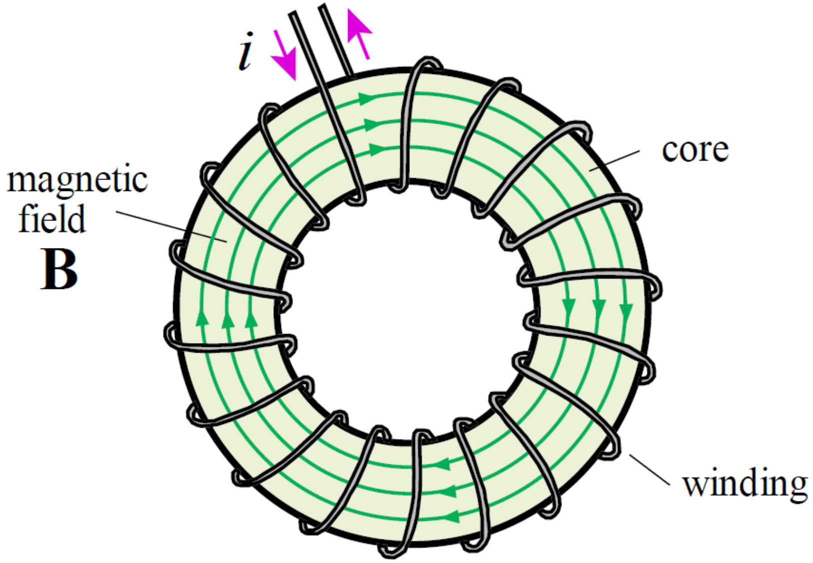
Magnetic field of a loop current

6-1 INTRODUCTION TO INDUCTOR

By winding several turns around a core, each turn will generate a magnetic filed when current flows. The magnetic field generated by each turn of wire add up and results in a much stronger magnetic field confined within the windings. This is how we make inductors.



Magnetic field of a solenoid inductor



Magnetic field of a toroid inductor

6-1 INTRODUCTION TO INDUCTOR

Magnetic flux (Φ) is a measurement of the total magnetic field which passes through a given area.

Considering a coil of N turns, where each turn forms a loop with exactly the same boundary, each turn will "link" the "same" flux Φ, all for a total **"flux linkage"** of **λ=NΦ**.

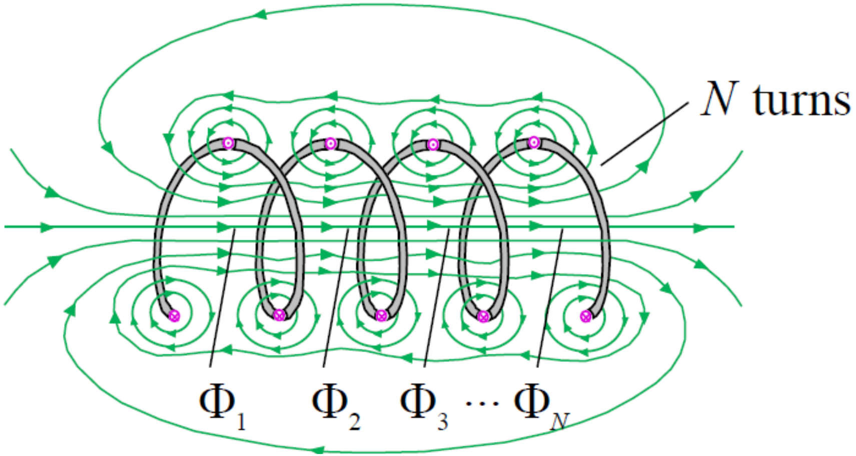
An ideal inductor is a structure where the flux linkage λ is directly proportional to the current *i* through it.

We define a constant, called the self inductance, ***L***, of the linear structure by the linear relationship:

$$\lambda = Li$$

The unit of inductance is henry, with symbol of H.

Note that the definition of capacitance is also a linear relationship: Q=CV.



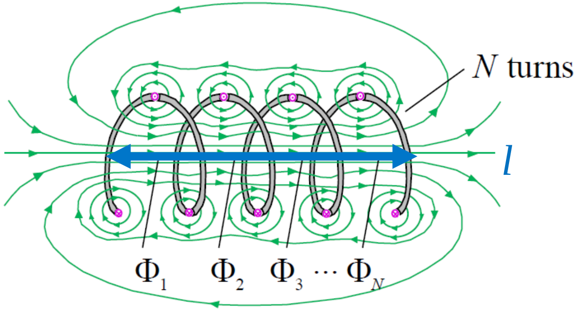
$$\lambda = \Phi_1 + \Phi_2 + \Phi_3 + \ldots + \Phi_N = N\Phi_{av}$$

6-1 INTRODUCTION TO INDUCTOR

We define a constant, called the self inductance, ***L***, of the structure by the linear relationship:

$$\lambda = Li \qquad\qquad L = \frac{\lambda}{i}$$

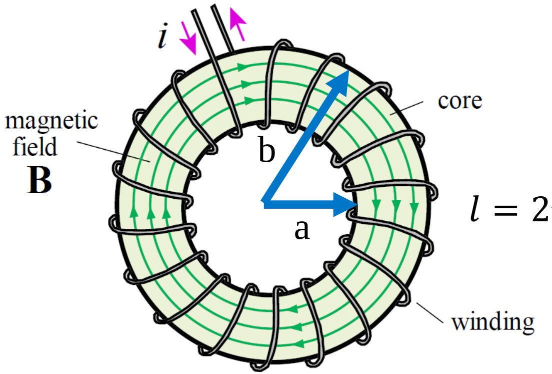
The unit of inductance is henry, with symbol of H.



Similar to the capacitance ***C*** of a capacitor, it should be noted that ***L* is a also purely geometric property**, and depends only on the conductor arrangements and the materials used in the construction.

For a **closely wound** toroid or solenoid, the inductance is approximately

$$L \approx N^2 \frac{\mu_r \mu_0 A}{l} = N^2 \frac{\mu A}{l}$$



$$l = 2\pi \frac{a+b}{2}$$

where N is the number of turns, A is the cross-sectional area, *l* is the length of the toroid or solenoid, μ is the permeability of the media.

μ_r is the relative permeability of the material and μ_0 is the permeability of free space where μ_0 = 1.25663706212×10^{-6} H/m.

48510 LEC 6 – CAPACITOR

Topic 6-2-1: The V-I characteristic of inductor

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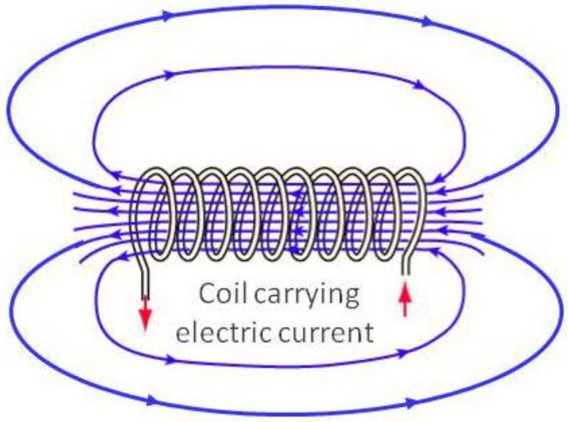
6-2-1 V-I CHARACTERISTIC OF CAPACITOR

We know the electrical current flow generates magnetic filed. For an ideal inductor, the flux linkage λ is directly proportional to the current i through it.

$$\lambda = Li$$

According to *Faraday's law of induction*, when the current flowing through the coil changes, **the time-varying magnetic field induces an electromotive force (e.m.f.) (voltage) in the conductor**. The induced voltage can be calculated via

$$v = -\frac{d\lambda}{dt}$$



The minus sign comes from the fact that the **induced voltage has a polarity (direction) which opposes the change in current** that created it, according to *Lenz's law*.

To make it easier, we *ignore the minus sign* and consider only the magnitude of voltage to get the V-I characteristic. Note that we can find out the polarity of voltage according to *Lenz's law* later.

Substituting λ into v, we get

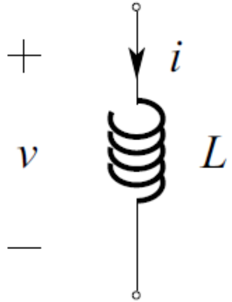
$$v = L\frac{di}{dt}$$

Giving the current through an inductor, we can find the voltage induced on the inductor.

6-2-1 V-I CHARACTERISTIC OF CAPACITOR

$$v = L\frac{di}{dt}$$

Giving the current through an inductor, we can find the voltage induced on the inductor.



The polarity of the voltage can be determined using Lenz's law. ***As a rule of thumb, following the direction of current, the voltage across the inductor always decreases.*** *(I will explain this later.)*

How the current changes with the voltage?

Take the integral of the two sides of the previous equation within a time period:

$$\int_{t_0}^{t} v\,dt = L\int_{t_0}^{t}\frac{di}{dt}dt \;\Rightarrow\; \frac{1}{L}\int_{t_0}^{t} v\,dt = \int_{t_0}^{t} di = i(t) - i(t_0)$$

$$\Rightarrow\; i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)$$

Giving the voltage across an inductor within a time period, we can find the current change during this period.

UTS:

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Topic 6-3-1: Inductor charging

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6-3-1 INDUCTOR CHARGING

Case 1: An inductor connected with a DC source

Before the switch is turned on (t < 0 s), $I_L = 0$, $V_L = 0$.

Once the switch is turned on (t = 0 s), I_L starts to increase, magnetic field starts to increase, which generates a EMF (voltage) to oppose this change, since the current is still very small at this instant, the voltage on the resistor is also very small. All the source voltage Vs is used to overcome the resist from inductor, so at this instant $V_L = 2$, $I_L = 0$.

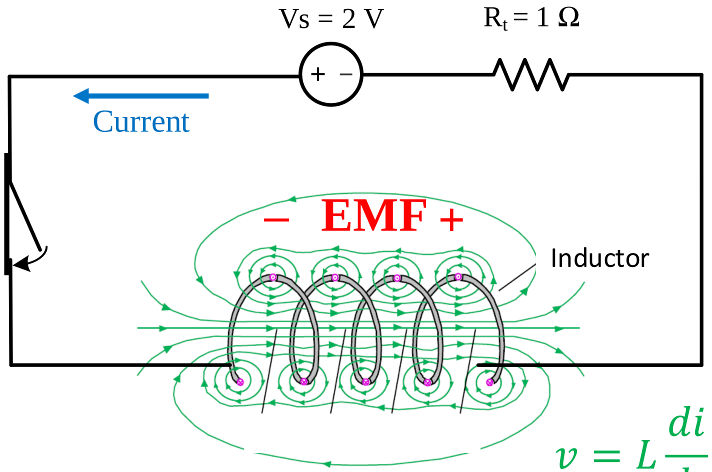
With time goes on, I_L keeps increasing, the voltage applied on the resistor keeps increasing, thus the voltage from the source left for the inductor decreases (V_L *decreases*).

In this case, will current still increase? will it stops changing? or will it reverse back?

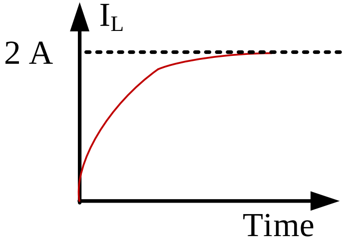
The current still increase but the increase is slower, $di/dt = V_L$ gets smaller. This will last for a short period of time before the steady state is reached (0 < t < t_s).

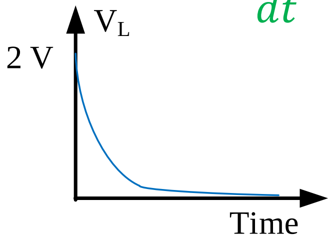
Finally, when current is large enough ($IR_t = Vs$), all the source voltage will be applied on the resistor, and there is no voltage left for inductor. Thus current will not change anymore, and the circuit reaches a steady state where $V_L = 0$, $V_R = 2$, $I = 2\,A$.

This process is also called "inductor charging".



$$v = L\frac{di}{dt}$$





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Topic 6-3-2: Inductor discharging

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6-3-2 INDUCTOR DISCHARGING

Case 2: An fully charged inductor connected with a resistor

Considering we have the inductor connected with a source for a while and the inductor has already reached the steady state, meaning $I_L = 2\ A$, $V_L = 0$. The inductor has a strong magnetic filed but the filed is not changing.

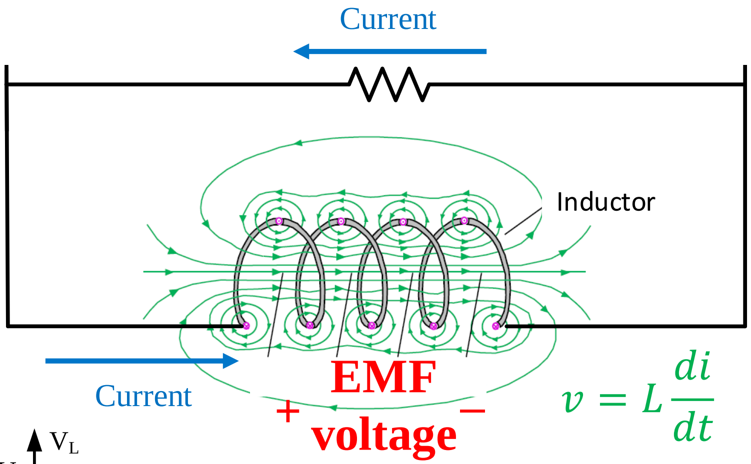
At t = 0 s, we remove the source and connect the inductor with a resistor. We lost the power source so the current will disappear, but not at a instant. At this instant, current is still 2 A.

With the time goes on, the current decreases, so as the magnetic field decreases. But the changing magnetic field will generate an EMF (voltage) to oppose this change. So the inductor will generate a voltage V_L serving as a temporary source.

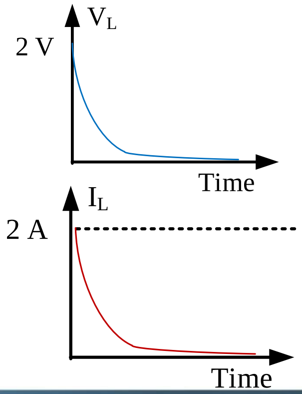
Since the current keeps decreasing, the magnetic field gets weaker, so as the voltage $V_{L.}$

After a short period of time, the magnetic field becomes 0, V_L becomes 0 and current also becomes zero. We say all the energy stored in the magnetic field is released to the resistor and a steady state is reached.

This process is also called "inductor discharging".



$$v = L\frac{di}{dt}$$



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Topic 6-4: Energy stored in inductor

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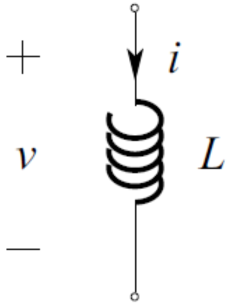
6-4 ENERGY STORED IN INDUCTOR

As long as there is current flowing through inductor, there is magnetic field streaming through the inductor. Energy is stored in the magnetic field. But how much?

According to the definition of power, $P = vi$



According to the V-I characteristic of inductor, $v = L\frac{di}{dt}$

Substituting *v* into P, we have $P = Li\frac{di}{dt}$

According to the definition of energy and power, the energy change in a period of time can be calculated by taking the integral of power in this time period:

$$W_L(t) - W_L(t_0) = \int_{t_0}^{t} P\,dt \;= L\int_{t_0}^{t} i\frac{di}{dt}dt \;= L\int_{i(t_0)}^{i(t)} i\,di \;= \frac{L}{2}[i^2(t) - i^2(t_0)]$$

We can always find a moment when the current passing through the inductor is 0. By setting this moment as t_0, then we $i(t_0) = 0$. No current means no magnetic field thus no energy in the inductor, i.e., $W_L(t_0) = 0$.

Therefore, we have energy stored in inductor as

$$\boxed{W_L(t) = \frac{L}{2}i^2(t)}$$

Energy stored in an inductor only depends on the inductance and the current at this time.

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Topic 6-5: Equivalent inductance

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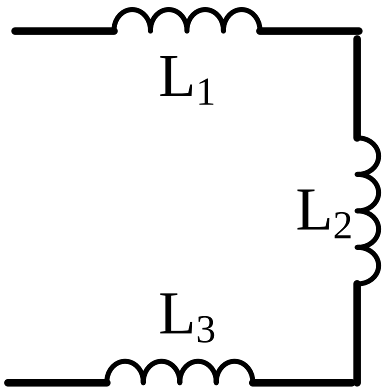
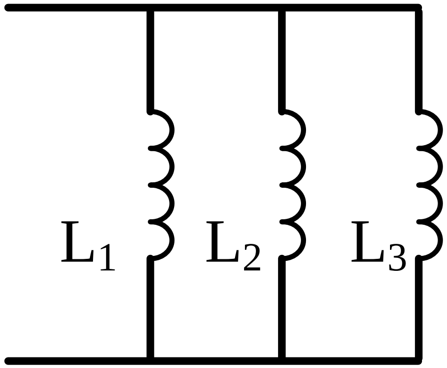
6-5 EQUIVALENT INDUCTANCE

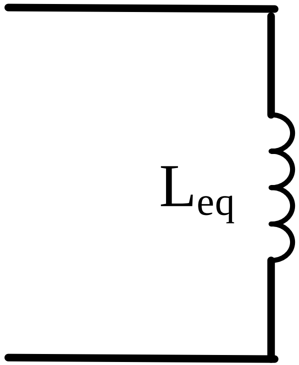
A combination of several inductors can be equivalent to one inductor. The method of calculating equivalent inductance is the same as calculating equivalent resistance.



Inductors connected in series



Inductors connected in parallel



Equivalent inductance

$$L_{eq} = L_1 + L_2 + L_3$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$