48510 LEC 7 – TRANSIENT ANALYSIS FOR INDUCTOR AND CAPACITOR

Topic 7-1-0: Review of inductor charging

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7-1-0 REVIEW OF INDUCTOR CHARGING

Review of inductor charging process

Before the switch is turned on (t < 0 s), $I_L = 0$, $V_L = 0$.

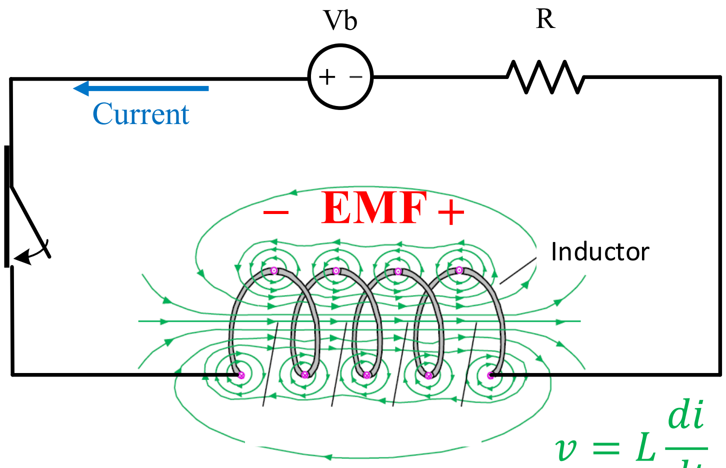
Once the switch is turned on (t = 0 s), I_L starts to increase, magnetic field starts to increase, which generates a EMF (voltage) to oppose this change, since the current is still very small at this instant, the voltage on the resistor is also very small. All the source voltage Vs is used to overcome the resist from inductor, so at this instant $V_L = Vb$, $I_L = 0$.

With time goes on, I_L keeps increasing, the voltage applied on the resistor keeps increasing, thus the voltage from the source left for the inductor decreases (V_L *decreases*).

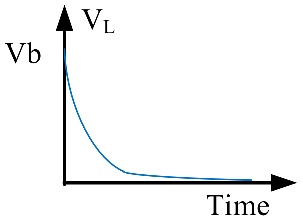
Finally, when current is large enough ($IR = Vb$), all the source voltage will be applied on the resistor, and there is no voltage left for inductor. Thus current will not change anymore, and the circuit reaches a steady state where $V_L = 0$, $V_R = Vb$, $I = Vb/R$.

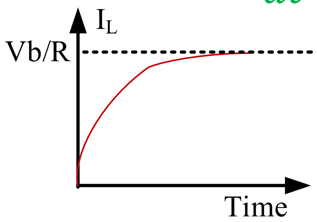
This process is also called "inductor charging".

How long it takes for the inductor to get fully charged?



$$v = L\frac{di}{dt}$$





48510 LEC 7 – TRANSIENT ANALYSIS FOR INDUCTOR AND CAPACITOR

Topic 7-1-1: Transient analysis for Inductor charging

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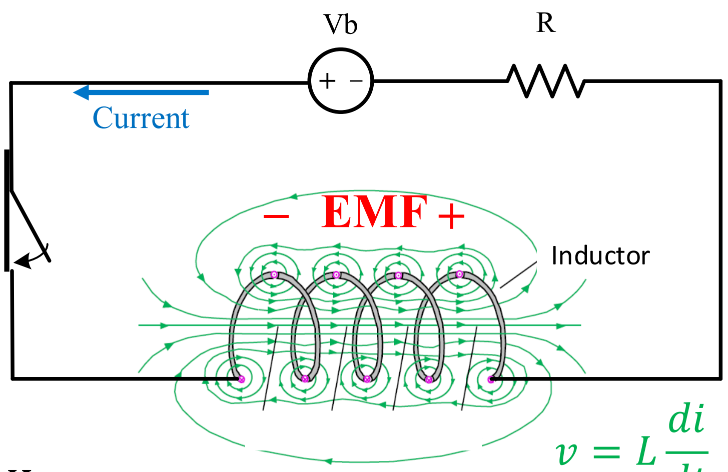
7-1-1 TRANSIENT ANALYSIS FOR INDUCTOR CHARGING

We know the voltage of the source is shared by the inductor and resistor.

$$V_b = V_R + V_L$$

$$V_b = iR + L\frac{di}{dt}$$

This is a first-order ordinary differential equation



$$v = L\frac{di}{dt}$$

The solution of this kind of equation for the variable is always in the following format

$$i = K_1 e^{st} + K_2$$

where K1, K2 and s are all unknown constants.

Substituting the standard form of *i* into the equation:

$$V_b = RK_1 e^{st} + RK_2 + LsK_1 e^{st} = (R + Ls)K_1 e^{st} + RK_2$$

$$\Rightarrow V_b - RK_2 = (R + Ls)K_1 e^{st}$$

Considering e^{st} must change with time (*s* can't be zero as the current changes with time),

The left side of the equation is a constant and the right side is changing with time, this could happen only with one premise:

$$\Rightarrow (R + Ls)K_1 = 0 \Rightarrow R + Ls = 0 \Rightarrow s = -\frac{R}{L}$$

$$V_b - RK_2 = 0 \Rightarrow K_2 = \frac{V_b}{R}$$

7-1-1 TRANSIENT ANALYSIS FOR INDUCTOR CHARGING

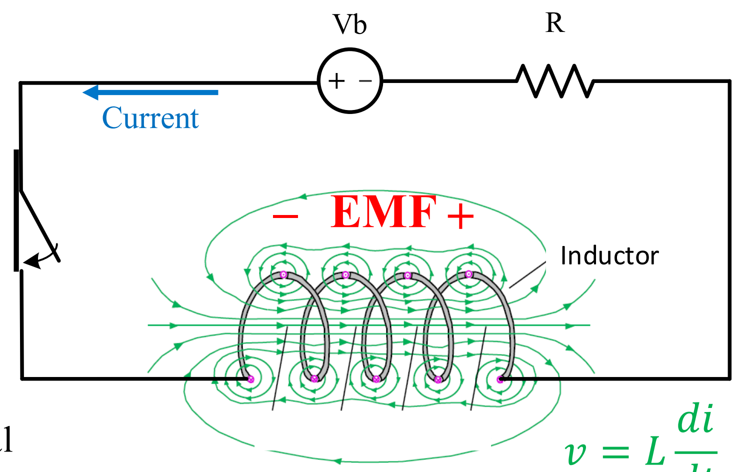
The solution of this kind of equation for the variable is always in the following format $i = K_1 e^{st} + K_2$

where K1, K2 and s are all unknown constants.

$$V_b = iR + L\frac{di}{dt} \Rightarrow V_b - RK_2 = (R + Ls)K_1 e^{st}$$

Considering e^{st} must change with time,

$$s = -\frac{R}{L} \qquad K_2 = \frac{V_b}{R}$$



$$v = L\frac{di}{dt}$$

We still need to find the value for K1, and this needs an additional equation. *To find this equation, we need to use a boundary condition.*

At $t = 0+$ (the instance when the switch just turned on),

$V_R = 0$ so $V_L = V_b$

($e^{st} = 1$ at $t = 0+$)

$$\Rightarrow V_L = LsK_1 e^{st} = V_b \Rightarrow LsK_1 = V_b \Rightarrow K_1 = \frac{V_b}{Ls}$$

$$\Rightarrow K_1 = -\frac{V_b}{R}$$

7-1-1 TRANSIENT ANALYSIS FOR INDUCTOR CHARGING

The solution of this kind of equation for the variable is always in the following format

$$i = K_1 e^{st} + K_2$$

where K1, K2 and s are all unknown constants.

$$s = -\frac{R}{L} \qquad K_2 = \frac{V_b}{R} \qquad K_1 = -\frac{V_b}{R}$$

$$\Rightarrow \quad i_L = -\frac{V_b}{R} e^{-t\frac{R}{L}} + \frac{V_b}{R}$$

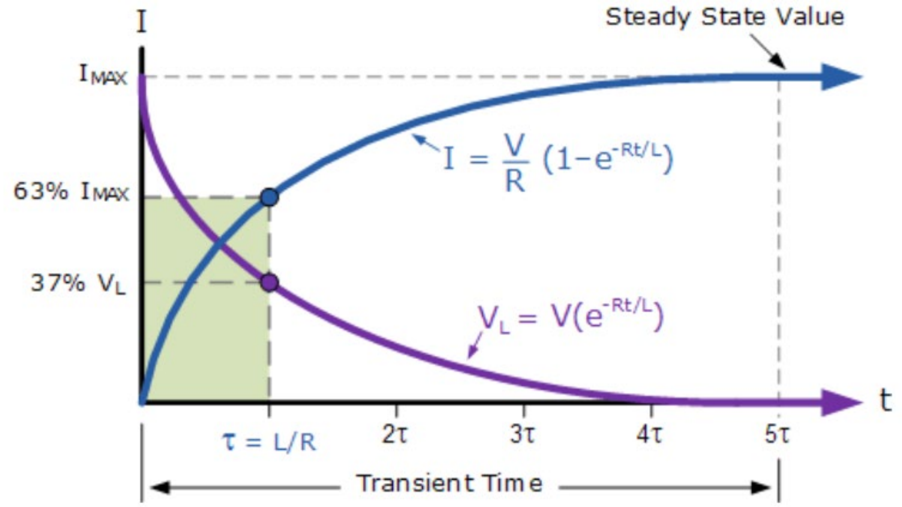
$$\Rightarrow \quad i_L = \frac{V_b}{R}(1 - e^{-t\frac{R}{L}}) = \frac{V_b}{R}(1 - e^{-\frac{t}{\tau}})$$

where $\tau = \frac{L}{R}$ is defined as the time constant.

Once we know the current, we can determine the voltage

$$v_L = L\frac{di}{dt} \quad \Rightarrow \quad v_L = -L\frac{V_b}{R}(-\frac{R}{L}e^{-t\frac{R}{L}})$$

$$\Rightarrow \quad v_L = V_b e^{-\frac{t}{\tau}}$$



At $t = \tau$, the current was reached to 63% of the maximum value and the voltage was reduced to 37% of the maximum value.

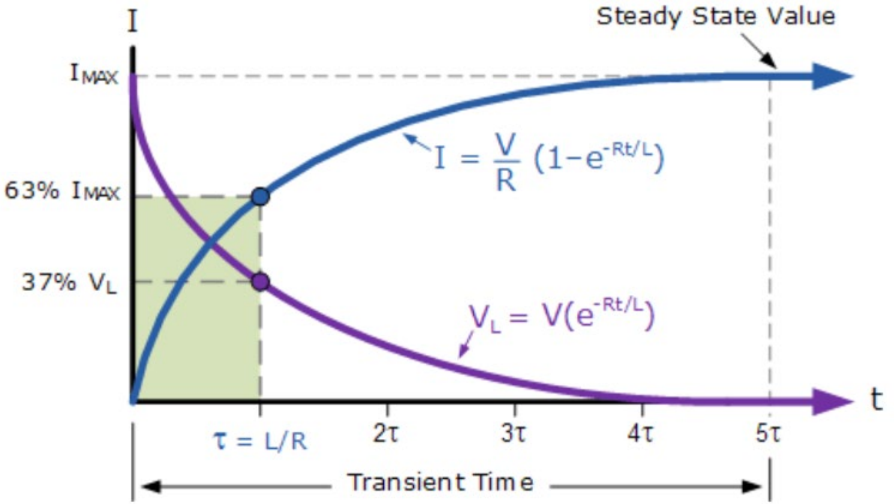
7-1-1 TRANSIENT ANALYSIS FOR INDUCTOR CHARGING

According to the fact that voltage of the source is shared by the inductor and resistor: $V_b = V_R + V_L$, we have derived the voltage and current change with time.

$$i_L = \frac{V_b}{R}(1 - e^{-\frac{t}{\tau}})$$

$$v_L = V_b e^{-\frac{t}{\tau}}$$

where $\tau = \frac{L}{R}$ is defined as the time constant.



The time required for the current flowing in the LR series circuit to reach its maximum steady state value is equivalent to about 5τ. Once the maximum steady state is reached, the inductance of the coil has reduced to zero acting more like a short circuit.

The transient time of any inductive circuit is determined by the relationship between the inductance and the resistance, but is not related to the source voltage.

The larger the inductance or the smaller the resistance, the slower the charging process for the inductor.

48510 LEC 7 – TRANSIENT ANALYSIS FOR INDUCTOR AND CAPACITOR

Topic 7-1-2: Transient analysis for Inductor discharging

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7-1-2 TRANSIENT ANALYSIS OF INDUCTOR DISCHARGING

Review of inductor discharging process

Considering we have an inductor fully charged to I_{Lmax}. At $t = 0$ s, we remove the source and connect the inductor with a resistor. We lost the power source so the current will disappear, but not at a instant. At this instant, current is still I_{Lmax}.

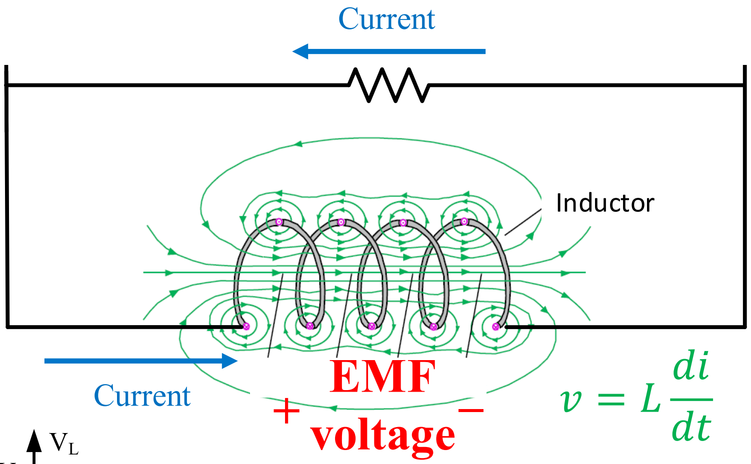
With the time goes on, the current decreases, so as the magnetic field decreases. But the changing magnetic field will generate an EMF (voltage) to oppose this change. So the inductor will generate a voltage V_L serving as a temporary source.

Since the current keeps decreasing, the magnetic field gets weaker, so as the voltage V_L.

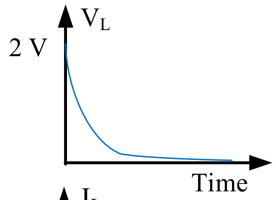
After a short period of time, the magnetic field becomes 0, V_L becomes 0 and current also becomes zero. We say all the energy stored in the magnetic field is released on the resistor and a steady state is reached.

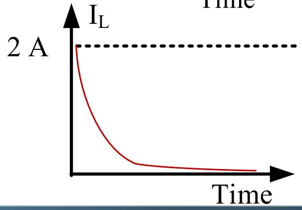
This process is also called "inductor discharging".

How long it takes for the inductor to get fully charged?



$$v = L\frac{di}{dt}$$





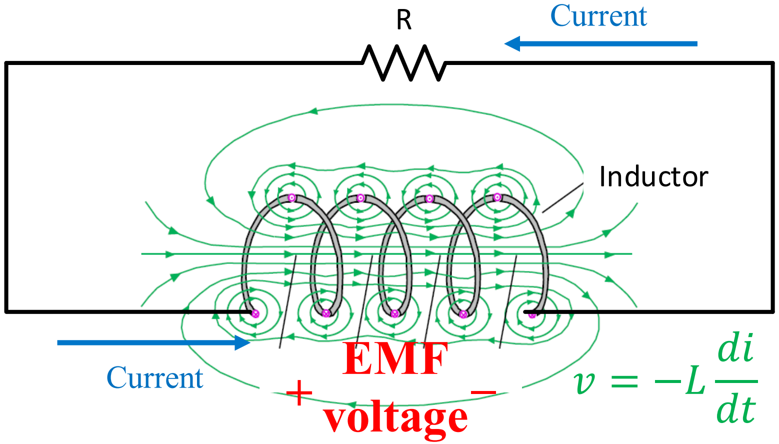
7-1-2 TRANSIENT ANALYSIS OF INDUCTOR DISCHARGING

We know the voltage of the source is shared by the inductor and resistor.

$$V_L = V_R$$

$$-L\frac{di}{dt} = iR$$

This is a first-order ordinary differential equation



During the discharging process, we need to include the minus sign in the voltage generated by inductor.

The solution of this kind of equation for the variable is always in the following format

$$i = K_1e^{st} + K_2$$

where K1, K2 and s are all unknown constants.

Substituting the standard form of *i* into the equation:

$$-LsK_1e^{st} = RK_1e^{st} + RK_2$$

$$\Rightarrow -(Ls + R)K_1e^{st} = RK_2$$

Considering e^{st} must change with time (*s* can't be zero as the current changes with time),

The left side of the equation is changing with time and the right side is a constant, this could happen only with one premise:

K1 can't be 0

$$\Rightarrow \begin{cases} (Ls + R)K_1 = 0 \Rightarrow Ls + R = 0 \Rightarrow s = -\frac{R}{L} \\ RK_2 = 0 \Rightarrow K_2 = 0 \end{cases}$$

7-1-2 TRANSIENT ANALYSIS OF INDUCTOR DISCHARGING

The solution of this kind of equation for the variable is always in the following format

$$i = K_1 e^{st} + K_2$$

where K1, K2 and s are all unknown constants.

$$V_L = V_R \Rightarrow -(Ls + R)K_1 e^{st} = RK_2$$

Considering e^{st} must change with time,

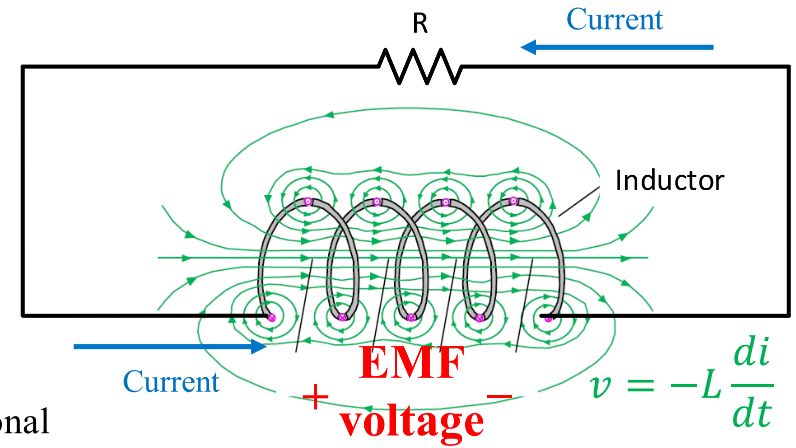
$$s = -\frac{R}{L} \qquad K_2 = 0$$

We still need to find the value for K1, and this needs an additional equation. *To find this equation, we need to use a boundary condition.*

At $t = 0+$ (the instant when the discharging starts),

$i_L = i_{Lmax}$ *(Note that i_{Lmax} is known parameter for a charged inductor as we describe the inductor using "the inductor is previously charged to a current of i_{Lmax}")*

$$\Rightarrow i_L = K_1 e^{st} = i_{Lmax} \Rightarrow K_1 = i_{Lmax}$$



7-1-2 TRANSIENT ANALYSIS OF INDUCTOR DISCHARGING

The solution of this kind of equation for the variable is always in the following format

$$i = K_1 e^{st} + K_2$$

where K1, K2 and s are all unknown constants.

$$s = -\frac{R}{L} \qquad K_2 = 0 \qquad K_1 = i_{Lmax}$$

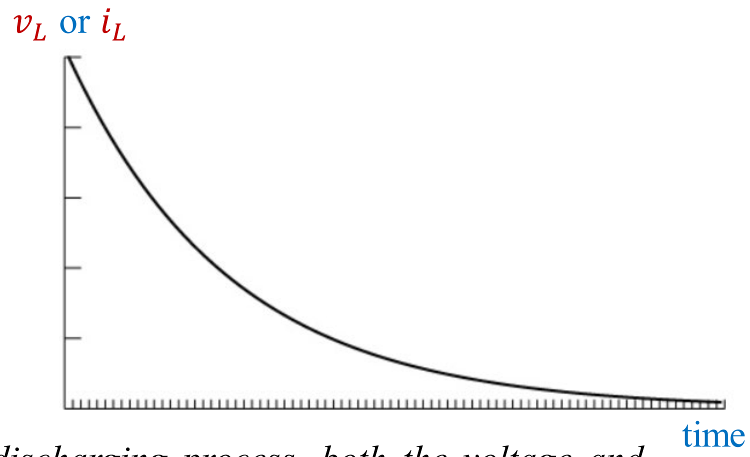
$$\Rightarrow \quad i_L = i_{Lmax} e^{-\frac{t}{\tau}}$$

where $\tau = \frac{L}{R}$ *is defined as the time constant.*

Once we know the current, we can determine the voltage

$$v_L = -L\frac{di}{dt} = -Li_{Lmax}\left(-\frac{1}{\tau}e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \quad v_L = i_{Lmax} R e^{-\frac{t}{\tau}}$$



In discharging process, both the voltage and current follow this trend with a different maximum value.

At $t = \tau$*, both the current and voltage was reduced to 37% of their maximum value.*

At $t = 5\tau$*, both the current and voltage was reduced to 0.*

48510 LEC 7 – TRANSIENT ANALYSIS FOR INDUCTOR AND CAPACITOR

Topic 7-2-1: Inductor response with DC and AC source

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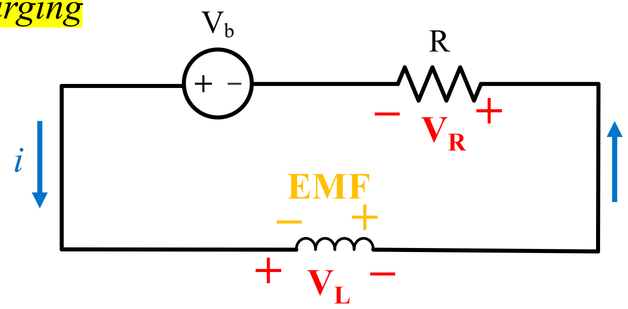
7-2-1 INDUCTOR RESPONSE WITH DC AND AC SOURCE

Charging



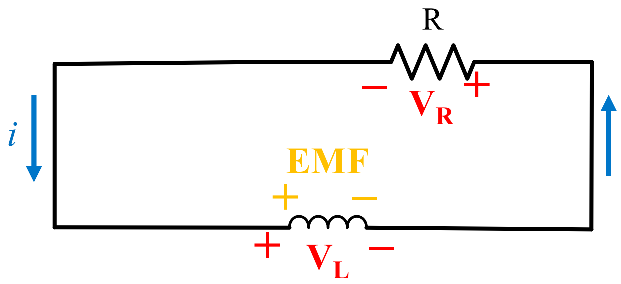
The EMF voltage generated by the inductor is negative, the source needs to split a part of its voltage to overcome this EMF.

$$v_L = -v_{emf} = L\frac{di}{dt}$$

$$i_L = \frac{V_b}{R}(1 - e^{-\frac{t}{\tau}})$$

$$v_{emf} = -v_L = -V_b e^{-\frac{t}{\tau}}$$

Discharging



The EMF voltage generated by the inductor is positive as the current is decreasing and it serves as the source of the circuit.

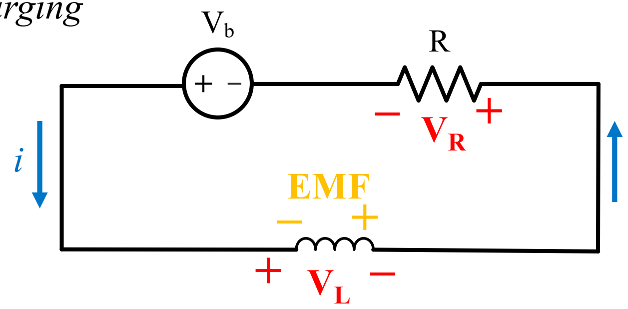
$$v_L = v_{emf} = -L\frac{di}{dt}$$

$$i_L = i_{Lmax} e^{-\frac{t}{\tau}}$$

$$v_{emf} = v_L = i_{Lmax} R e^{-\frac{t}{\tau}}$$

7-2-1 INDUCTOR RESPONSE WITH DC AND AC SOURCE

Charging

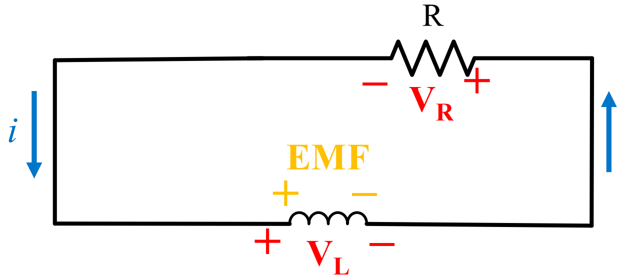


$$i_L = \frac{V_b}{R}(1 - e^{-\frac{t}{\tau}})$$

$$v_{emf} = -v_L = -V_b e^{-\frac{t}{\tau}}$$

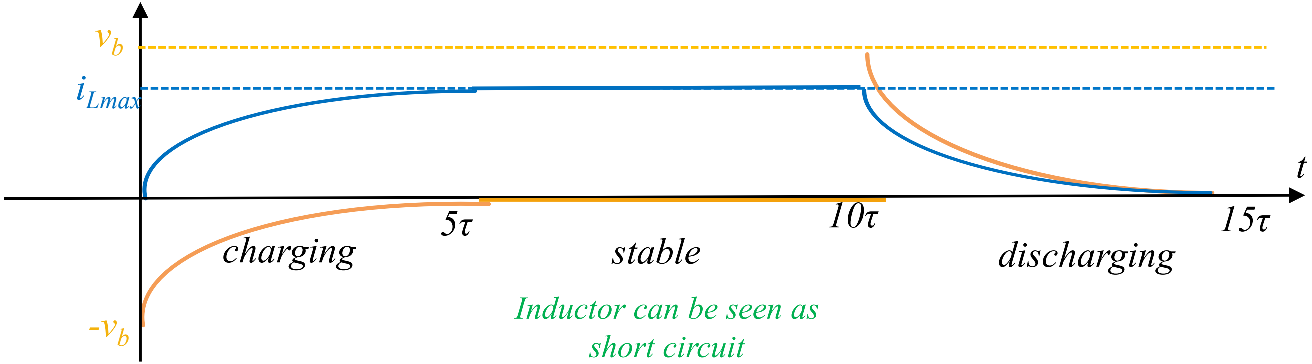
Considering a case when we charge and discharge an inductor with DC source.

Discharging



$$i_L = i_{Lmax} e^{-\frac{t}{\tau}}$$

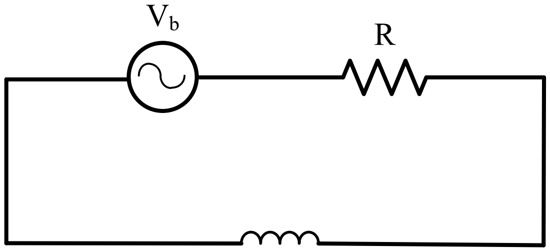
$$v_{emf} = v_L = i_{Lmax} R e^{-\frac{t}{\tau}}$$

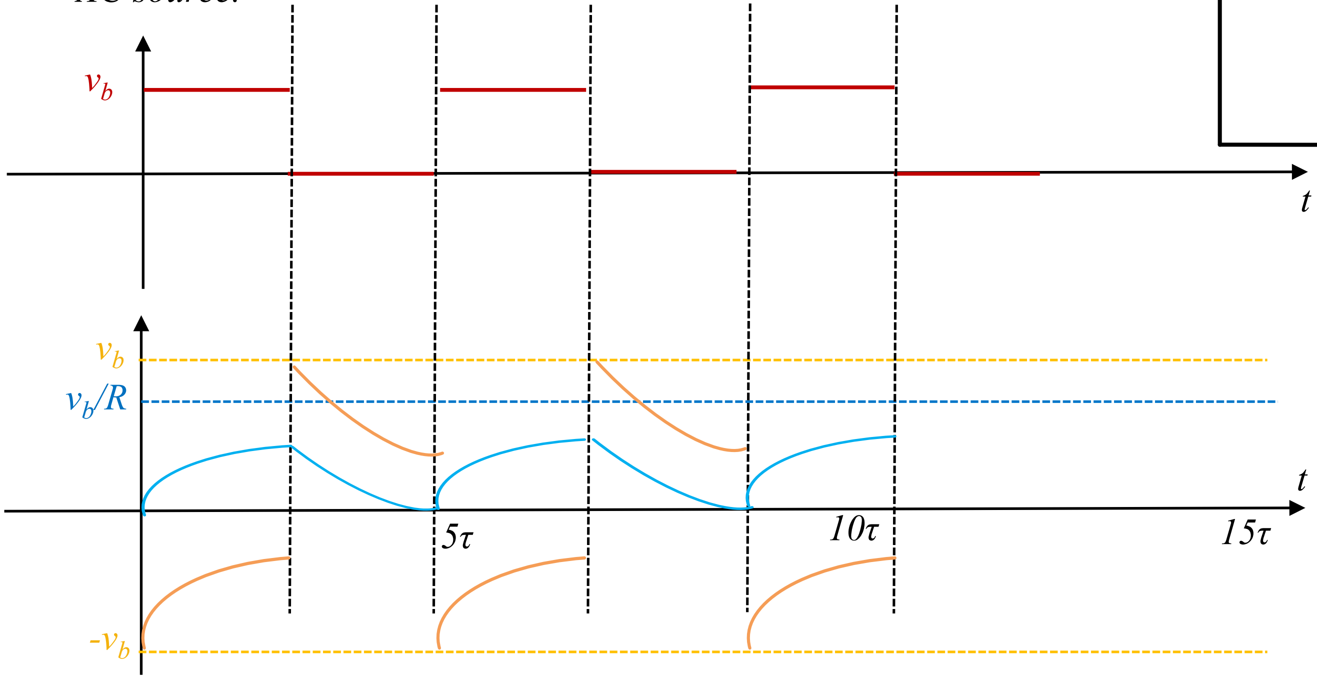


$$\tau = \frac{L}{R} \text{ (time constant)}$$

7-2-1 INDUCTOR RESPONSE WITH DC AND AC SOURCE

Considering the case when the inductor is connected to an AC source.





If the voltage source changes fast, before the inductor got fully charged, it already started discharging. Therefore, the maximum current it can have is smaller than the DC case. And the voltage will never reduce to zero. The source will always need to split some voltage on the inductor.

Considering an extreme case, if the source voltage changes extremely fast, the current can be charged to only a very small value then it starts to decrease. In this case, the current is close to 0 all the time. And the inductor can be seen as open circuit.

THINK. CHANGE. DO

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48510 LEC 7 – TRANSIENT ANALYSIS FOR INDUCTOR AND CAPACITOR

Topic 7-3-1: Transient analysis for capacitor charging

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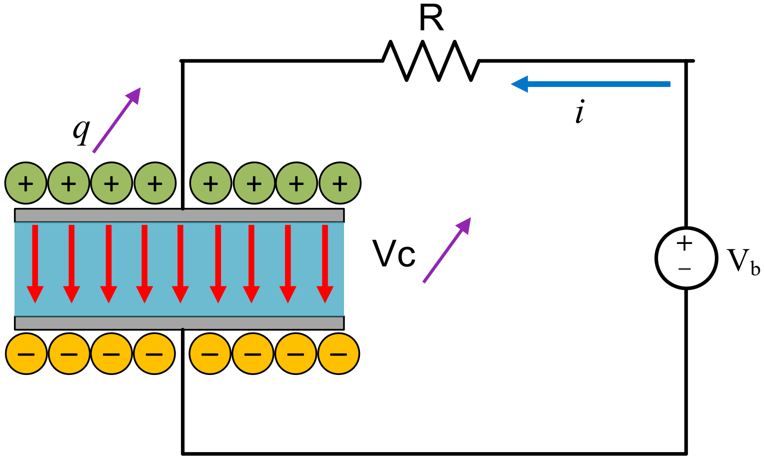
7-3-1 TRANSIENT ANALYSIS FOR CAPACITOR CHARGING

We know the voltage of the source is shared by the inductor and resistor.

$$V_b = v_c + v_R$$

$$V_b = v_c + RC\frac{dv_c}{dt}$$

This is a first-order ordinary differential equation



The solution of this kind of equation for the variable is always in the following format

$$v_c = K_1 e^{st} + K_2$$

where K1, K2 and s are all unknown constants.

Substituting the standard form of v_c into the equation:

$$V_b = K_1 e^{st} + K_2 + RCsK_1 e^{st} = (RCs + 1)K_1 e^{st} + K_2$$

$$i = C\frac{dv_c}{dt}$$

$$\Rightarrow V_b - K_2 = (RCs + 1)K_1 e^{st}$$

Considering e^{st} must change with time (s can't be zero as the voltage changes with time),

The left side of the equation is a constant and the right side is changing with time, this could happen only with one premise:

$$\Rightarrow \begin{cases} (RCs + 1)K_1 = 0 \Rightarrow RCs + 1 = 0 \Rightarrow s = -\frac{1}{RC} \\ V_b - K_2 = 0 \Rightarrow K_2 = V_b \end{cases}$$

7-3-1 TRANSIENT ANALYSIS FOR CAPACITOR CHARGING

The solution of this kind of equation for the variable is always in the following format

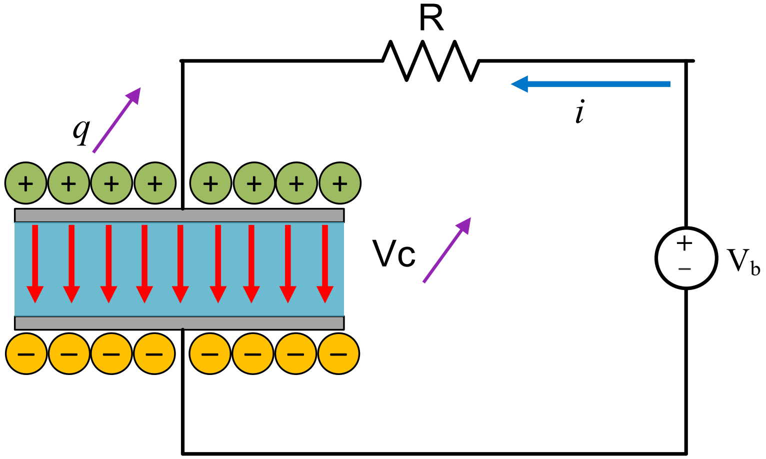
$$v_c = K_1 e^{st} + K_2$$

where K1, K2 and s are all unknown constants.

$$V_b = v_c + v_R \Rightarrow V_b - K_2 = (RCs + 1)K_1 e^{st}$$

Considering e^{st} must change with time,

$$s = -\frac{1}{RC} \quad K_2 = V_b$$



$$i = C\frac{dv_c}{dt}$$

We still need to find the value for K1, and this needs an additional equation. *To find this equation, we need to use a boundary condition.*

At *t = 0+* (the voltage on the capacitor is zero as there is no charge),

$$v_c(t = 0+) = K_1 e^{st} + K_2 = 0$$

$$\Rightarrow K_1 + V_b = 0$$

$$\Rightarrow K_1 = -V_b$$

$$\Rightarrow v_c = -V_b e^{-\frac{t}{RC}} + V_b$$

$$i_c = C\frac{dv_c}{dt} = \frac{V_b}{R} e^{-\frac{t}{RC}}$$

define $\tau = RC$ as the time constant.

$$\Rightarrow v_c = V_b(1 - e^{-\frac{t}{\tau}})$$

$$i_c = \frac{V_b}{R} e^{-\frac{t}{\tau}}$$

7-3-1 TRANSIENT ANALYSIS FOR CAPACITOR CHARGING

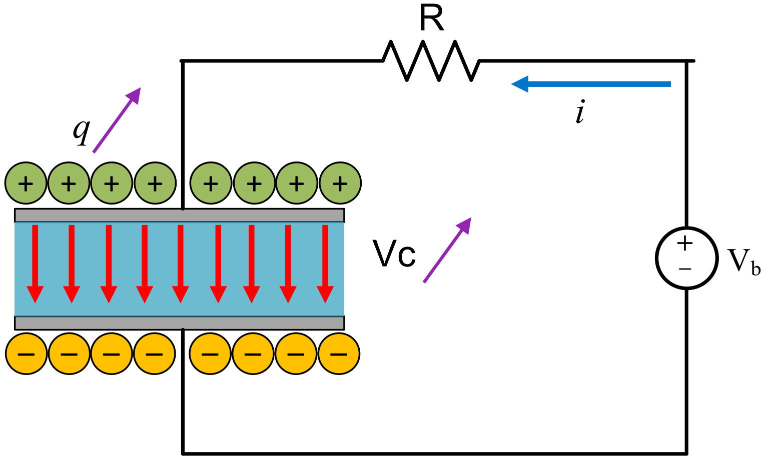
Capacitor charging

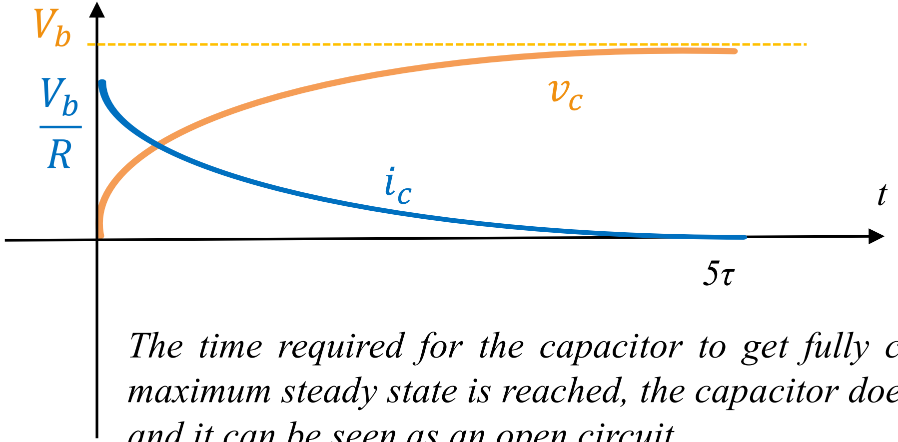
$$v_c = V_b(1 - e^{-\frac{t}{\tau}})$$

$$i_c = \frac{V_b}{R} e^{-\frac{t}{\tau}}$$

where $\tau = RC$ is the time constant.

The transient time of any capacitive circuit is determined by RC, but is not related to the source voltage. The larger the capacitance or the resistance, the slower the charging process.





At $t = \tau$, the current was reduced to 37% of the maximum value and the voltage was increased to 63% of the maximum value.

The time required for the capacitor to get fully charged is about 5τ. Once the maximum steady state is reached, the capacitor does not allow DC current to flow and it can be seen as an open circuit.

THINK.
CHANGE.
DO

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48510 LEC 7 – TRANSIENT ANALYSIS FOR INDUCTOR AND CAPACITOR

Topic 7-3-2: Transient analysis for capacitor discharging

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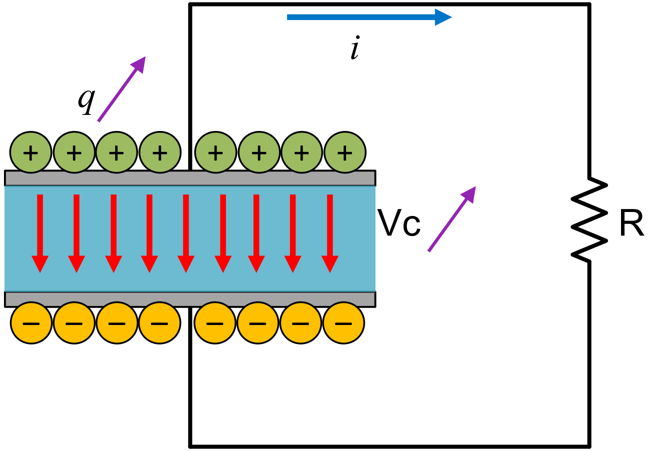
7-3-2 TRANSIENT ANALYSIS OF CAPACITOR DISCHARGING

During the capacitor discharging, it serves as a voltage source.

$$v_C = v_R$$

$$v_C = RC\frac{dv_C}{dt}$$

This is a first-order ordinary differential equation



$$i = C\frac{dv_c}{dt}$$

The solution of this kind of equation for the variable is always in the following format

$$v_C = K_1e^{st} + K_2$$

where K1, K2 and s are all unknown constants.

Substituting the standard form of *i* into the equation:

$$K_1e^{st} + K_2 = RCK_1se^{st}$$

$$\Rightarrow (RCs - 1)K_1e^{st} = K_2$$

Considering e^{st} must change with time (as the current changes with time),

The left side of the equation is changing with time and the right side is a constant, this could happen only with one premise:

K1 can't be 0

$$\Rightarrow (RCs - 1)K_1 = 0 \Rightarrow RCs - 1 = 0 \Rightarrow s = -\frac{1}{RC}$$

$$K_2 = 0$$

7-3-2 TRANSIENT ANALYSIS OF CAPACITOR DISCHARGING

The solution of this kind of equation for the variable is always in the following format

$$v_C = K_1 e^{st} + K_2$$

where K1, K2 and s are all unknown constants.

$$v_C = v_R \Rightarrow (RCs - 1)K_1 e^{st} = K_2$$

Considering e^{st} must change with time,

$$s = -\frac{1}{RC} \quad K_2 = 0$$

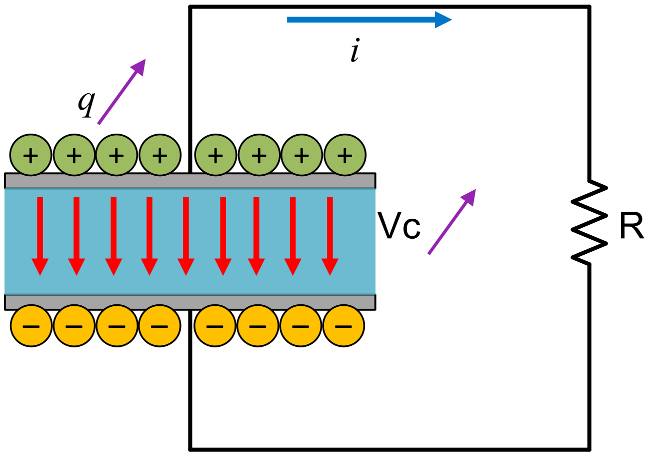
We still need to find the value for K1, and this needs an additional equation. *To find this equation, we need to use a boundary condition.*

At $t = 0+$ (the instant when the discharging starts),

$v_c = v_{Cmax}$ *(Note that v_{Cmax} is known parameter which is the initial voltage across the capacitor when the discharging starts.)*

$$\Rightarrow v_C(t = 0) = K_1 e^{st} = v_{Cmax}$$

$$\Rightarrow K_1 = v_{Cmax}$$



$$i = C\frac{dv_c}{dt}$$

7-3-2 TRANSIENT ANALYSIS OF CAPACITOR DISCHARGING

The solution of this kind of equation for the variable is always in the following format

$$v_C = K_1 e^{st} + K_2$$

where K1, K2 and s are all unknown constants.

$$s = -\frac{1}{RC} \quad K_2 = 0 \quad K_1 = v_{Cmax}$$

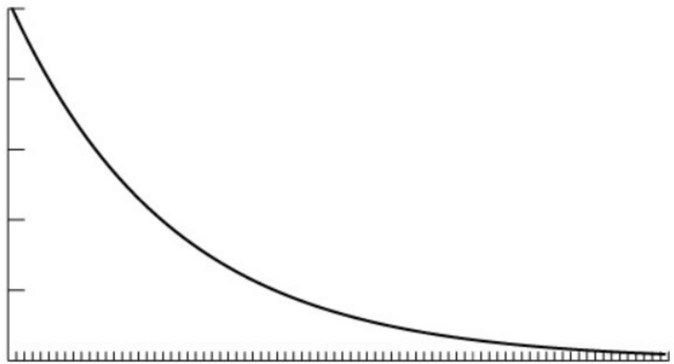
$$\Rightarrow v_C = v_{Cmax} e^{-\frac{t}{\tau}}$$

where $\tau = \frac{1}{RC}$ is defined as the time constant.

Once we know the voltage, we can determine the current

$$i_C = C\frac{dv}{dt} = -Cv_{Cmax}(-\frac{1}{\tau}e^{-\frac{t}{\tau}})$$

$$\Rightarrow i_C = \frac{v_{Cmax}}{R}e^{-\frac{t}{\tau}}$$



In discharging process, both the voltage and current follow this trend with a different maximum value.

At $t = \tau$, both the current and voltage was reduced to 37% of their maximum value.

At $t = 5\tau$, both the current and voltage was reduced to 0.

48510 LEC 7 – TRANSIENT ANALYSIS FOR INDUCTOR AND CAPACITOR

Topic 7-4-1: Capacitor response with DC and AC source

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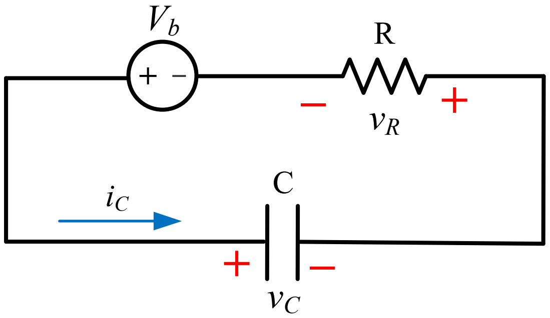
7-4-1 CAPACITOR RESPONSE WITH DC AND AC SOURCE

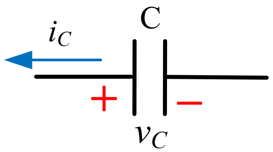
Charging



$$i_c = \frac{V_b}{R}e^{-\frac{t}{\tau}}$$

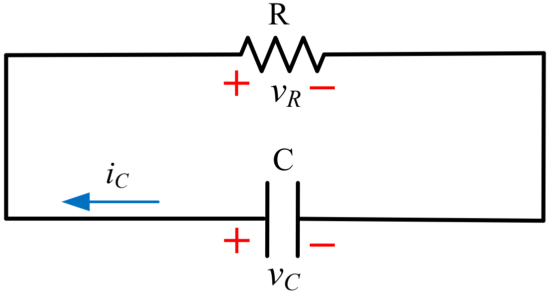
$$v_c = V_b(1 - e^{-\frac{t}{\tau}})$$

Open



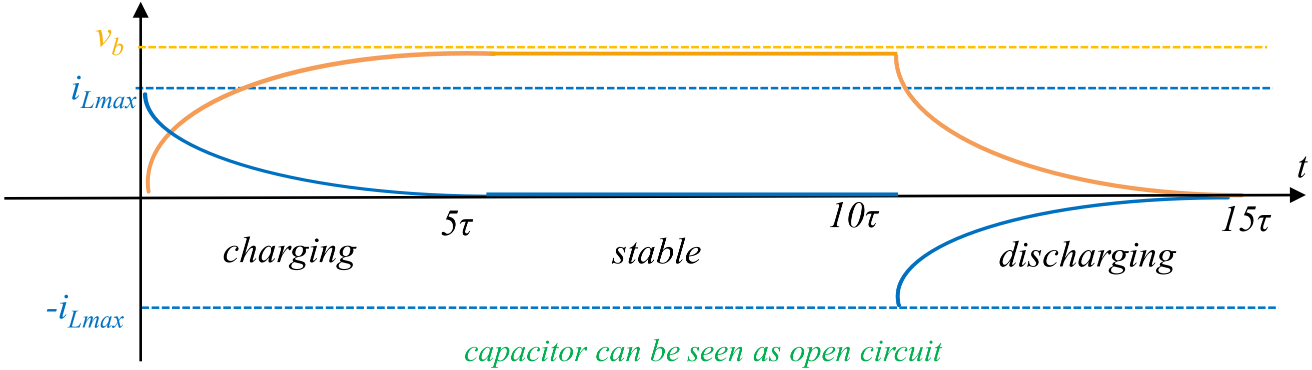
Considering a case when we charge and discharge a capacitor with DC source.

Discharging



$$i_C = -\frac{v_{Cmax}}{R}e^{-\frac{t}{\tau}}$$

$$v_C = v_{Cmax}e^{-\frac{t}{\tau}}$$

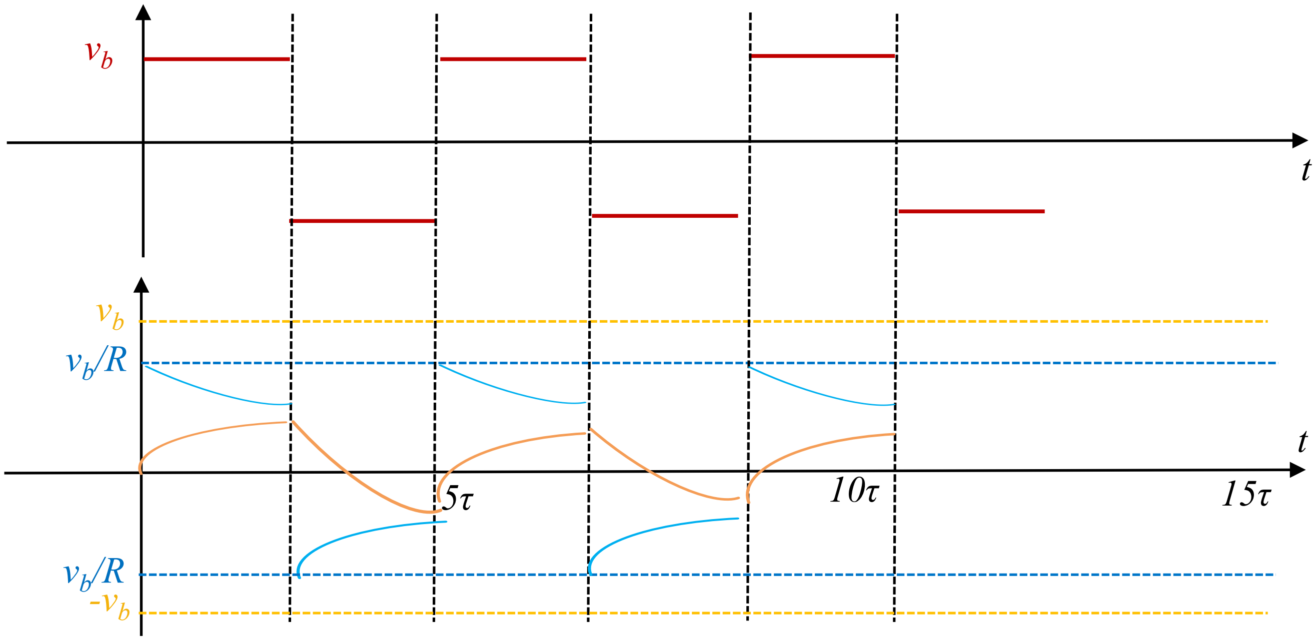


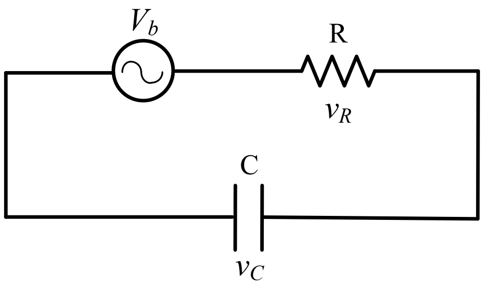
$$\tau = \frac{1}{RC} \text{ (time constant)}$$

capacitor can be seen as open circuit

7-4-1 CAPACITOR RESPONSE WITH DC AND AC SOURCE

Considering the case when the capacitor is connected to an AC source.





If the voltage source changes fast, before the capacitor got fully charged, it already started discharging and charging in the other direction. Therefore, the maximum voltage it can have is smaller than the Vb. And the current will never reduce to zero as it keeps charging and discharging.

Blue curves represent current, yellow curves represent voltage

Considering an extreme case, if the source voltage changes extremely fast, before any charges can be accumulated on the capacitor, then it starts to decrease, therefore the voltage on the capacitor will keep to a very small value (nearly zero), and the current will always be quite large. It behaves like a short circuit.