48510 LEC 5 – CAPACITOR

Topic 5-1: Introduction to capacitor

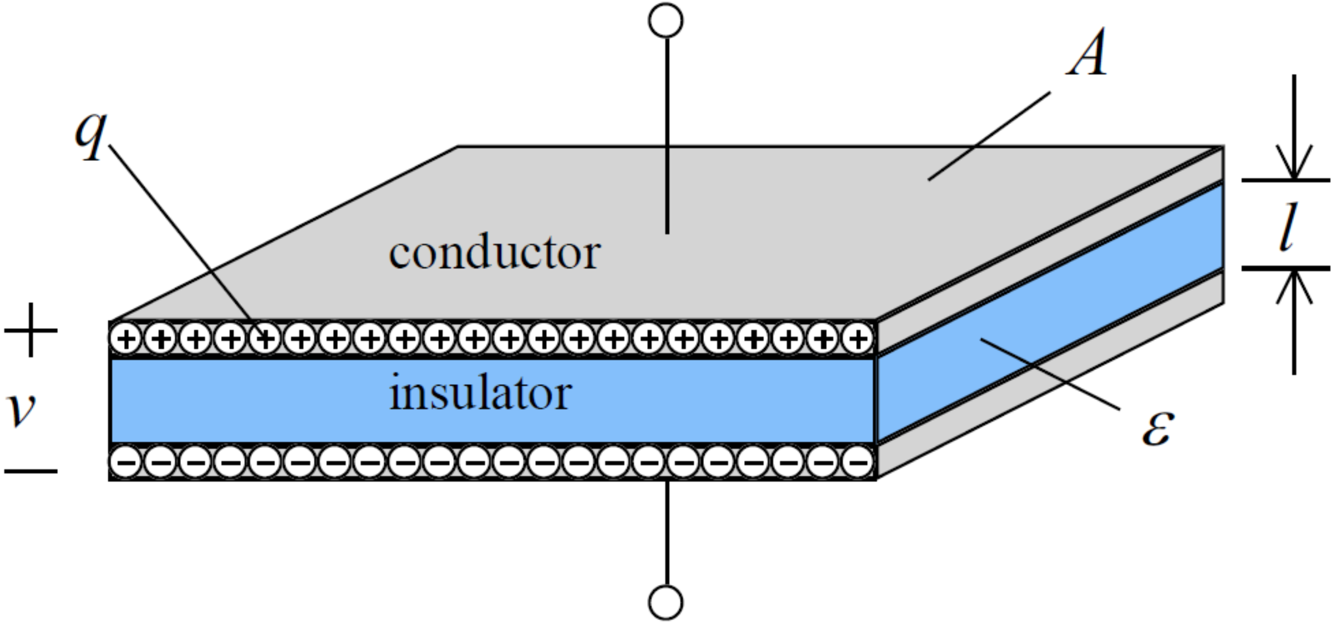
DR CAN DING

Lecturer

can.ding@uts.edu.au

5-1 INTRODUCTION TO CAPACITOR

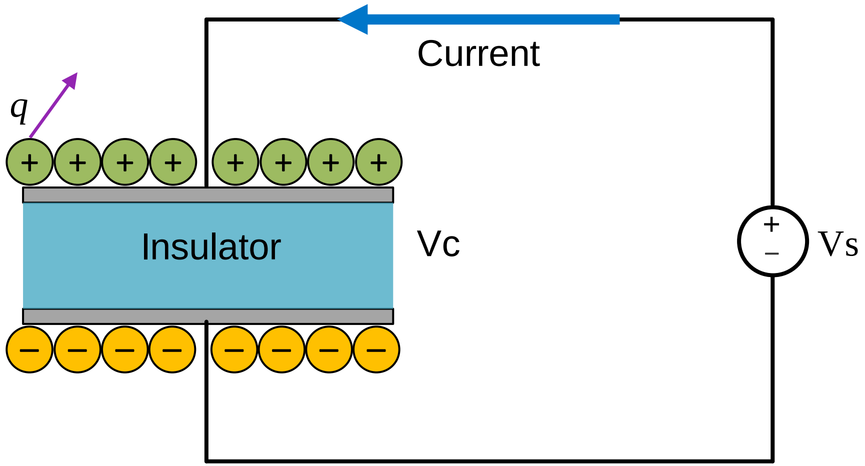
The most widely used capacitor is the parallel plate capacitor formed by two conductive plates separated by an insulator layer:



5-1 INTRODUCTION TO CAPACITOR

When connecting a capacitor to a voltage source, positive and negative charges will accumulated on the two conducting plates of the capacitor. This process is defined as charging the capacitor.

The charges on the plates cannot increase forever, when it will stop?



5-1 INTRODUCTION TO CAPACITOR

When connecting a capacitor to a voltage source, positive and negative charges will accumulated on the two conducting plates of the capacitor. This process is defined as charging the capacitor.

With time goes on, q increases

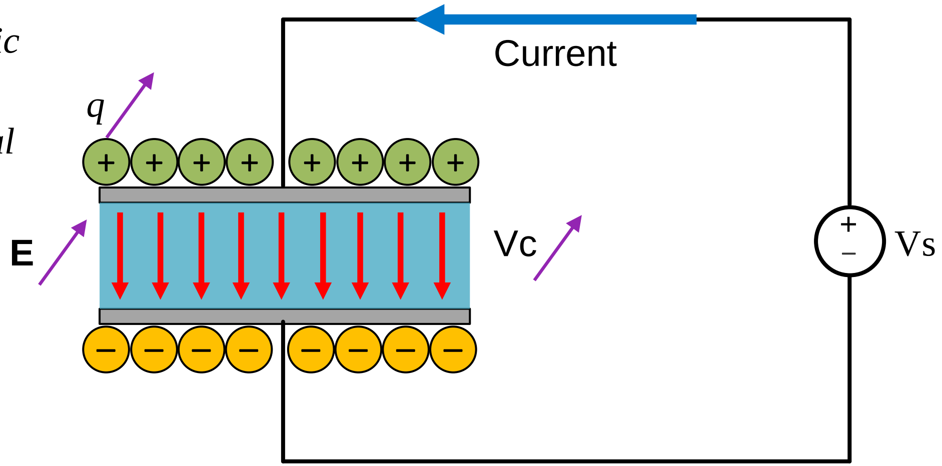
The charges on the two plates generates electric field ***E****. q increases,* ***E*** *increases*

Along the electric field line, there is a potential change (voltage difference) of Vc.

With ***E*** *increases, Vc increases*

When Vc = Vs, it reaches a balance

After balance is reached, current = 0, and the charges on the plates reach the maximum value of Q.



Ideally, the amount of charge Q deposited on the plates is proportional to the voltage V impressed across them.

We define a constant called the capacitance, C, of the structure by the linear relationship:

$$Q = CV \qquad C = \frac{Q}{V}$$

Unit of capacitance is F (farad)

5-1 INTRODUCTION TO CAPACITOR

We define a constant called the capacitance, C, of the structure by the linear relationship:

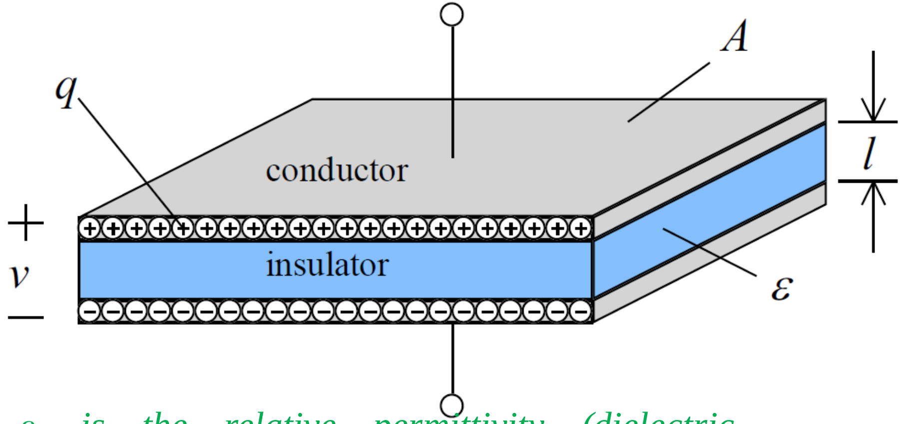
$$Q = CV, C = \frac{Q}{V}$$

It should be noted that ***C is a purely geometric property****, and depends only on the conductor arrangements and the materials used in the construction.*

Specifically, the capacitance between two large, closely-spaced, parallel conducting plates is

$$C = \frac{\varepsilon_r \varepsilon_0 A}{l} = \frac{\varepsilon A}{l}$$ *Unit is F (farad)*

where A is the area of either of the two parallel plates, l is the distance between them, ε is the permittivity of the media.



ε_r is the relative permittivity (dielectric constant) of a material and ε_0 is the permittivity of the vacuum.

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m (Farad per meter)

For example, the relative permittivity of paper is about 1.4.

48510 LEC 5 – CAPACITOR

Topic 5-2-1: The V-I characteristic of capacitor

DR CAN DING

Lecturer

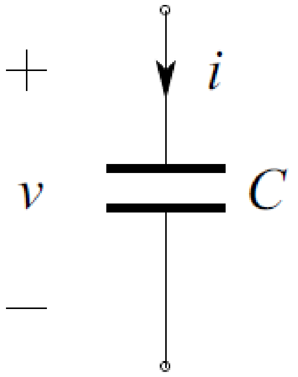
can.ding@uts.edu.au

5-2-1 V-I CHARACTERISTIC OF CAPACITOR

What is the V-I characteristic of a capacitor? How the current flowing through the capacitor changes with the voltage and vice versa?

According to the definition of current, i.e., the ***net rate of flow of electric charge through a surface or into a control volume***:

$$i(t) = \frac{dq}{dt}$$



According to the definition of capacitance, the charge on the capacitor is determined by the voltage and capacitance:

$$Q = VC$$

Substitute Q into i, we have

$$i(t) = C\frac{dV(t)}{dt}$$

Giving the voltage across the capacitor, we can find the current.

As noticed from this equation, if the voltage across the capacitor is a constant, the current is 0. This is because the voltage is a constant means the amount of charges on the two plates does not change. No changes entering or leaving the capacitor means current is 0.

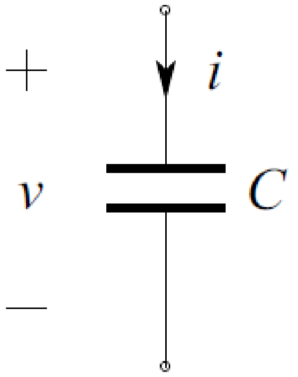
5-2-1 V-I CHARACTERISTIC OF CAPACITOR

What is the V-I characteristic of a capacitor? How the current flowing through the capacitor changes with the voltage and vice versa?

$$i(t) = C\frac{dV(t)}{dt}$$

Giving the voltage across the capacitor, we can find the current.

How the voltage changes with the current entering/leaving capacitor?



$$\Rightarrow \quad dv(t) = \frac{i(t)dt}{C}$$

$$\Rightarrow \quad \int_{t_0}^{t} dv(t) = \frac{1}{C}\int_{t_0}^{t} i(t)dt = V(t) - V(t_0)$$

$$\Rightarrow \quad V(t) = \frac{1}{C}\int_{t_0}^{t} i(t)dt + V(t_0)$$

Given the current entering/leaving the capacitance at a time period, we are able to find the voltage change on a capacitor during this period.

48510 LEC 5 – CAPACITOR

Topic 5-2-2: The V-I characteristic of capacitor example

DR CAN DING

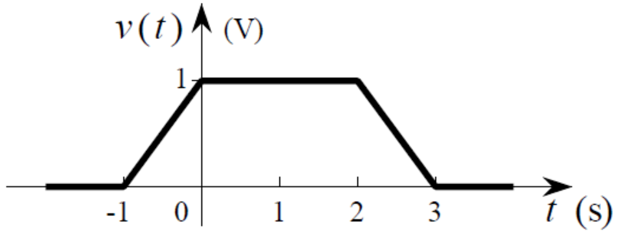
Lecturer

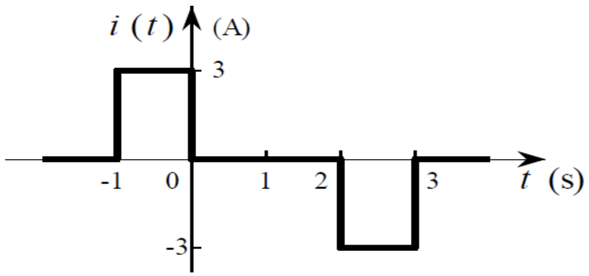
can.ding@uts.edu.au

5-2-2 V-I CHARACTERISTIC EXAMPLE

Given the voltage, find out the current

The voltage waveform across a 3 F capacitor is shown below:





V-I characteristic of a capacitor

$$i(t) = C\frac{dV(t)}{dt} \qquad V(t) = \frac{1}{C}\int_{t_0}^{t} i(t)dt + V(t_0)$$

When $t \leq -1, v(t)$ is constant $\quad i(t) = C\frac{dv(t)}{dt} = 0$

When $-1 < t < 0, \quad i(t) = C\frac{dv(t)}{dt} = 3 * 1 = 3A$

When $0 \leq t < 2, \quad i(t) = 0$

When $2 \leq t < 3, \quad i(t) = C\frac{dv(t)}{dt} = 3 * (-1) = -3A$

When $t \geq 3, \quad i(t) = 0$

5-2-2 V-I CHARACTERISTIC EXAMPLE

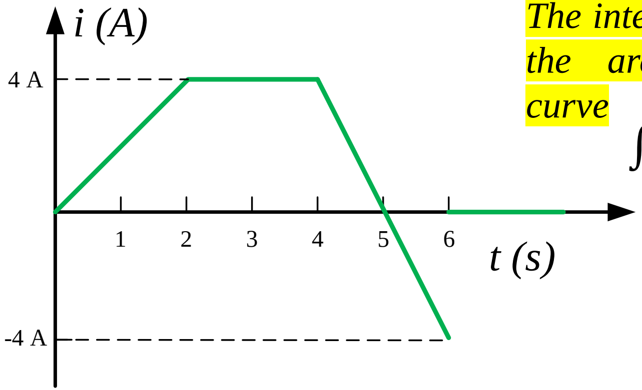
Given the current, find out the voltage

V-I characteristic of a capacitor

The current waveform through a 2 F capacitor is given as follows, if we know the voltage at $t = 0$ s is 2 V, find out the voltage at $t = 7$ s.

$$i(t) = C\frac{dV(t)}{dt}$$

$$V(t) = \frac{1}{C}\int_{t_0}^{t} i(t)dt + V(t_0)$$



The integral is actually the area under the curve

$$V(t=7) = \frac{1}{2}\int_0^7 i(t)dt + 2$$

$$\int_0^7 i(t)dt = \int_0^2 i(t)dt + \int_2^4 i(t)dt + \int_4^5 i(t)dt + \int_5^6 i(t)dt + \int_6^7 i(t)dt$$

$$2 \times 4 \div 2 = 4 \qquad 2 \times 4 = 8 \qquad 1 \times 4 \div 2 = 2 \qquad 1 \times (-4) \div 2 = -2 \qquad 0$$

$$V(t=7) = \frac{1}{2}\int_0^7 i(t)dt + 2$$
$$= \frac{1}{2}(4 + 8 + 2 - 2 + 0) + 2$$
$$= 8$$

UTS:

48510 LEC 5 – CAPACITOR

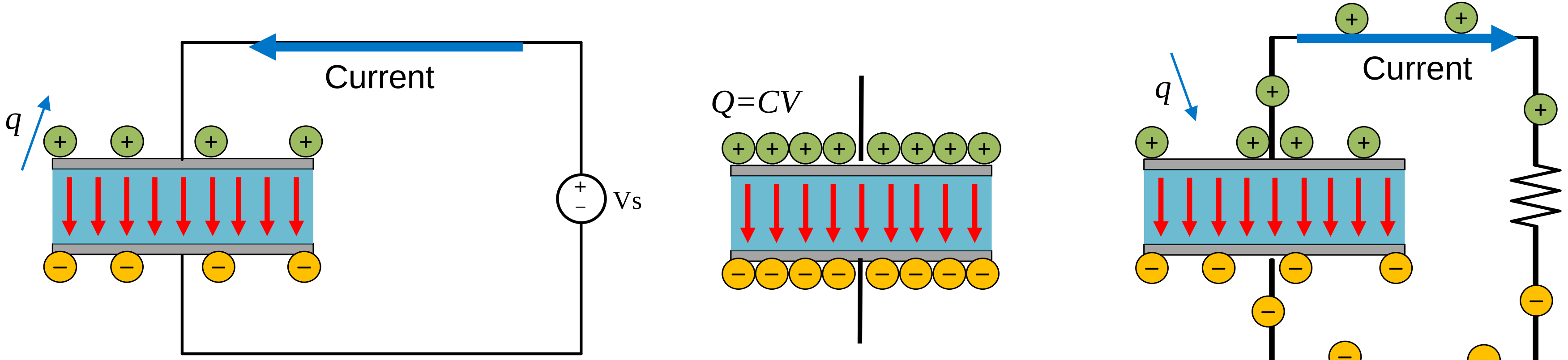
Topic 5-3: Energy stored in capacitor

DR CAN DING
Lecturer
can.ding@uts.edu.au

5-3 ENERGY STORED IN CAPACITOR

By connecting a capacitor to a DC (direct current) source, the current takes charges to capacitor and distributed on the two plates. **The distribution of the charges generates electric field and the energy is stored in the electric field.**

After the capacitor is fully charged, disconnect the capacitor with the source, the charges on the two plates have no place to go due to the open circuit. In this case, both the charges, the voltage and the electric field will not change and the energy is well stored in the capacitor.

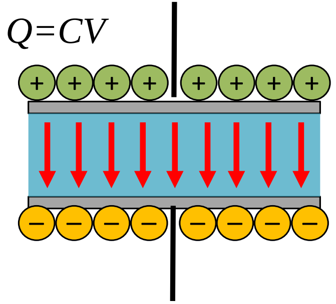


Charging ***Storing*** ***Discharging***

If we connect the charged capacitor with a resistor, the voltage across the capacitor will lead to current and the charges will leave the capacitor. This process is called discharging.

5-3 ENERGY STORED IN CAPACITOR

The higher the voltage across the capacitor, the more charges on the capacitor, thus more energy is stored in the capacitor. The question is how much energy is stored in a capacitor?



Assuming the capacitor is charging, the power delivered to the capacitor is

$$P = vi$$

According to the V-I characteristic, $$i(t) = C\frac{dv(t)}{dt}$$

Substitute i into P, we have $$P = Cv\frac{dv(t)}{dt}$$

According to the definition of energy and power, the energy change in a period of time can be calculated by taking the integral of power in this time period:

$$W_c(t) - W_c(t_0) = \int_{t_0}^{t} P\, d(t)$$

$$\int_{t_0}^{t} P\, d(t) = \int_{t_0}^{t} Cv\frac{dv(t)}{dt}d(t) = C\int_{v(t_0)}^{v(t)} v dv(t) = \frac{1}{2}C[v^2(t) - v^2(t_0)]$$

Energy stored in a capacitor only depends on the capacitance and the voltage at this time

Note that, if we assume the capacitor has 0 voltage at t_0, meaning that the capacitor has no energy before the charging starts. Then we know

$$v(t_0) = 0 \qquad W_c(t_0) = 0$$

By substituting $v(t_0)$ and $W_c(t_0)$ into the previous equation, we have

$$W_c(t) = \frac{1}{2}Cv^2(t)$$

48510 LEC 5 – CAPACITOR

Topic 5-4: Equivalent capacitance

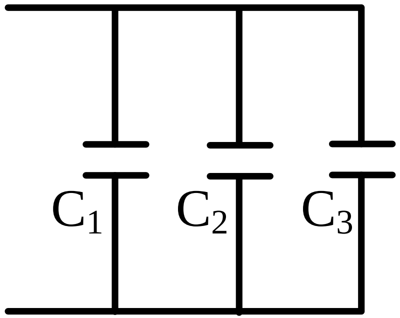
DR CAN DING

Lecturer

can.ding@uts.edu.au

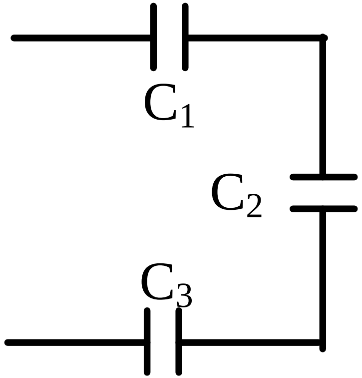
5-4 EQUIVALENT CAPACITANCE

Like resistor, a combination of capacitors can be equivalent to one capacitor as well.



Capacitor connected in parallel

$$C_{eq} = C_1 + C_2 + C_3$$

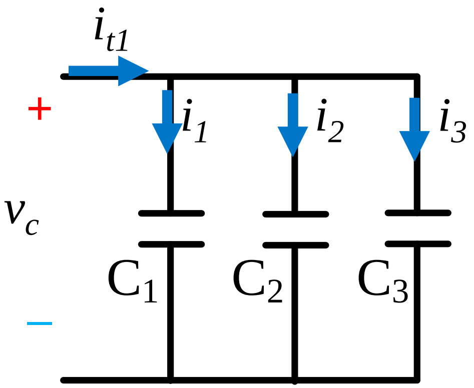


Capacitor connected in series

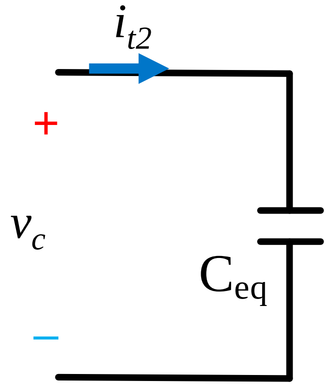
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

5-4 EQUIVALENT CAPACITANCE

Capacitor connected in parallel



Equivalent capacitance



If the two circuits are equivalent to each other, we know when the same voltage is applied on the two circuits, the resultant current should be the same.

$$i_{t1} = i_{t2}$$

KCL $\quad i_{t1} = i_1 + i_2 + i_3$

$$i_{t2} = C_{eq}\frac{dv_c}{dt}$$

V-I characteristic

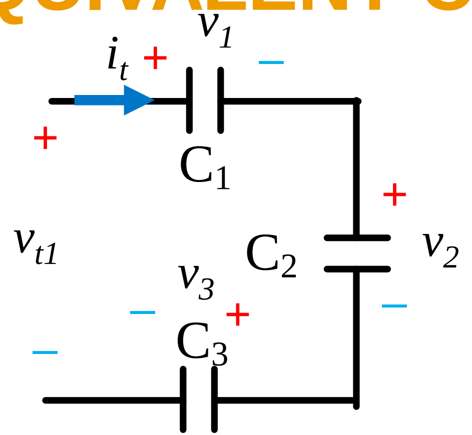
$$i_1 = C_1\frac{dv_c}{dt} \quad i_2 = C_2\frac{dv_c}{dt} \quad i_3 = C_3\frac{dv_c}{dt}$$

$$i_{t1} = (C_1 + C_2 + C_3)\frac{dv_c}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3$$

5-4 EQUIVALENT CAPACITANCE

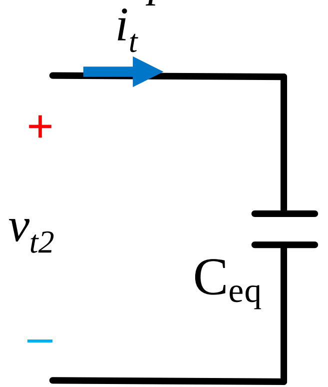
Capacitor connected in series



If the two circuits are equivalent to each other, we know when the same current flowing through two circuits, the resultant voltage should be the same.

$$v_{t1} = v_{t2}$$

Equivalent capacitance



$$v_{t2} = \frac{1}{C_{eq}}\int_{t_0}^{t} i(t)dt + v_{t2}(t_0)$$

KVL $\quad v_{t1} = v_1 + v_2 + v_3$

V-I characteristic

$$v_1 = \frac{1}{C_1}\int_{t_0}^{t} i(t)dt + v_1(t_0)$$

$$v_2 = \frac{1}{C_2}\int_{t_0}^{t} i(t)dt + v_2(t_0)$$

$$v_3 = \frac{1}{C_3}\int_{t_0}^{t} i(t)dt + v_3(t_0)$$

$$v_{t1} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)\int_{t_0}^{t} i(t)dt + v_{t1}(t_0)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$