48510 LEC 9 – AC WAVEFORMS AND SINUSOIDS

Topic 9-1-1: Introduction to sinusoidal waveform 1 – *peak value, peak-to-peak value, period, frequency, and angular frequency*

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9-1-1 SINUSOIDAL WAVEFORM 1

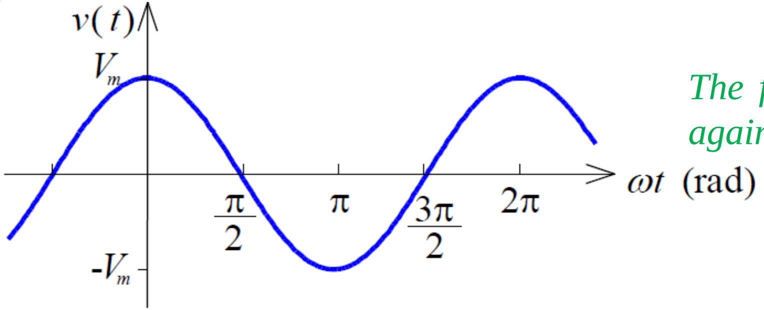
The sinusoid is the most important function in electrical and electronic engineering. The current generated by power station is sinusoid. The electromagnetic wave, which is the base of telecommunication, is also sinusoid.

Consider the sinusoidal voltage: $V(t) = V_m \cos(\omega t)$

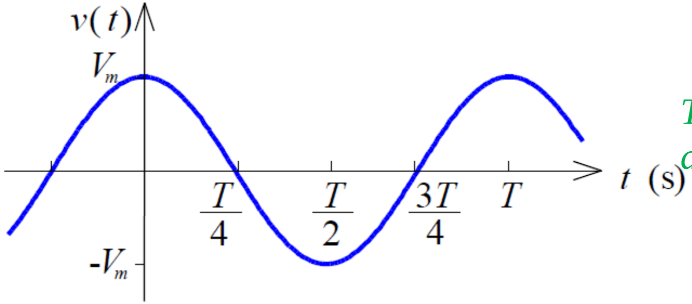
V_m is the peak value (V);

$V_{pp} = 2V_m$ is the peak-to-peak value (V);

The function repeats itself every 2π (radians), or every T (seconds). Therefore, the period is 2π (radians) or T (seconds). In most cases, we prefer to use T in seconds as the time period.



The function plot against radians



The function plot against time

What's the relationship between T and ω?

When $t = T$, $\omega t = 2\pi$, thus we have

$$\omega T = 2\pi \Rightarrow \omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$$

$f = \frac{1}{T}$ is the frequency (Hz);

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi}$$

$\omega = 2\pi f$ is the angular frequency (rad/s);

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Topic 9-1-2: Introduction to sinusoidal waveform 2 – *phase angle*

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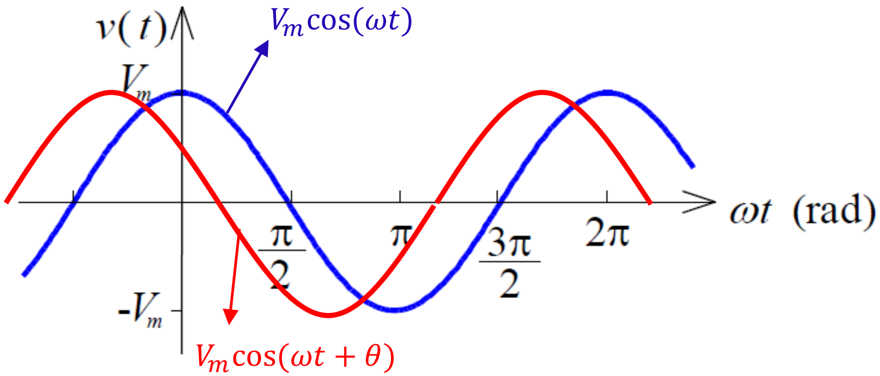
9-1-2 SINUSOIDAL WAVEFORM 2 – PHASE ANGLE

A more general form of sinusoidal waveform is:

$\boldsymbol{V(t) = V_m \cos(\omega t + \theta)}$, **where $\boldsymbol{\theta}$ is the phase angle**.

Compared to $V_m \cos(\omega t)$, it translates to the left by θ degree.



Phase relationship:

Since corresponding points on $V_m \cos(\omega t + \theta)$ occur θ rad earlier compared to $V_m \cos(\omega t)$, we say that $V_m \cos(\omega t + \theta)$ leads $V_m \cos(\omega t)$ by θ rad.

Conversely, $V_m \cos(\omega t)$ lags $V_m \cos(\omega t + \theta)$ by θ rad.

Also, we can say $V_m \cos(\omega t - \theta)$ lags $V_m \cos(\omega t)$ by θ rad.

$$V_1(t) = V_{m1} \cos(\omega t + \theta_1)$$

$$V_2(t) = V_{m2} \cos(\omega t + \theta_2)$$

If $\theta_1 > \theta_2$, $V_1(t)$ leads $V_2(t)$ by $\theta_1 - \theta_2$ degrees.

If $\theta_1 < \theta_2$, $V_1(t)$ lags $V_2(t)$ by $\theta_1 - \theta_2$ degrees.

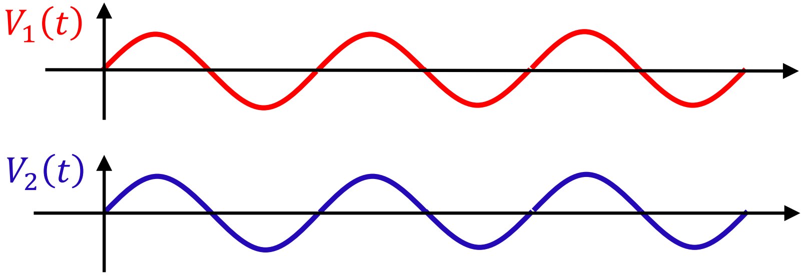
Note we only concern phase relationships between two sinusoidal waveforms when they have the same frequency.

9-1-2 SINUSOIDAL WAVEFORM 2 – PHASE ANGLE

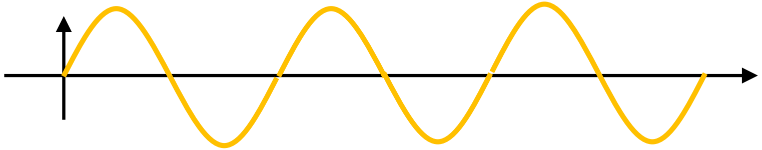
Specially, for two sinusoidal signals:

$$V_1(t) = V_{m1}\cos(\omega t + \theta_1) \qquad V_2(t) = V_{m2}\cos(\omega t + \theta_2)$$

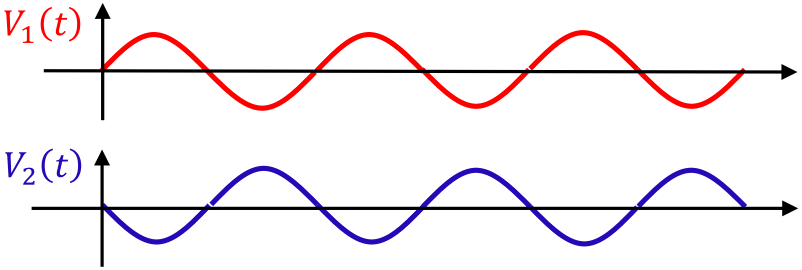
if the phase angles are equal ($\theta_1 = \theta_2$), they are said to be ***in phase***; if the difference between two phase angles are 180^0 or π in radian ($\theta_1 - \theta_2 = \pm 180^0\ or\ \pm\pi$), they are said to be ***out-of-phase***.



$$V_1(t) + V_2(t) = (V_{m1} + V_{m2})\cos(\omega t + \theta_1)$$



In-phase signals add up with each other



$$V_1(t) + V_2(t) = (V_{m1} - V_{m2})\cos(\omega t + \theta_1)$$

Out-of-phase signals cancel with each other

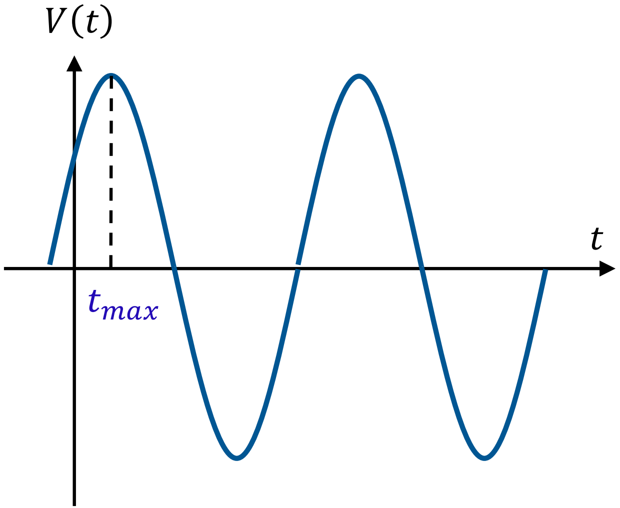
9-1-2 SINUSOIDAL WAVEFORM 2 – PHASE ANGLE

We always use this standard function to describe sinusoidal AC signals.

$V(t)$ or $I(t) = A_m\cos(\omega t + \theta)$, where A_m is the amplitude, ω is the angular frequency and θ is the phase.

Given the plot of the function, it is straightforward to find A_m and $\omega = \frac{2\pi}{T}$,

but how to determine θ?



Find t_{max}, the time at which positive peak occurs.

We know when positive peak occurs, $\omega t_{max} + \theta = 0$.

Therefore, $\theta = -\omega t_{max}$

Since $\omega = \frac{2\pi}{T}$,

$$\theta = -2\pi\frac{t_{max}}{T} \text{ (in radian)} \quad \text{or} \quad \theta = -360^0\frac{t_{max}}{T} \text{ (in degree)}$$

Note that we always want θ to be as small as possible in the standard form, therefore, you always need to find the positive peak near the t=0 and assume the time of that peak to be t_{max}.

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Topic 9-1-3: Introduction to sinusoidal waveform 3 – *phase angle example*

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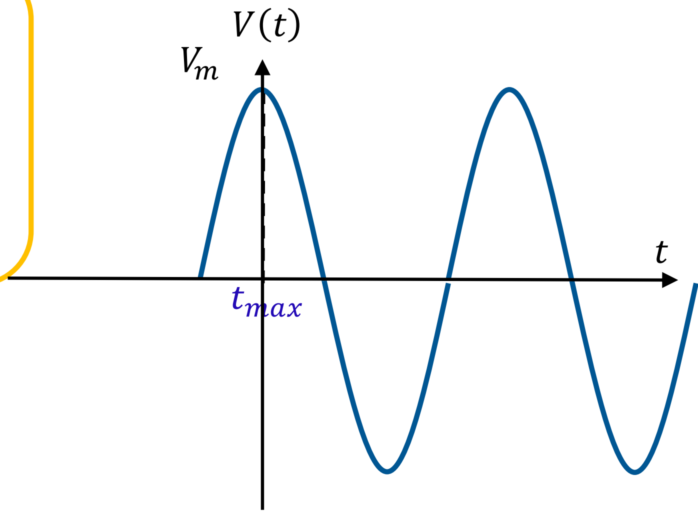
9-1-3 PHASE ANGLE EXAMPLES

Examples to find θ of a sinusoid $V(t)$ *or* $I(t) = A_m\cos(\omega t + \theta)$

Find t_{max}, the time at which positive peak occurs.

$$\theta = -2\pi\frac{t_{max}}{T} \text{ (in radian)} \quad \text{or} \quad \theta = -360^0\frac{t_{max}}{T} \text{ (in degree)}$$

Note that we always want the absolute value of θ to be as small as possible in the standard form, therefore, you always need to find the positive peak near the t=0.



Example 1:

$t_{max} = 0$

$\theta = 0^0$

$V(t) = V_m\cos(\omega t)$

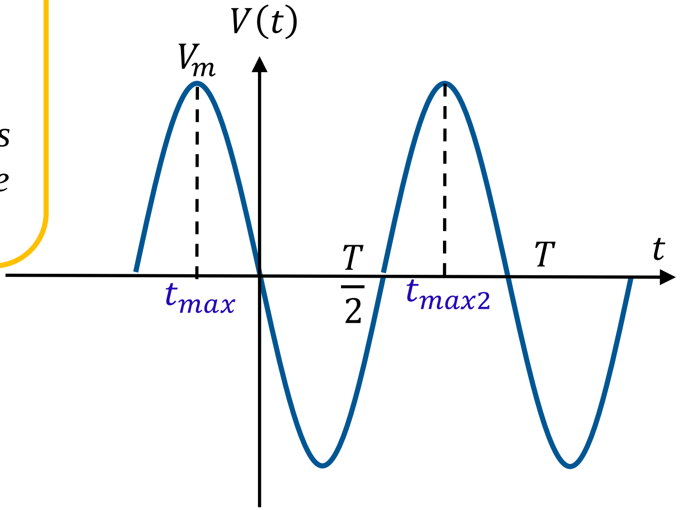
9-1-3 PHASE ANGLE EXAMPLES

Examples to find θ of a sinusoid $V(t)$ *or* $I(t) = A_m \cos(\omega t + \theta)$

Find t_{max}, the time at which positive peak occurs.

$\theta = -2\pi \frac{t_{max}}{T}$ *(in radian)* or $\theta = -360^0 \frac{t_{max}}{T}$ *(in degree)*

Note that we always want the absolute value of θ to be as small as possible in the standard form, therefore, you always need to find the positive peak near the t=0.



Example 2:

$t_{max} = -\frac{T}{4}$

$\theta = -360^0 \frac{-1}{4} = 90^0$

$V(t) = V_m \cos(\omega t + 90^0)$

or

$t_{max} = \frac{3T}{4}$

$\theta = -360^0 \frac{3}{4} = 270^0$

$V(t) = V_m \cos(\omega t - 270^0)$

Both of these two results are correct, but the left one is recommended.

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Topic 9-1-4: Introduction to sinusoidal waveform 4 – *average value and RMS value*

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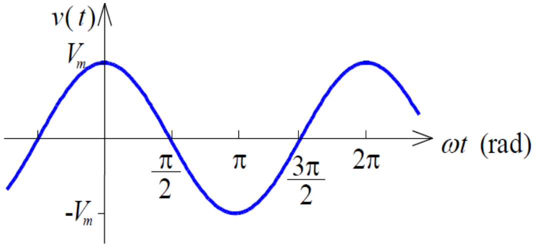
9-1-4 AVERAGE VALUE

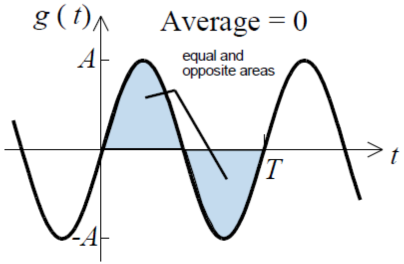
If you measure a waveform with a DC meter (or use a multimeter on the DC setting), by design the meter will measure the average value.



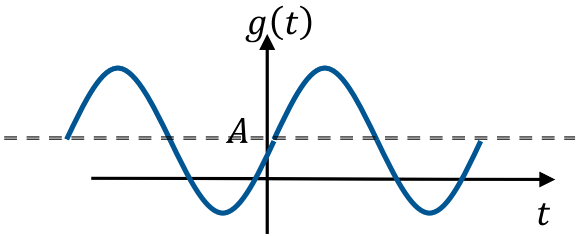
For periodic waveforms we have the **average value over a specific time interval as:**

$$G = \frac{1}{T}\int_{t_0}^{t_0+T} g(t)dt$$

where T is the period of the waveform and t_0 is an arbitrary time that can be chosen for mathematical convenience. Thus, instead of determining the average over the entire (infinite) time extent of a periodic waveform, we only have to determine the average over one period.



*If a periodic waveform is symmetric about the horizontal axis, then the **average value is zero**, since the areas are "equal but opposite".*



If a periodic waveform is symmetric about a* horizontal line *g* = *A, then the average value is A.

One could calculate the average value of a sinusoid using

(Vp_positive+Vp_negative)/2

9-1-4 RMS VALUE

RMS value is a measure of the effectiveness of a source in **delivering power** to a resistive load.

*Australia **standard voltage is 230 V**. This voltage is the RMS value.*

The power delivered by the AC supply at any instant of time is $P = VI = \frac{V^2}{R}$

For AC source, the voltage $V(t) = V_m \cos(\omega t)$

Therefore, $P = \frac{V_m^2}{R} cos^2(\omega t)$

Since $cos^2(\omega t) = \frac{1}{2}[1 + \cos(2\omega t)]$

$$P = \frac{V_m^2}{2R} + \frac{V_m^2}{2R} cos(2\omega t)$$

The effective value of this term is 0 as $cos(\omega t)$ changes with time and the average value is 0.

Therefore,

$$P = \frac{V_m^2}{2R}$$

By definition the average power delivered by AC source is $P = \frac{V_{RMS}^2}{R}$ ⟹ $V_{RMS}^2 = \frac{V_m^2}{2}$

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

SUMMARY OF SINUSOIDAL WAVEFORM

Standard form of the sinusoidal voltage:

$$V(t) = V_m \cos(\omega t + \theta)$$

V_m is the peak value (V);

V_{pp} is the peak-to-peak value (V);

V_{ave} is the average value (V) = $\frac{(V_{P+} + V_{P-})}{2} = 0$;

V_{RMS} is effective root mean square (RMS) value (V) = $\frac{V_m}{\sqrt{2}}$;

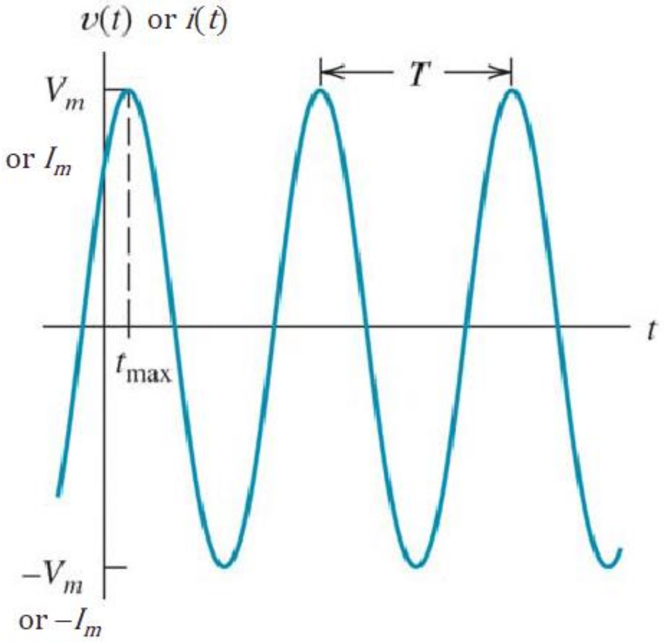
$P = \frac{V_{RMS}^2}{R} = I_{RMS}^2 R$ is average power delivered to a resistor by a sinusoidal voltage or current;

T is the period (s);

$f = \frac{1}{T}$ is the frequency (Hz);

$\omega = 2\pi f$ is the angular frequency (rad/s);

θ is the phase angle (rad or degree).



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Topic 9-2-1: Complex number

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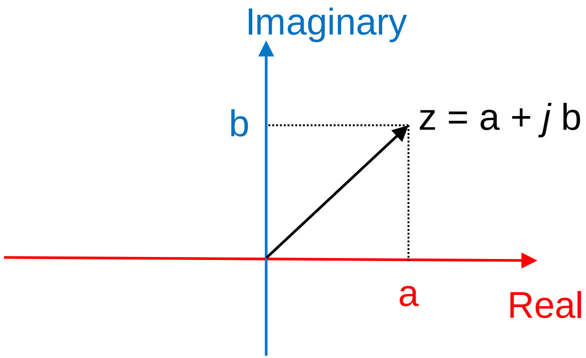
9-2-1 COMPLEX NUMBER

3.5 + *j* 2, 70000 + *j* 0, 0 - *j* 1

- Real Part — 3.5, 70000, -0.2
- Imaginary Part — $\sqrt{-1} *$ Real number → *j* (*i*) — *j* 3.5, *j* 70000, -*j* 0.2



Complex number on Complex plane

9-2-1 COMPLEX NUMBER

Some useful definitions:

1) A pure imaginary number (for example j 6), is also a complex number, but with zero real part.

2) A pure real number (for example 5), can be thought of as a complex number with zero imaginary part .

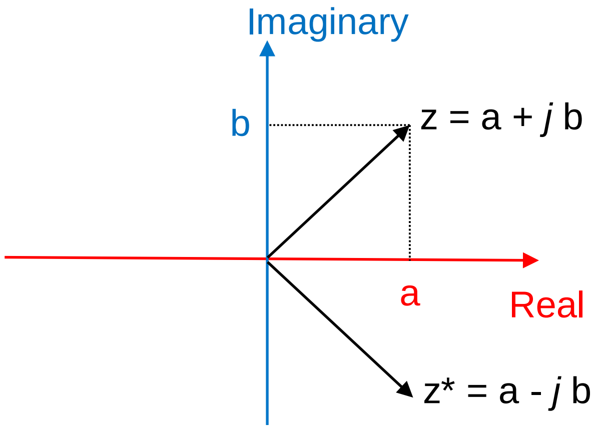
3) For the complex number z = a + j b, the Modulus (Magnitude) of z is $|z| = \sqrt{a^2 + b^2}$.

4) Geometrically, the modulus of a complex number is the distance from the origin to its location in the complex plane.

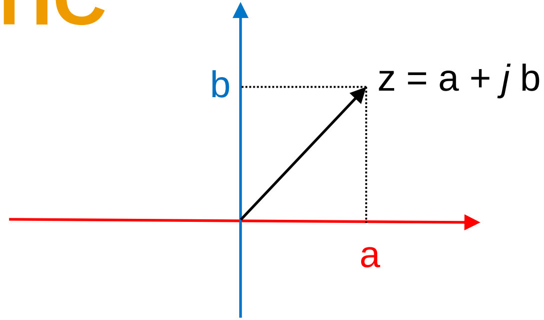
5) The complex conjugate of z is $\bar{z}$ or z* = a - j b.

6) The complex conjugate of a complex number is its reflection in the real axis.

7) The product of a complex number and its conjugate $|zz^*| = |z|^2$ is a real number.



9-2-1 COMPLEX NUMBER ARITHMETIC



$z_1 = a + jb, z_2 = c + jd$ Rectangular form

Add

$$z_1 + z_2 = (a + c) + j(b + d)$$

Subtract

$$z_1 - z_2 = (a - c) + j(b - d)$$

Multiplication

$$z_1 z_2 = ac + jad + jbc + j^2 bd$$
$$= (ac - bd) + j(ad + bc)$$

Division

$$\frac{z_1}{z_2} = \frac{a + jb}{c + jd}$$

$$= \frac{(a + jb) \times (c - jd)}{(c + jd) \times (c - jd)}$$

Multiplication (numerator)

Multiplication/ Modulus (denominator)

$$= \frac{(ac + bd) + j(bc - ad)}{c^2 + d^2}$$

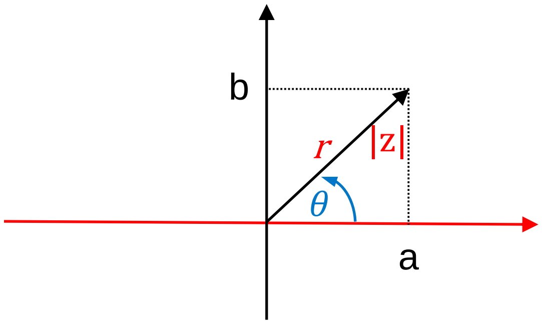
No *j* in the denominator

$$= \frac{ac + bd}{c^2 + d^2} + j\frac{bc - ad}{c^2 + d^2}$$

9-2-1 COMPLEX NUMBER ARITHMETIC

$z = a + jb$ Rectangular form

$z = r\angle\theta$ Polar form



Convert rectangular form to polar form

$$r = |z| = \sqrt{a^2 + b^2}$$

$$\theta = arctan\frac{b}{a}$$

Convert polar form to rectangular form

$$z = r\angle\theta$$
$$= r\cos\theta + jr\sin\theta$$

Polar form arithmetic

$z_1 = r_1 \angle \theta_1, z_2 = r_2 \angle \theta_2,$

Add

Convert to rectangular form first

Subtract

Convert to rectangular form first

Multiplication

$$z_1 z_2 = r_1 r_2 \angle (\theta_1 + \theta_2)$$

Division

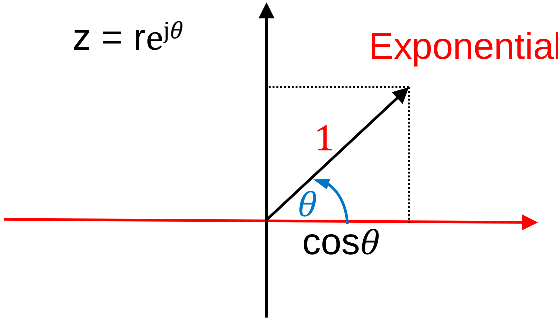
$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle (\theta_1 - \theta_2)$$

9-2-1 COMPLEX NUMBER ARITHMETIC

z = a + *j* b   Rectangular form

z = r∠θ Polar form

z = re$^{j\theta}$ Exponential form



$$e^{j\theta} = \cos(\theta) + j sin(\theta)$$ Euler's law

$$(e^{j\theta})^* = e^{-j\theta} = \cos(\theta) - j sin(\theta)$$

$$|e^{j\theta}| = |e^{-j\theta}| = \sqrt{cos^2(\theta) + sin^2(\theta)} = 1$$

Exponential form

$$z_1 = r_1 e^{j\theta_1}, z_2 = r_2 e^{j\theta_2}$$

Add

Convert to rectangular form first

Subtract

Convert to rectangular form first

Multiplication

$$z_1 z_2 = r_1 r_2 e^{j(\theta_1+\theta_2)}$$

Division

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{j(\theta_1-\theta_2)}$$

9-2-1 COMPLEX NUMBER ARITHMETIC

$z = a + j\, b$ Rectangular form

$z = r\angle\theta$ Polar form

$z = re^{j\theta}$ Exponential form

Add

rectangular form

Subtract

rectangular form

Multiplication

Polar form or exponential form

Division

Polar form or exponential form

48510 LEC 9 – AC WAVEFORMS AND SINUSOIDS

Topic 9-2-2: Complex number supplementary

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9-2-1 COMPLEX NUMBER ARITHMETIC

Conversion from rectangular form to polar/exponential form

Rectangular form $z = a + jb$

Polar form $z = r\angle\theta$

Exponential form $z = re^{j\theta}$

To find out r and θ.

$$r = |z| = \sqrt{a^2 + b^2}$$

$$\theta = arctan\frac{b}{a}$$

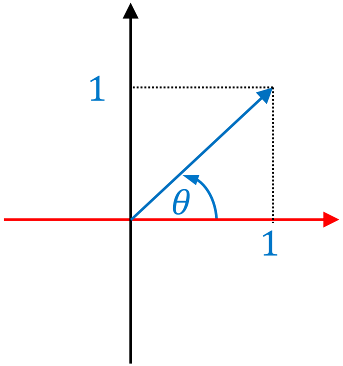
Obtain the phase angle can be tricky!

Because there are more than 1 solutions for arctan function.

Case 1:

$$z_1 = a + jb = 1 + j$$

$$\theta = arctan\frac{b}{a} = \arctan(1) = 45^0$$

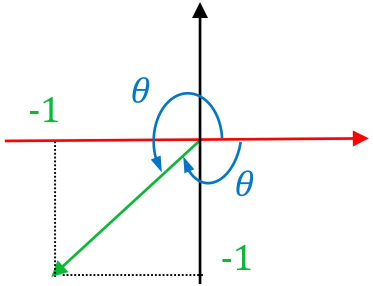


Case 2:

$$z_2 = a + jb = -1 - j$$

$$\theta = arctan\frac{b}{a} = \arctan(1) = 45^0?$$

$$= 225^0 \text{ or } = -135^0$$



9-2-1 COMPLEX NUMBER ARITHMETIC

Conversion from rectangular form to polar/exponential form

Rectangular form $z = a + jb$

Polar form $z = r\angle\theta$

Exponential form $z = re^{j\theta}$

To find out r and θ.

$$r = |z| = \sqrt{a^2 + b^2}$$

$$\theta = arctan\frac{b}{a}$$

Obtain the phase angle can be tricky!

Because there are more than 1 solutions for arctan function.

$$tan\theta = \tan(\theta \pm \pi)$$

$$arctanX = \theta \ or \ \theta \pm \pi$$

The θ obtained by calculator is the one from range of -90 to 90 degrees.

One needs to observe the complex number in the complex plane to find out which quadrant it locates, then you can determine the right θ!