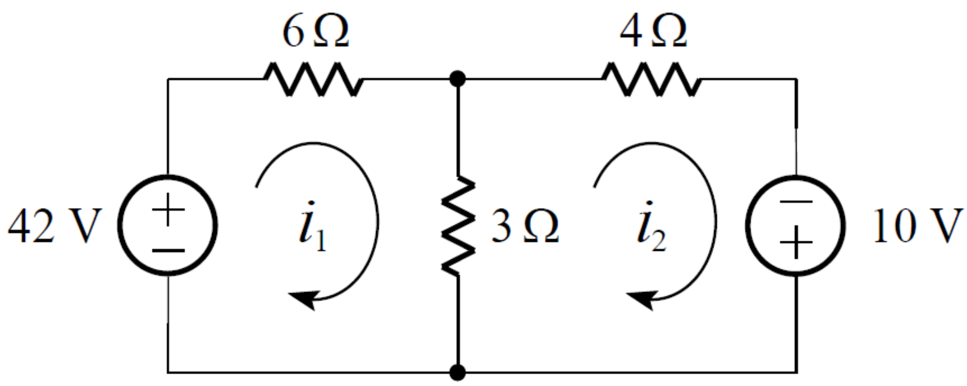
Worksolutions

A. Mesh Analysis

1. Find currents i1 and i2.

A two-mesh circuit is shown below.



Solutions:

We apply KVL to each mesh. For the left-hand mesh:

$$42 - 6i_1 - 3(i_1 - i_2) = 0$$
$$9i_1 - 3i_2 = 42$$

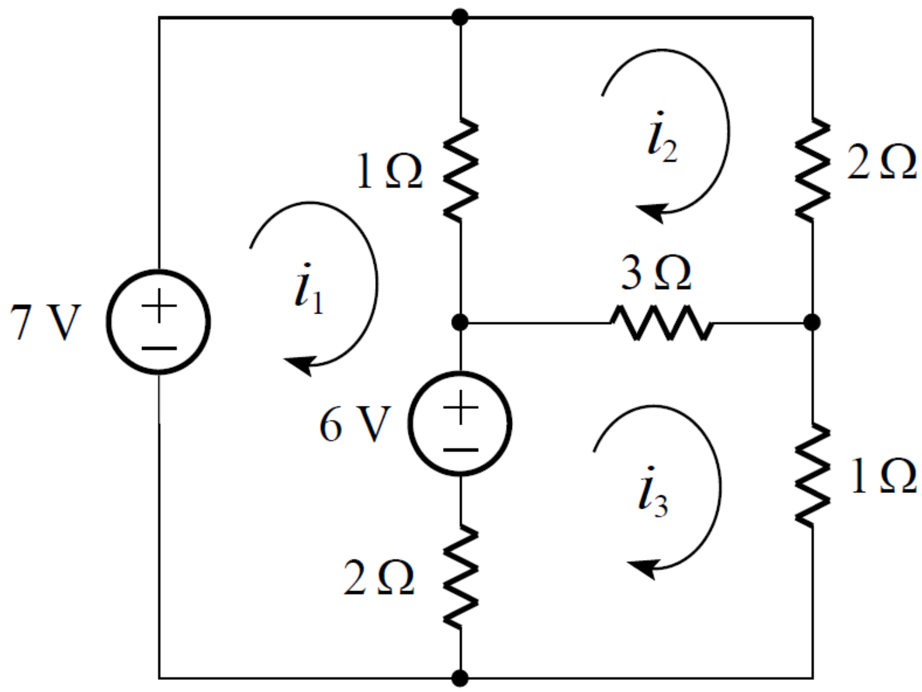
For the right-hand mesh:

$$-3(i_2 - i_1) - 4i_2 + 10 = 0$$
$$-3i_1 + 7i_2 = 10$$

The solution is obtained by solving simultaneously: $i_1 = 6\text{ A}$ and $i_2 = 4\text{ A}$.

2. Find currents below:

Consider the five-node, three-mesh circuit shown below.



Solutions:

The three required mesh currents are assigned as indicated, and we methodically apply KVL about each mesh:

$$7 - 1(i_1 - i_2) - 6 - 2(i_1 - i_3) = 0$$
$$-1(i_2 - i_1) - 2i_2 - 3(i_2 - i_3) = 0$$
$$-2(i_3 - i_1) + 6 - 3(i_3 - i_2) - 1i_3 = 0$$

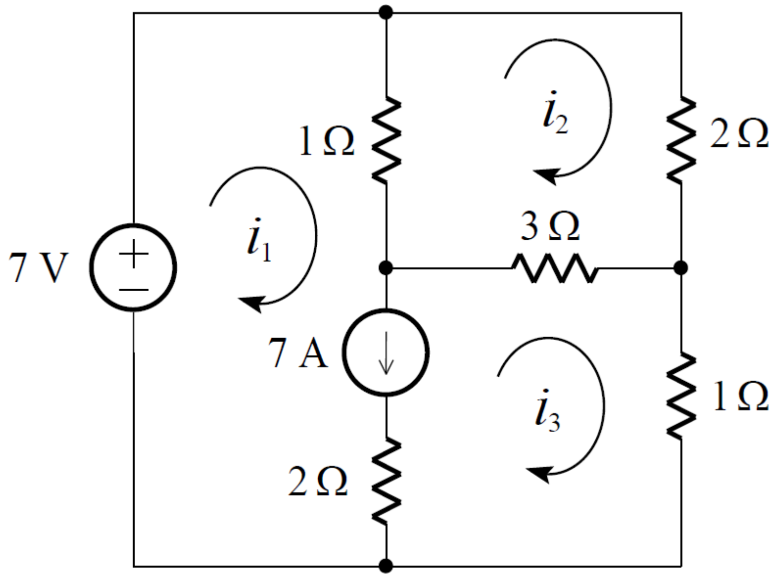
Simplifying and writing as a matrix equation:

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

i1=3A, i2=2A, i3=3A

3. Find the currents below.

Consider the circuit shown below in which a 7 A independent current source is in the common boundary of two meshes.

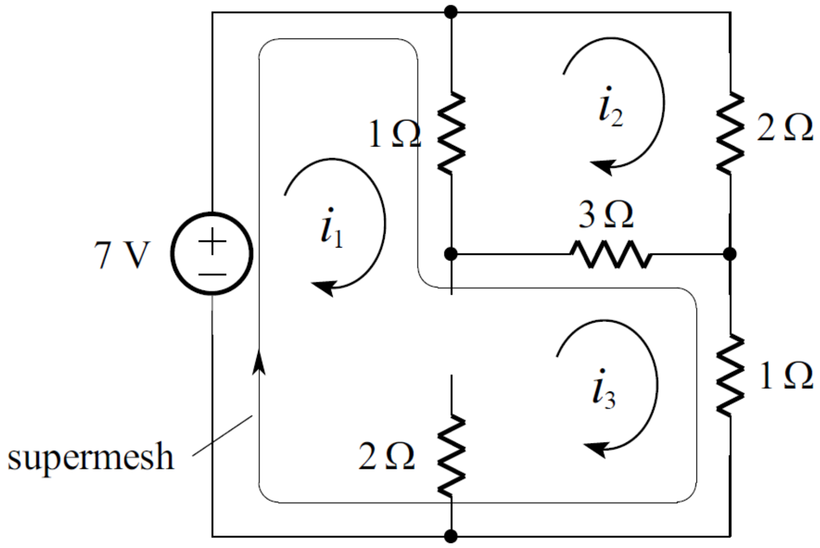


Solutions:

For the independent current source, we relate the source current to the mesh currents:

$$i_1 - i_3 = 7$$

We then mentally open-circuit the current source, and form a supermesh whose interior is that of meshes 1 and 3:



Applying KVL about the supermesh:

$$7 - 1(i_1 - i_2) - 3(i_3 - i_2) - 1i_3 = 0$$

Around mesh 2 we have:

$$-1(i_2 - i_1) - 2i_2 - 3(i_2 - i_3) = 0$$

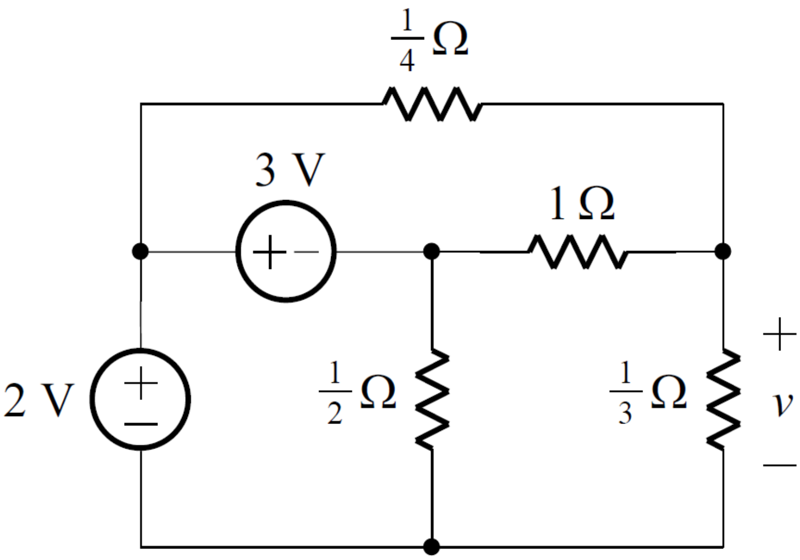
Rewriting these last three equations in matrix form, we have:

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -4 & 4 \\ -1 & 6 & -3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 0 \end{bmatrix}$$

i1=9A, i2=2.5A, i3=2A

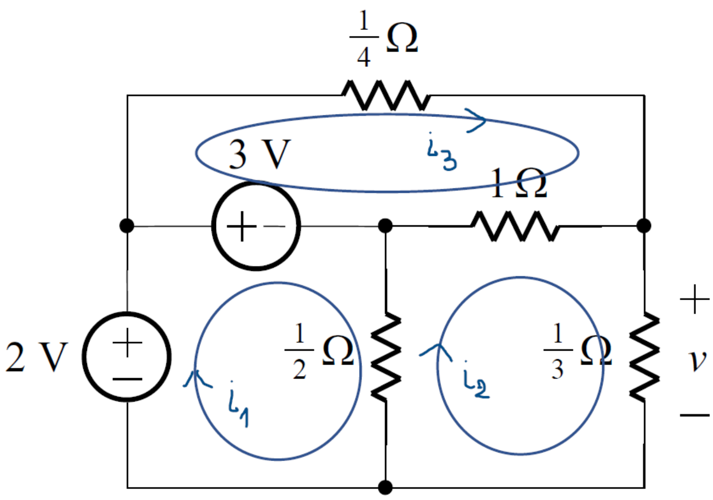
4.

Use mesh analysis to find v in the circuit shown below:



Solutions:

Use mesh analysis to find v in the circuit shown below:



KVL in mesh 1:

$$-2+3+\frac{1}{2}(i_1-i_2)=0$$

KVL in mesh 2:

$$\frac{1}{2}(i_2-i_1)+1(i_2-i_3)+\frac{1}{3}i_2=0$$

KVL in mesh 3:

$$-3+\frac{1}{4}i_3+1(i_3-i_2)=0$$

$$\begin{cases} \frac{1}{2}i_1 - \frac{1}{2}i_2 & = -1 \\ -\frac{1}{2}i_1 + \frac{11}{6}i_2 - i_3 = 0 \\ -i_2 + \frac{5}{4}i_3 = 3 \end{cases}$$

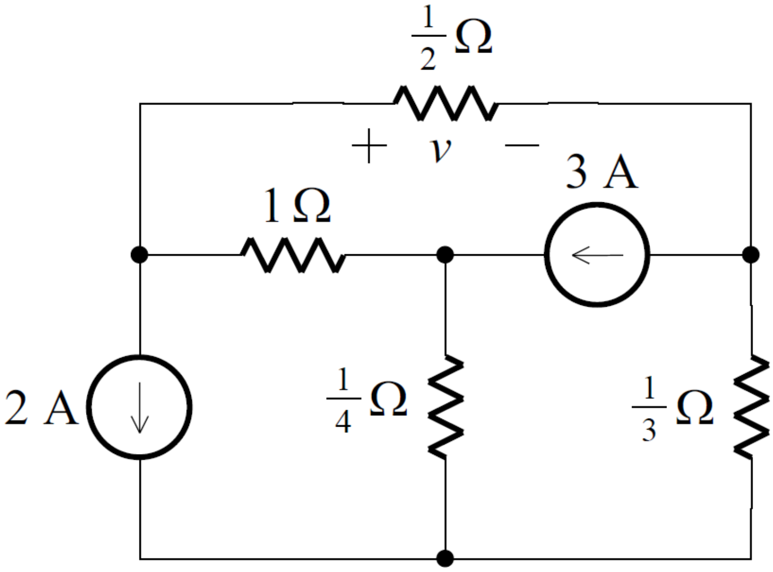
$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{11}{6} & -1 \\ 0 & -1 & \frac{5}{4} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix}$$

$$I = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0.625 \\ 2.625 \\ 4.5 \end{bmatrix} A$$

$$v = i_2 * \frac{1}{3}\Omega = \frac{7}{8}\,V = 0.875\,V$$

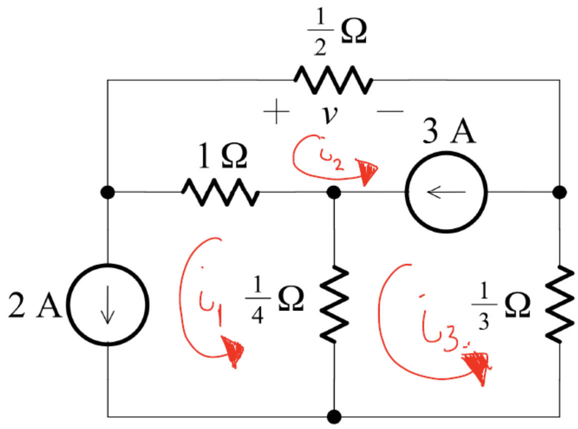
5.

Use mesh analysis to find v in the circuit shown below:



Solutions:

Use mesh analysis to find v in the circuit shown below:



We need to find an equation to describe the voltage around each loop. Using the same process as question 2, **KVL** is again used.

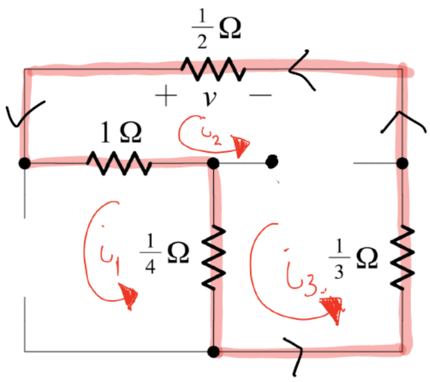
First we note that in this question, there are 2 current sources. This is advantageous because it can directly tell us the current seen in the loops which contain a current source

That is:

$$i_1 = 2A. \quad (1)$$

$$i_3 - i_2 = 3A. \quad (2)$$

However, in order to solve equation 2, we need use create a **SUPER MESH**. To do this, we open circuit all current sources:



We can now see there is only one loop for current to flow in. Writing the KVL equation:

$$\frac{1}{3}i_3 + \frac{1}{2}i_2 + (i_2 - i_1)(1) + (i_3 - i_1)\left(\frac{1}{4}\right) = 0 \quad (3)$$

from (1), $i_1 = 2A$.

$$\frac{1}{3}i_3 + \frac{1}{2}i_2 + i_2 - 2 + \frac{1}{4}i_3 - \frac{1}{2} = 0 \quad (3)$$

$$\frac{3}{2}i_2 + \frac{7}{12}i_3 = 2\tfrac{1}{2}. \quad (3)$$

and $i_3 - i_2 = 3 \quad (2)$

$$i_3 = 3 + i_2. \quad (2)$$

sub (2) into (3)

$$\frac{3}{2}i_2 + \frac{7}{12}(3 + i_2) = 2\tfrac{1}{2}.$$

$$\frac{25}{12}i_2 = 2\tfrac{1}{2} - \frac{21}{12}$$

$$i_2 = \frac{9}{25}.$$

We now have a value for i2, so we can now find v using ohms law:
Note that i2 is flowing in the opposite direction than the polarity of v so we must use -i2

$$v = -i_2 R$$

$$v = -\frac{9}{25} \times \frac{1}{2}$$

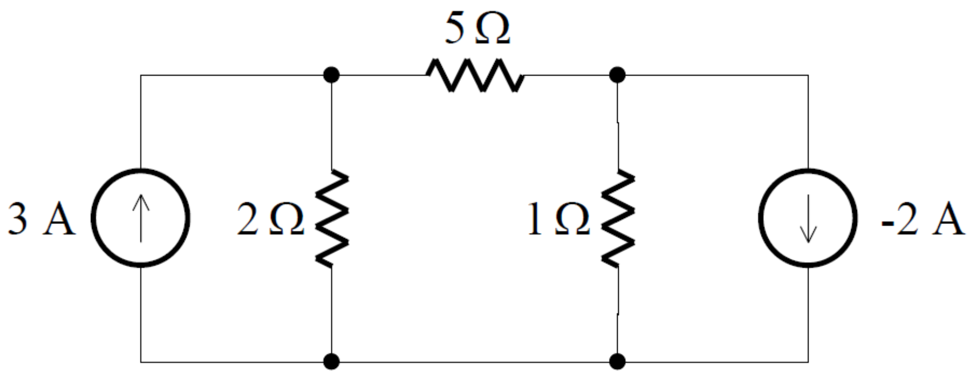
$$v = \frac{-9}{50} V$$

$$v = -0.18 V.$$

B. Nodal Analysis

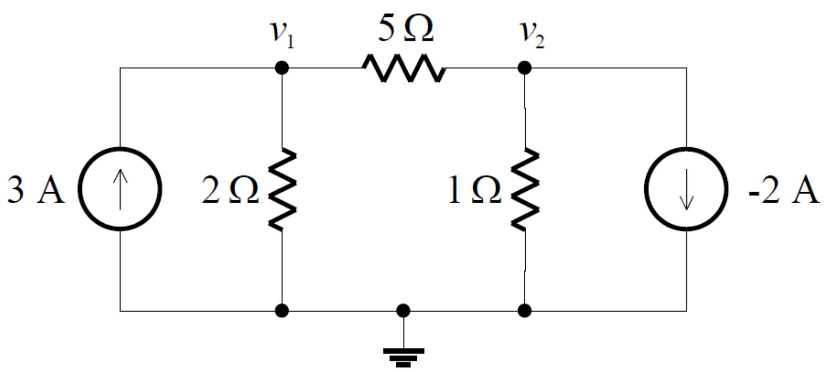
6.

We apply nodal analysis to the following 3-node circuit:



Solutions:

Following the steps above, we assign a reference node and then assign nodal voltages:

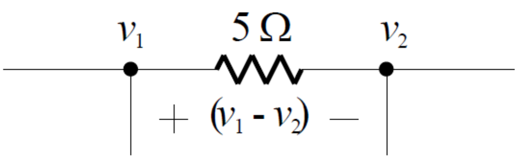


We chose the bottom node as the reference node, but either of the other two nodes could have been selected. A little simplification in the resultant equations is obtained if the node to which the greatest number of branches is connected is identified as the reference node.

In many practical circuits the reference node is one end of a power supply which is generally connected to a metallic case or chassis in which the circuit resides; the chassis is often connected through a good conductor to the Earth. Thus, the metallic case may be called “ground”, or “earth”, and this node becomes the most convenient reference node.

To avoid confusion, the reference node will be called the “common” unless it has been specifically connected to the Earth (such as the outside conductor on a digital storage oscilloscope, function generator, etc.).

Note that the voltage across any branch in a circuit may be expressed in terms of nodal voltages. For example, in our circuit the voltage across the $5\,\Omega$ resistor is $(v_1 - v_2)$ with the positive polarity reference on the left:



We must now apply KCL to nodes 1 and 2. We do this be equating the total current *leaving* a node to zero. Thus:

$$\frac{v_1}{2} + \frac{v_1 - v_2}{5} - 3 = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{1} + (-2) = 0$$

Simplifying, the equations can be written:

$$0.7v_1 - 0.2v_2 = 3$$

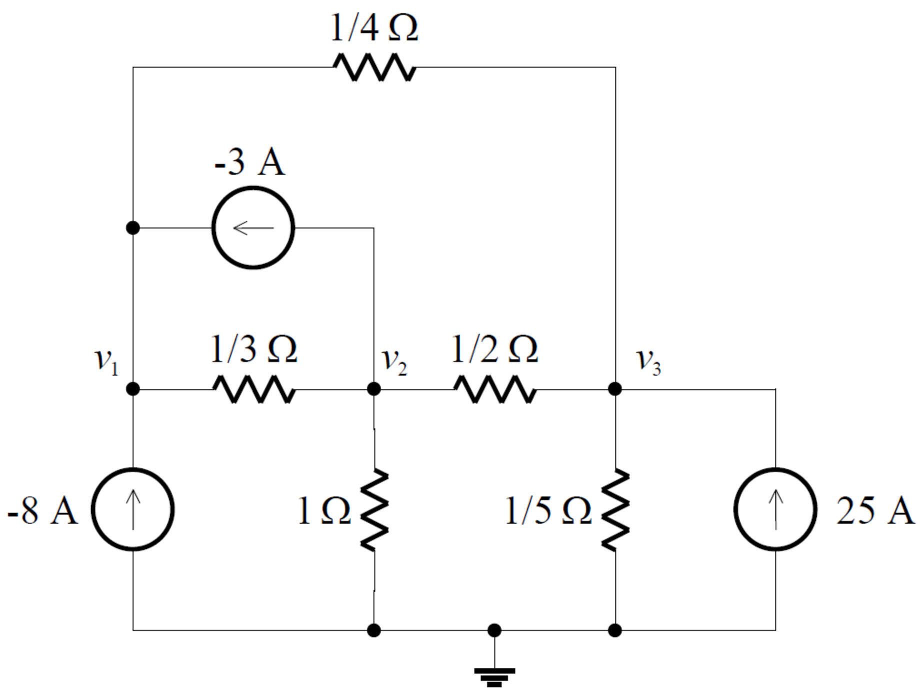
$$-0.2v_1 + 1.2v_2 = 2$$

V1=5V and V2=2.5V

Everything is now known about the circuit – any voltage, current or power in the circuit may be found in one step. For example, the voltage at node 1 with respect to node 2 is $(v_1 - v_2) = 2.5\ \mathrm{V}$, and the current directed downward through the $2\,\Omega$ resistor is $v_1/2 = 2.5\ \mathrm{A}$.

7. Please find v1, v2, v3.

A circuit is shown below with a convenient reference node and nodal voltages specified.



Solutions:

We sum the currents leaving node 1:

$$3(v_1 - v_2) + 4(v_1 - v_3) - (-8) - (-3) = 0$$
$$7v_1 - 3v_2 - 4v_3 = -11$$

At node 2:

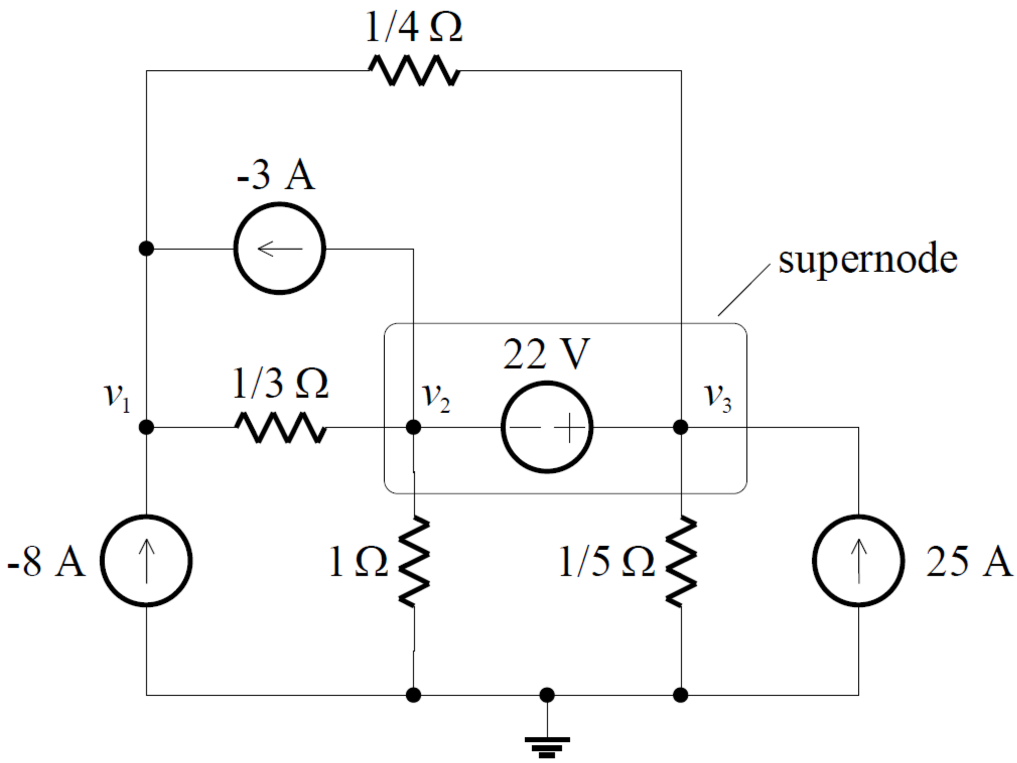
$$3(v_2 - v_1) + 1v_2 + 2(v_2 - v_3) - 3 = 0$$
$$-3v_1 + 6v_2 - 2v_3 = 3$$

At node 3:

$$4(v_3 - v_1) + 2(v_3 - v_2) + 5v_3 - 25 = 0$$
$$-4v_1 - 2v_2 + 11v_3 = 25$$

V1=1V, V2=2V, V3=3V

8. Find v1, v2, v3



Solutions:

KCL at node 1 remains unchanged:

$$3(v_1 - v_2) + 4(v_1 - v_3) - (-8) - (-3) = 0$$
$$7v_1 - 3v_2 - 4v_3 = -11$$

We find six branches connected to the supernode around the 22 V source (suggested by a broken line in the figure). Beginning with the $1/3\,\Omega$ resistor branch and working clockwise, we sum the six currents leaving this supernode:

$$3(v_2 - v_1) + (-3) + 4(v_3 - v_1) + (-25) + 5v_3 + 1v_2 = 0$$
$$-7v_1 + 4v_2 + 9v_3 = 28$$

We need one additional equation since we have three unknowns, and this is provided by KVL between nodes 2 and 3 inside the supernode:

$$v_3 - v_2 = 22$$

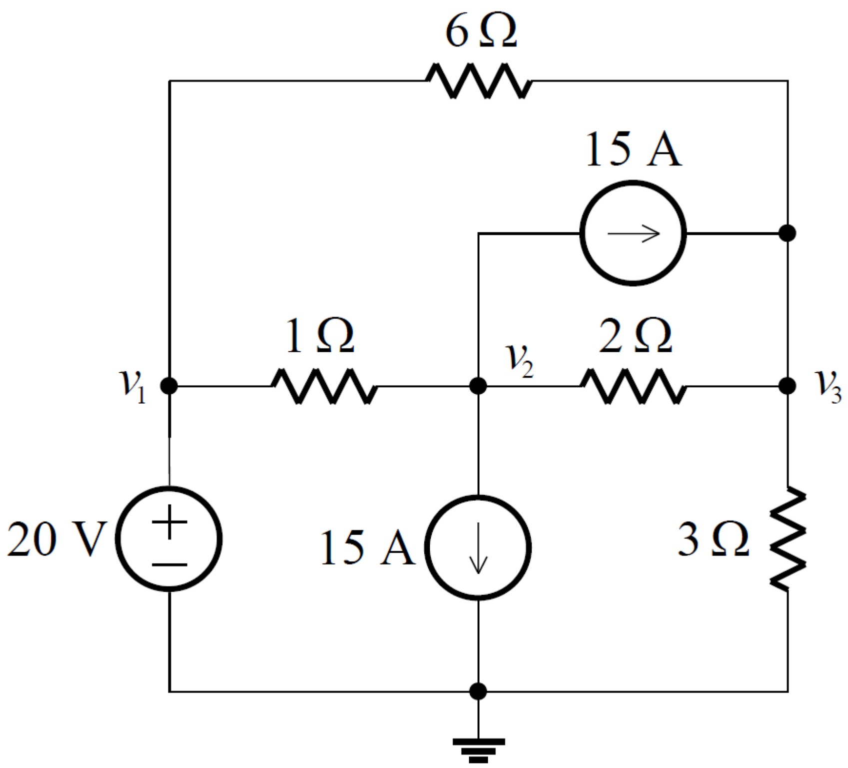
Rewriting these last three equations in matrix form, we have:

$$\begin{bmatrix} 7 & -3 & -4 \\ -7 & 4 & 9 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 28 \\ 22 \end{bmatrix}$$

The solution turns out to be $v_1 = -4.5\text{ V}$, $v_2 = -15.5\text{ V}$ and $v_3 = 6.5\text{ V}$.

9.

Use nodal analysis to find the node voltages for the circuit given below:



where, $R_1 = 1\Omega$, $R_2 = 6\Omega$, $R_3 = 2\Omega$ and $R_4 = 3\Omega$.

Solutions:

In order to find the voltages at each node, we must use nodal analysis and recognise that the circuit contains an independent voltage source. The reference node, or *common*, in the circuit will be assigned to the 0V reference denoted by the ground symbol at the bottom of the circuit.

Often, when using nodal analysis on circuits that contain BOTH independent current source and independent voltage sources may lead to implementing the concept of a ***supernode***. Using this concept is not necessary to solve this problem - we will find out why when formulating our nodal equations.

Voltage at Node 1:

The potential difference between node 1 and the reference node is simply equal to V_1. The voltage at v_1 is not constrained by any other node voltage, therefore there is no need for a supernode.

$$v_1 = 20V$$

Next, we apply KCL to the currents leaving node 2 and 3 (assuming current leaving the node is positive):

KCL at Node 2:

$$\frac{(v_2 - v_1)}{R_1} + \frac{(v_2 - v_3)}{R_3} + 15 + 15 = 0$$

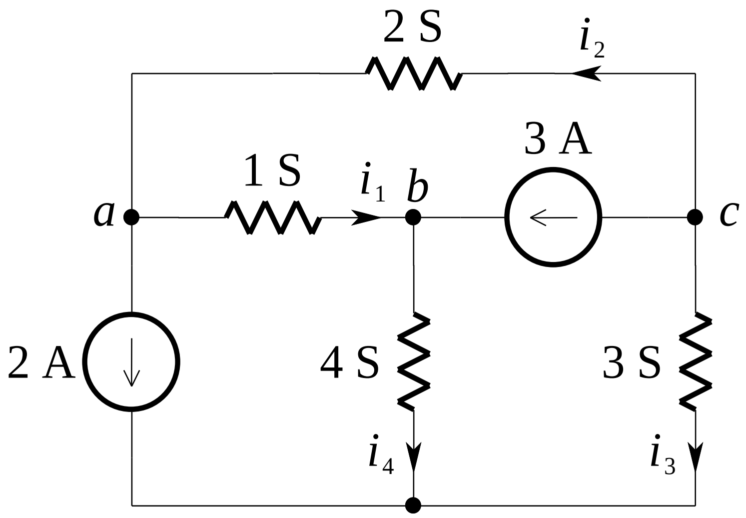
KCL at Node 3:

$$\frac{(v_3 - v_1)}{R_2} + \frac{(v_3 - v_2)}{R_3} + \frac{v_3}{R_4} - 15 = 0$$

$$v_1 = 20V,\ v_2 = -0.667V \text{ and } v_3 = 18V$$

10

Consider the circuit shown below:



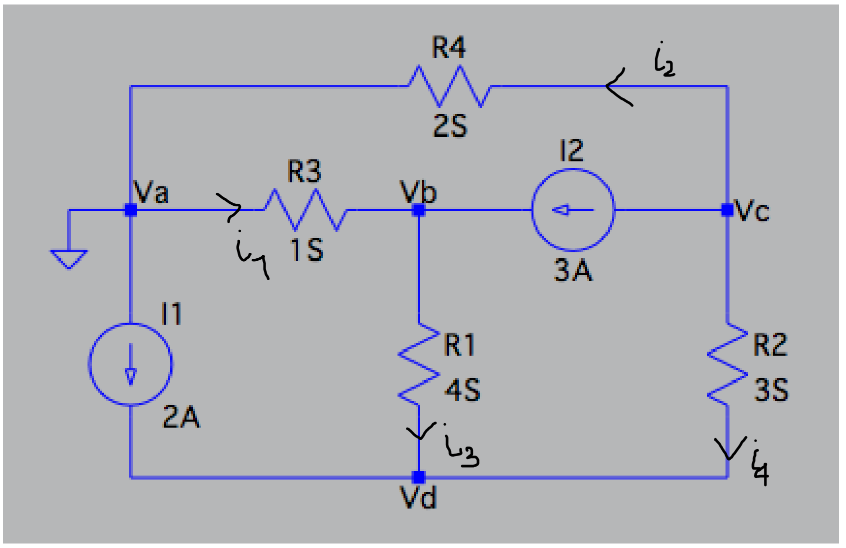
Using nodal analysis, find i_1, i_2, i_3 and i_4 when:

(a) Node *a* is the reference node.

(b) Node *b* is the reference node.

(c) Node *c* is the reference node.

a)



At node b:

$$1(V_b - 0) + 4(V_b - V_d) - 3 = 0$$

At node c:

$$2(V_c - 0) + 3 + 3(V_c - V_d) = 0$$

At node d:

$$-2 + 4(V_d - V_b) + 3(V_d - V_c) = 0$$

=>

$$\begin{cases} 5V_b \qquad - 4V_d = 3 \\ \qquad 5V_c - 3V_d = -3 \\ -4V_b - 3V_c + 7V_d = 2 \end{cases}$$

=>

$$\begin{cases} V_b = 1.64\ V \\ V_c = 0.18\ V \\ V_d = 1.3\ V \end{cases}$$

=>

$$i_1 = \frac{V_a - V_b}{R3} = -1.64\ V * 1S = -1.64\ A$$

(The negative sign indicate that the current would run from Vb to Va.)

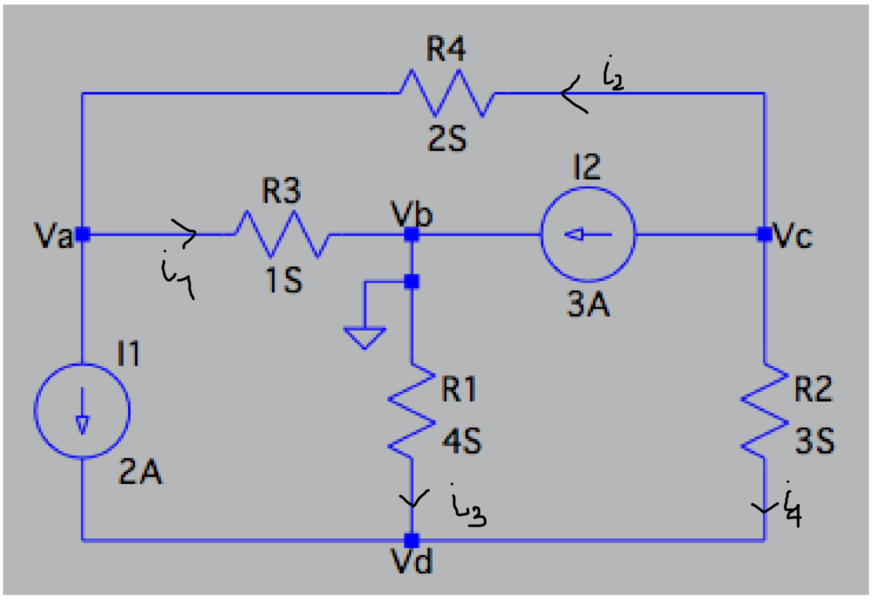
$$i_2 = \frac{V_c - V_a}{R4} = 0.18V * 2S = 0.36\ A$$

$$i_3 = \frac{(V_b - V_d)}{R1} = 0.34\ V * 4S = 1.36\ A$$

$$i_4 = \frac{V_c - V_d}{R2} = -1.12\ V * 3S = -3.36\ A$$

(The negative sign indicate that the current would run from Vd to Vc.)

b)



At node a:

$$2(V_a - V_c) + 1(V_a - 0) + 2 = 0$$

At node c:

$$2(V_c - V_a) + 3 + 3(V_c - V_d) = 0$$

At node d:

$$-2 + 4(V_d - 0) + 3(V_d - V_c) = 0$$

=>

$$\begin{cases} 3V_a \quad - 2V_c \qquad\qquad = -2 \\ -2V_a + 5V_c - 3V_d = -3 \\ \qquad -3V_c + 7V_d = 2 \end{cases}$$

=>

$$\begin{cases} V_a = \ -1.64\ V \\ V_c = -1.46\ V \\ V_d = -0.34\ V \end{cases}$$

=>

$$i_1 = \frac{V_a - V_b}{R3} = -1.64V * 1S = -1.64\ A$$

(The negative sign indicate that the current would run from Vb to Va.)

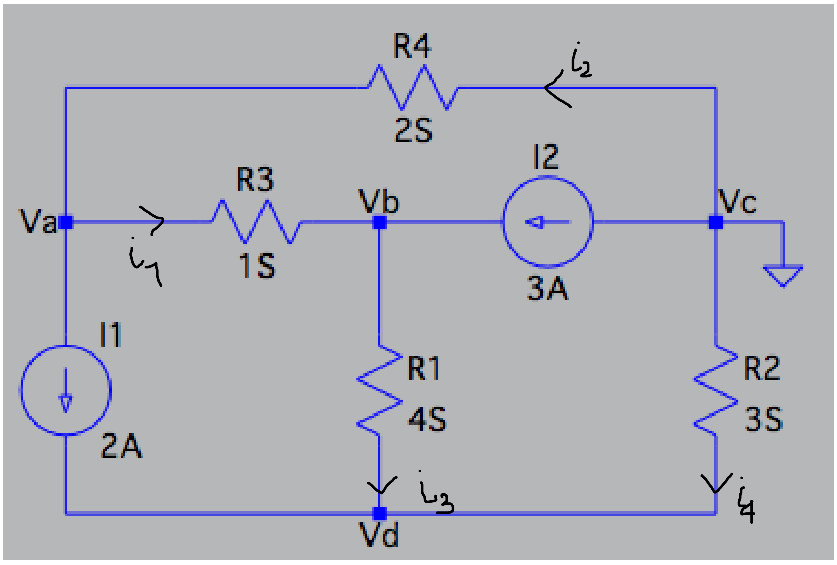
$$i_2 = \frac{V_c - V_a}{R4} = 0.18V * 2S = 0.36\ A$$

$$i_3 = \frac{(V_b - V_d)}{R1} = -(-0.34V) * 4S = 1.36\ A$$

$$i_4 = \frac{V_c - V_d}{R2} = -1.12V * 3S = -3.36\ A$$

(The negative sign indicate that the current would run from Vd to Vc.

c)



At node a:

$$2(V_a - 0) + 1(V_a - V_b) + 2 = 0$$

At node b:

$$1(V_b - V_a) - 3 + 4(V_b - V_d) = 0$$

At node d:

$$-2 + 4(V_d - V_b) + 3(V_d - 0) = 0$$

=>

$$\begin{cases} 3V_a \quad - V_b \qquad\quad = -2 \\ -V_a + 5V_b - 4V_d = 3 \\ \qquad -4V_b + 7V_d = 2 \end{cases}$$

=>

$$\begin{cases} V_a = -0.18\ V \\ V_b = 1.46\ V \\ V_d = 1.12\ V \end{cases}$$

=>

$$i_1 = \frac{V_a - V_b}{R3} = -1.64V * 1S = -1.64\ A$$

(The negative sign indicate that the current would run from Vb to Va.)

$$i_2 = \frac{V_c - V_a}{R4} = -(-0.18V) * 2S = 0.36\ A$$

$$i_3 = \frac{(V_b - V_d)}{R1} = 0.34V * 4S = 1.36\ A$$

$$i_4 = \frac{V_c - V_d}{R2} = -1.12V * 3S = -3.36\ A$$

(The negative sign indicate that the current would run from Vd to Vc.)