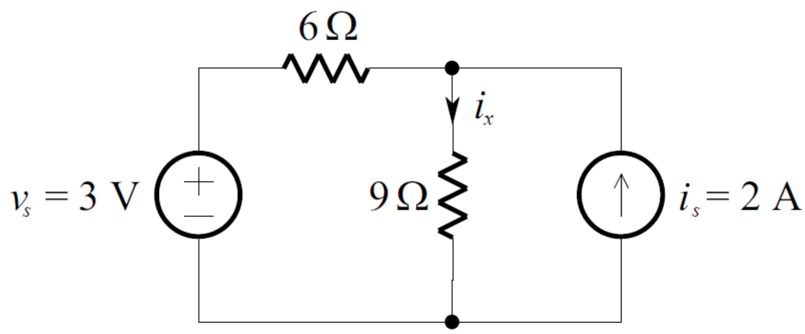
Work solutions – Week 4

Superposition:

We use superposition in the following circuit to write an expression for the unknown branch current i_x.



1.

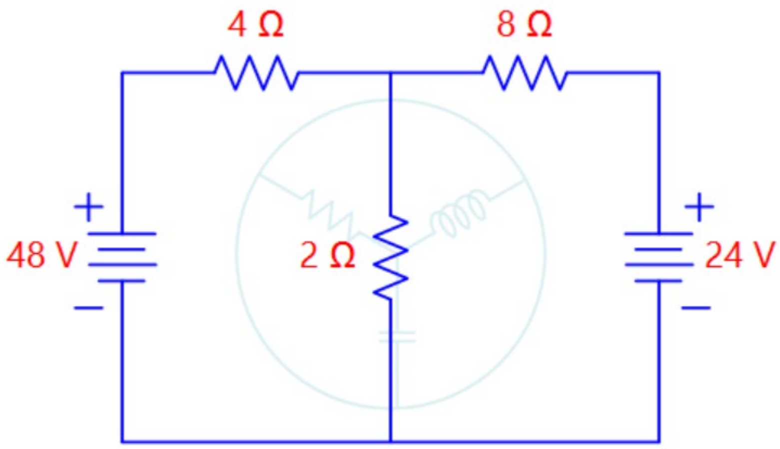
Solutions:

We first set the current source equal to zero (an open-circuit) and obtain the portion of i_x due to the voltage source as 0.2 A. Then if we let the voltage source be zero (a short-circuit) and apply the current divider rule, the remaining portion of i_x is seen to be 0.8 A.

We may write the answer in detail as:

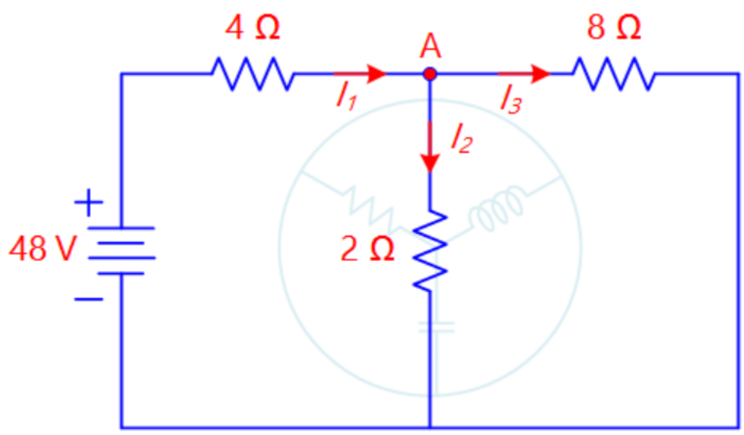
$$i_x = i_x\big|_{i_s=0} + i_x\big|_{v_s=0} = \frac{3}{6+9} + \frac{6}{6+9}2 = 0.2 + 0.8 = 1\,\text{A}$$

2. Find the current through 2 ohm resistor using superposition method.



Solutions:

At first, find the current through 2Ω resistor with 48V source acting alone. Hence replace the 24 V source by a short circuit.



Here current I_2 flows through the load resistor.

To find the load current, find the total current supplied by the source (I_1) with its total resistance. Then apply current division rule and find the current through 2Ω resistor with 48V source acting alone.

Calculations for this step is as follows

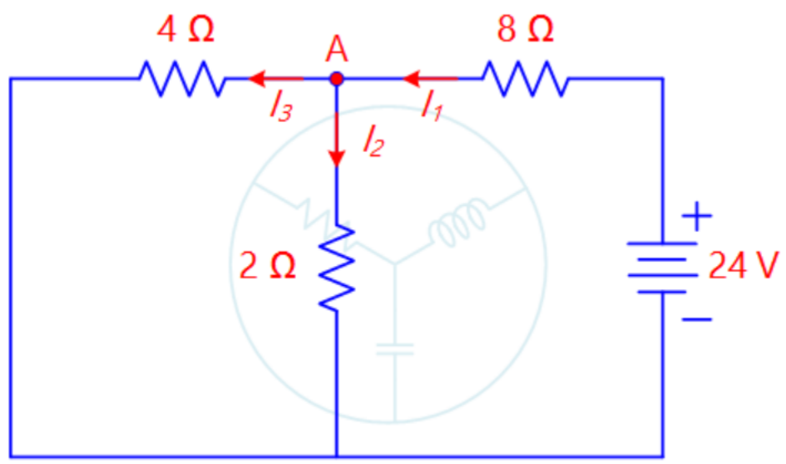
$$R_{eq} = 4 + \frac{8 \times 2}{8+2} = 4 + 1.6 = 5.6\ \Omega$$

$$I_{total} = \frac{48}{5.6} = 8.57\ A$$

$$I_{2\Omega(48V)} = 8.57 \times \frac{8}{8+2} = 6.86\ A$$

So, the current supplied by the 48V source is 6.86 Amperes.

Now consider the 24V source alone and replace 48 V source by a short circuit.



Now find the total resistance of the circuit and by find the total current supplied by the source.

Then apply current division rule at node 'A' and by find the current through 2 Ω resistor while 24V source acting alone.

$$R_{eq} = 8 + \frac{4 \times 2}{4+2} = 8 + 1.33 = 9.33\ \Omega$$

$$I_{total} = \frac{24}{9.33} = 2.57\ A$$

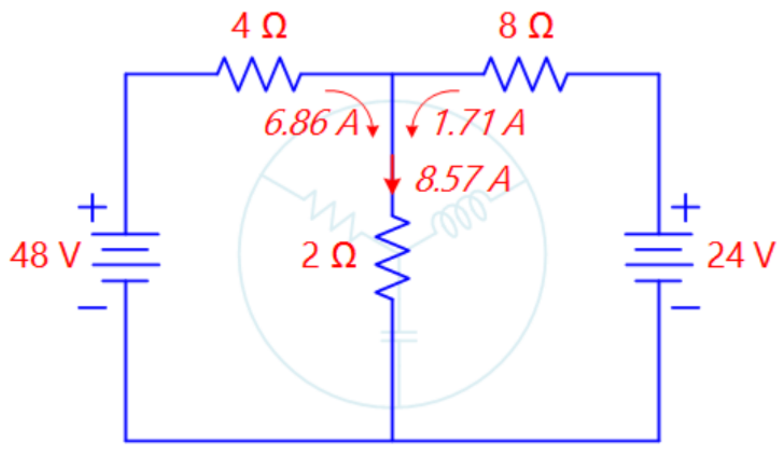
$$I_{2\Omega(24V)} = 2.57\frac{4}{4+2} = 1.71\ A$$

Here, the current supplied by the 24V source is 1.71 Amperes.

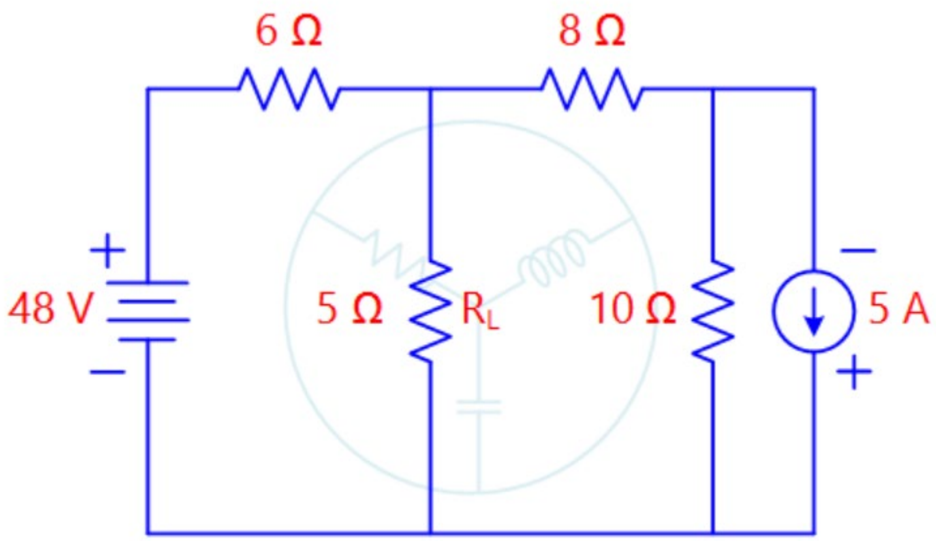
Finally, add the two currents considering their direction.

Here the two currents are flowing into the 2Ω resistor with the same direction. So the total current flowing through 2Ω will be the algebraic sum of $I_{2\Omega(48V)}$ and $I_{2\Omega(24V)}$.

$$I_{2\Omega} = I_{2\Omega(48V)} + I_{2\Omega(24V)} = 6.86 + 1.71 = 8.57\ Amperes$$



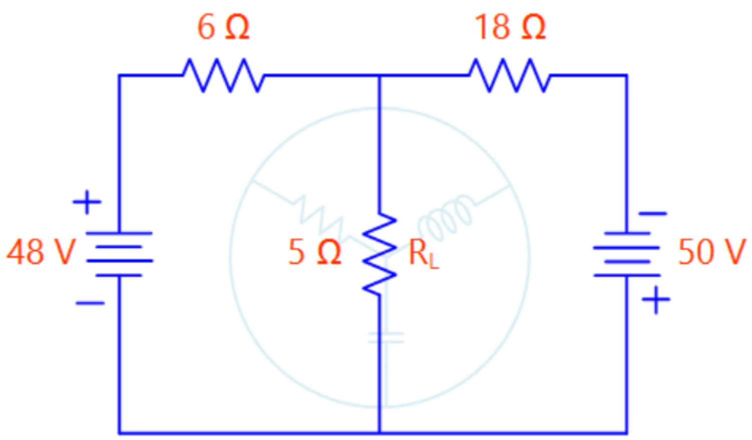
3. Find the current through 5 ohm resistor using superposition method.



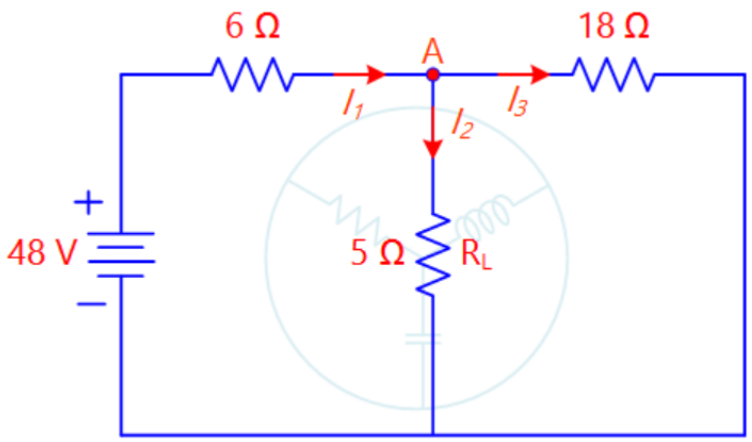
Solutions:

If you look the above circuit, it has a current and a voltage source. To reduce the complexity of the problem convert 5A current source in to its equivalent voltage source.

After the source Transformation, the circuit can be redrawn as given below.



At start, consider the 48V and replace 50V source by a short circuit. Now find the current through the 5Ω resistor by finding the total current and then applying current division rule at node 'A'.



The calculation steps are as follows.

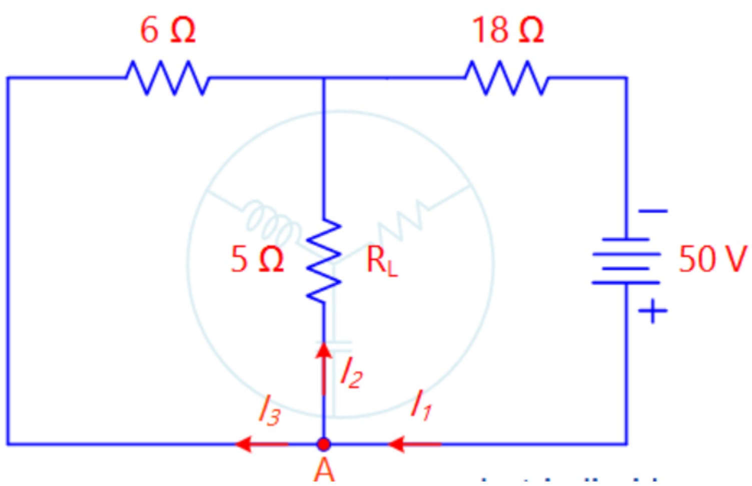
$$R_{eq} = 6 + \frac{5 \times 18}{5 + 18} = 6 + 3.91 = 9.91\ \Omega$$

$$I_{total} = \frac{48}{9.91} = 4.84\ A$$

$I_{5\Omega}$= 4.84*(18/18+5)=3.79A

Then, the current supplied by 48V voltage source is 3.79 A.

Now, consider the 50V voltage source only and replace 48V source by a short circuit. Now find the total current supplied by the 50V source and apply current division rule at node 'A' to find the load current.



The load current is calculated as follows,

$$R_{eq} = 18 + \frac{6 \times 5}{6+5} = 18 + 2.73 = 20.73\ \Omega$$

$$I_{total} = \frac{50}{20.73} = 2.41\ A$$

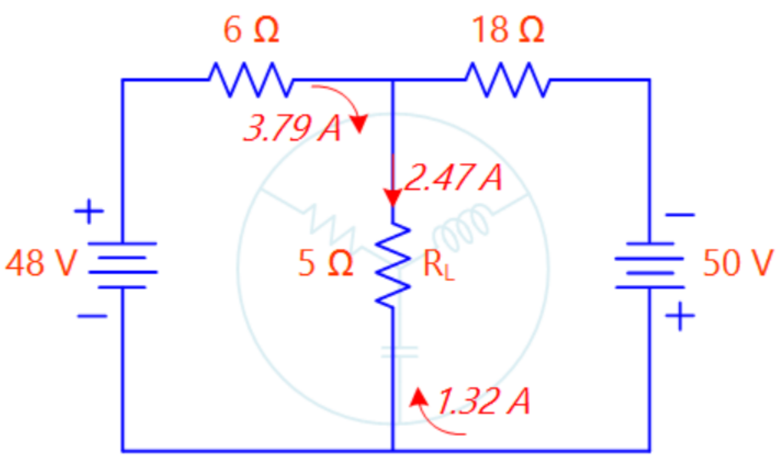
$$I_{5\Omega(50V)} = 2.41 \times \frac{6}{6+5} = 1.32\ Amperes$$

So the current supplied by 50V voltage source is 1.32 A.

Now perform algebraic sum of the current flows through the load resistor. The current by 48V source flows in to the load. Since the 50V source terminals are reversed, the current supplied by 50V source flows into the load in the opposite direction.

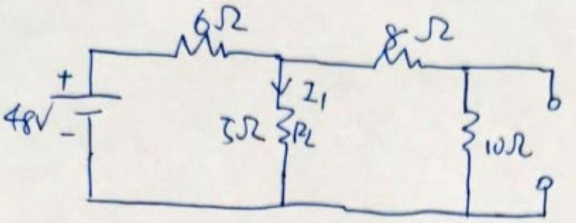
So the final current that flows through the load resistor can be calculated as,

$$I_{5\Omega} = I_{5\Omega(48V)} - I_{5\Omega(50V)} = 3.79 + (-1.32) = 2.47\ Amperes$$



4. For the above question, let us use another method to solve it. We don't do the source transformation.

Consider 48 V source only



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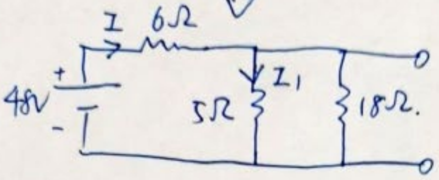
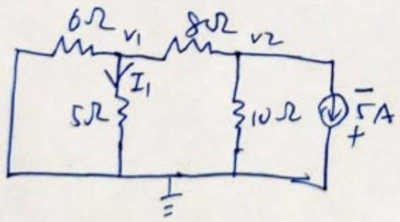


$$I = 48 \div \left(\frac{5\times18}{5+18} + 6\right) = 4.84\ A$$

$$I_1 = I \times \frac{18}{5+18} = \frac{48}{\left(\frac{5\times18}{5+18}+6\right)} \times \frac{18}{5+18}$$

$$= 3.79.$$

consider the current source only



using nodal analysis.

$$\begin{cases} \frac{V_1}{6} + \frac{V_1}{5} + \frac{V_1 - V_2}{8} = 0 \\ \frac{V_2 - V_1}{8} + \frac{V_2}{10} + 5 = 0 \end{cases}$$

$$\begin{cases} V_1 = -6.57\ V \Rightarrow I_1 = -1.32A \\ V_2 = \ldots \end{cases}$$

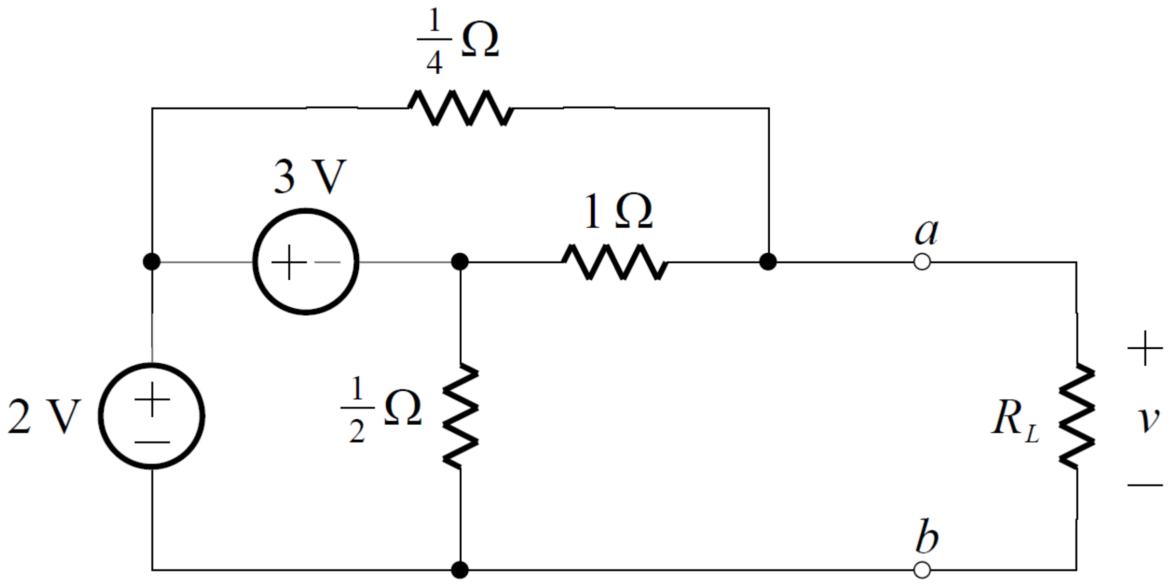
Since I_1 in both circuits are the same direction,

So $I_{total} = 3.79 - 1.32 = 2.47\ A$.

Thevenin's Theorem and Norton's Theorem

5.

Consider the circuit shown below:



(a) Find the Thévenin equivalent of the circuit to the left of terminals a and b.

(b) Find v when $R_L = \frac{1}{3}\,\Omega$.

(c) What value of R_L absorbs the maximum power?

(d) For the value of R_L determined in part (c), find the power absorbed by R_L.

Solutions:

Please be noted that, ¼ Ω resistor and the 1 Ω resistor are not connected to the voltage source as only one polarity end of them (A point) is connected with one end of the source, the other polarity end (B point) of the resistors are open in the air. So we don't need to consider these two resistors in circuit 1. So the 2V voltage is applied to the ½ resistor only. So the Vth1 is equal to 2V.

(a) Using superposition to calculate V_{th}, it can be divided into two circui

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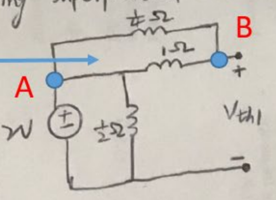

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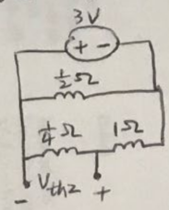
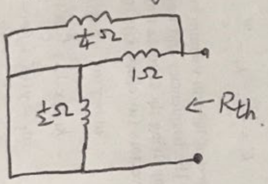


$V_{th1} = 2V$, $V_{th2} = \frac{-\frac{1}{4}}{1+\frac{1}{4}} \cdot 3 = -0.6V$

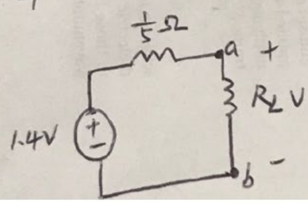
$\therefore V_{th} = V_{th1} + V_{th2} = 1.4V$

We can use the following circuit to calculate R_{th}:



$R_{th} = 1 // \frac{1}{4} = \frac{1}{5}\Omega$

∴ The Thévenin equivalent circuit should be:



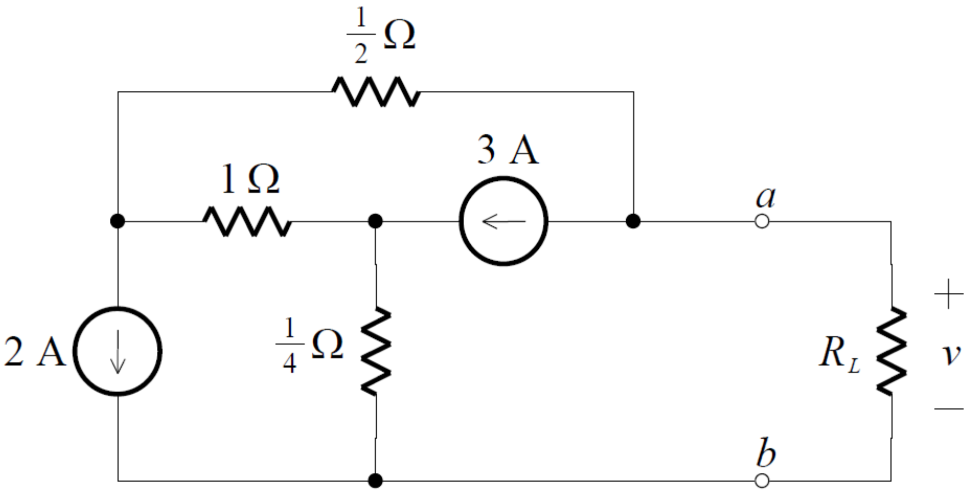
(b) when $R_L = \frac{1}{3}\Omega$. $V = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{5}} \cdot 1.4 = \frac{7}{8} V$

(c) For P_{Lmax} , $R_L = R_{th} = \frac{1}{5}\Omega$

(d) $P_{Lmax} = \frac{1}{5} \cdot \left(\frac{1.4}{\frac{1}{5}+\frac{1}{5}}\right)^2 = 2.45 W.$

6.

Consider the circuit shown below:



(a) Find the Thévenin equivalent of the circuit to the left of terminals a and b.

(b) Find v when $R_L = \frac{1}{3}\,\Omega$.

(c) What value of R_L absorbs the maximum power?

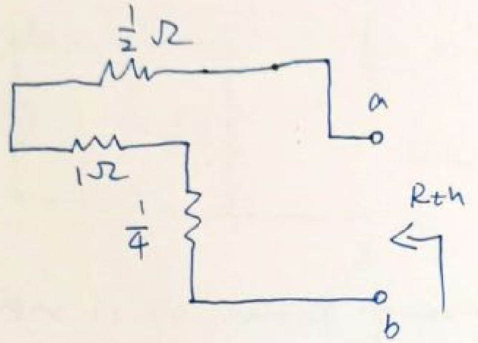
(d) For the value of R_L determined in part (c), find the power absorbed by R_L.

Solution:

(a)

1. Thevenin resistance.

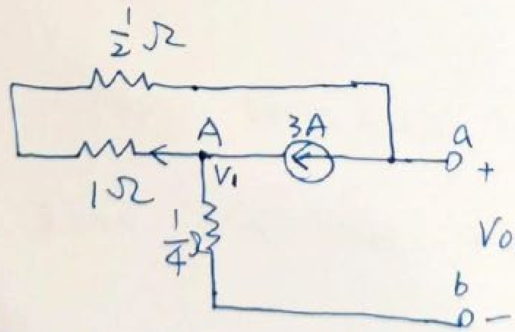
Remove power sources by short circuit voltage source, open circuit current source.



$$R_{th} = \frac{1}{2} + 1 + \frac{1}{4} = \frac{7}{4}\,\Omega.$$

2. use superposition to determine V_{oc}.

first, consider 3A source, open circuit the other one.



$V_{oc1} = V_a - V_b$.

since there is no current going through the $\frac{1}{4}\,\Omega$ resistor.

$V_b = V_1$

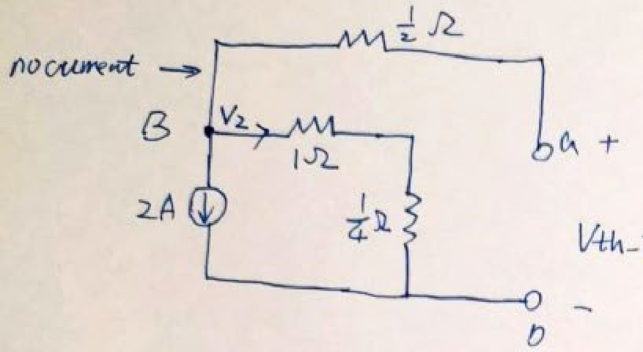
at point A, using KCL

$$\frac{V_1 - V_a}{1 + \frac{1}{2}} = 3 \Rightarrow V_1 - V_a = 4.5\,V.$$

Thus $V_{oc1} = V_a - V_b = V_a - V_1 = -4.5\,V$

Second, consider the 2A current source.

the circuit is below.



$V_{th-2} = V_a - V_b$

There is no current flowing through $\frac{1}{2}\Omega$ resistor.

$V_a = V_2$.

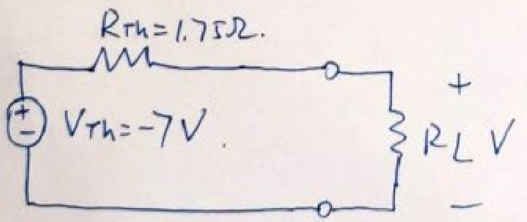
At point B using KCL

$$\frac{V_2 - V_b}{1+\frac{1}{4}} + 2 = 0 \Rightarrow V_2 - V_b = -2.5V$$

Thus, $V_{th-2} = V_a - V_b = V_2 - V_b = -2.5V$

Thus, $V_{Th} = V_{th-1} + V_{th-2} = -4.5 - 2.5 = -7 V$.

3. Thevenin equivalent circuit



(b) When $R_L = \frac{1}{3}\Omega$ $\quad V = -7 \times \frac{\frac{1}{3}}{\frac{1}{3}+\frac{7}{4}} = -1.12V$.

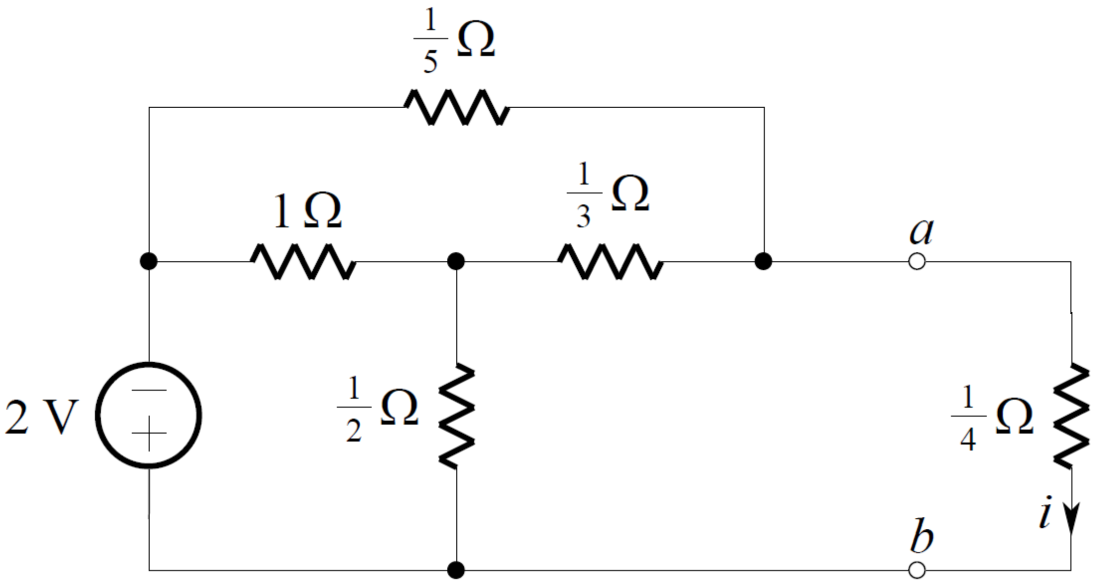
(c) When $R_L = R_{Th} = 1.75\Omega$, R_L has maximum electric power.

$V_{RL} = -7 \times \frac{1.75}{1.75+1.75} = -3.5V$. $\quad P = U_{RL} \times I_{RL} = U_{RL} \times \frac{U_{RL}}{R_L} = \frac{(U_{RL})^2}{R_L} = \frac{(-3.5)^2}{1.75}$

$= 7W$

7.

Consider the circuit shown below:

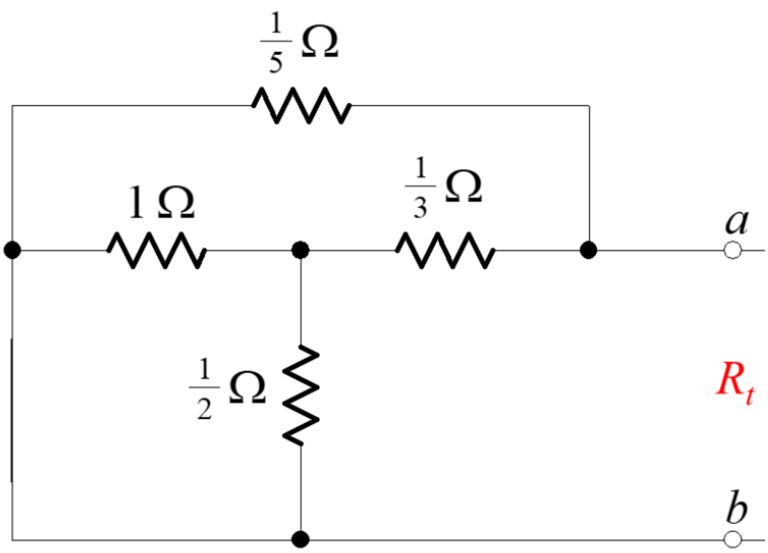


(a) Find the Norton equivalent of the circuit to the left of terminals a and b.

(b) Use the result of part (a) to find i.

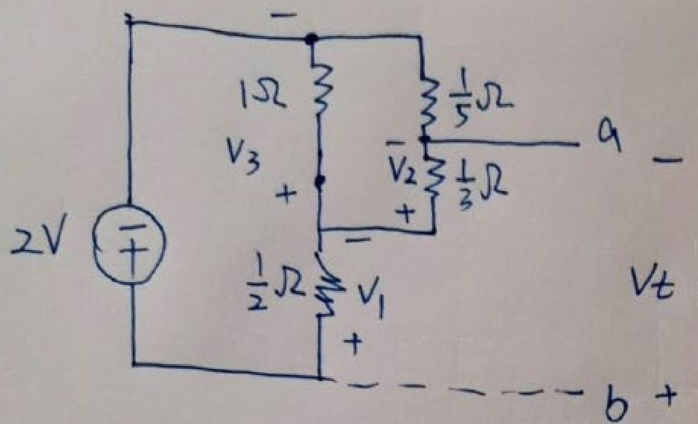
Solution:

(a) First, find the Thevenin resistance R_t by zeroing the voltage source;



R_t = 1/5 Ω // [(1 Ω // 1/2 Ω) + 1/3 Ω] = 1/5 Ω // (1/3 Ω + 1/3 Ω) = 2 / 13 Ω.

Second, find the Thevenin voltage V_t across the open terminals a and b;

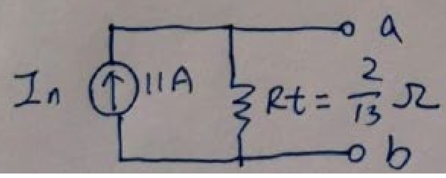


$$V_1 = 2V \times \frac{\frac{1}{2}\Omega}{\left[\frac{1}{2}\Omega + \underbrace{1\Omega \,||\, (\frac{1}{5}\Omega + \frac{1}{3}\Omega)}_{\frac{8}{23}}\right]} = 2 \times \frac{\frac{1}{2}}{\frac{39}{46}} = \frac{46}{39} V$$

$$V_2 = \underbrace{\left(2V - \frac{46}{39}V\right)}_{V_3} \times \left[\frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{5}}\right] = \frac{20}{39} V.$$

Thus, $V_t = V_1 + V_2 = \frac{22}{13} V$

$I_n = I_{sc} = V_t / R_t = 11$ A. The norton equivalent circuit is below:



(b) Given the Norton equivalent circuit above, use the current divider rule:

$i = -I_n \times [1/(1/4\ \Omega)] / [1/R_t + 1/(1/4\ \Omega)] = -88/21$ A