

Topics for Today

- CS Theory
 - Stack and heap recap
 - Tree data structure
 - Depth and Breadth first searches
- This week's lab
 - Designing a graph data structure
 - Finding connected components
 - Solving a sliding tile puzzle
 - Knight moves (chess)

Stack and Heap

Stack

- Fast allocation
- Only exists in scope
- Automatically deleted
- Fast deletion

Heap

- Uses `new` keyword
- Slow allocation
- Programmer decides lifetime
- Manually deleted
- Can cause memory leaks

```
Node stackNode{ val_: 1, head};
```

```
Node* heapNode = new Node{ val_: 2, head};
```

Designing a Graph Data Structure

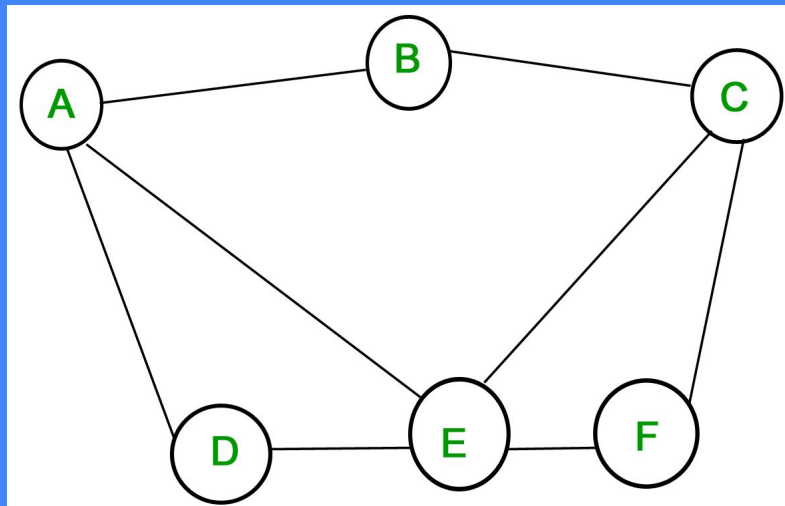
Most of the remainder of the course will be dealing with graph data structures. Let's take some time to think about how we can represent a graph through a data structure.

A graph is a collection of nodes, and the edges between the nodes.

So you need to think of a way of storing these two things.

And for this activity we'll look at ways of writing the following core functions:

- Add edge (u, v)
- Is edge (u, v)
- Adjacent To (v) - *get list edges on node v*
- Display - *print the graph*
- Constructor(number of nodes, directed?)



Designing a Graph Data Structure

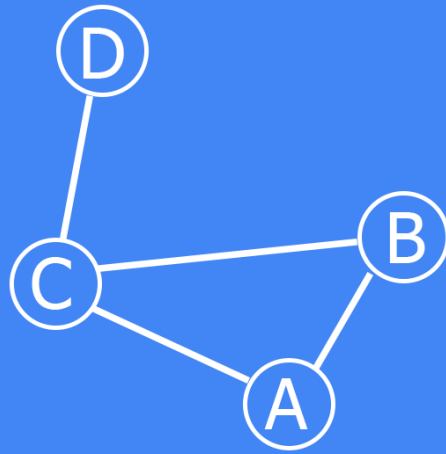
The approaches we will mostly be using in this course are:

1. Adjacency matrix - `vector<vector<bool>>`
A vector of vectors, forming a matrix. Each node gets a row and a column, cells in which represent the edges in our graph. The matrix is always the same size, $O(1)$ look up on edges.
2. Adjacency list - `vector<list<int>>` / `vector<set<int>>`

A vector of lists/sets, each node has a set, containing the nodes it has an edge to. Tends to take up less space than the matrix, but has $O(n)/O(\log n)$ edge lookup.

	A	B	C	D
A	[0, 1, 1, 0]			
B	[0, 0, 1, 0]			
C	[1, 1, 0, 1]			
D	[0, 0, 1, 0]			

A	{B, C}
B	{A, C}
C	{A, D}
D	{C}



Now go and implement a graph using one of these, we will be using it in the following exercise.

Depth and Breadth First Searches

Depth-First Search

- Explores options to exhaustion before moving on
- Stack data structure

			3	6		
			2	5		
			1	4		
			0			

Breadth-First Search

- Explores options widely, looking at all options before moving in towards extremities.
- Queue data structure
- Vertices are entered into queue in order of their distance to start node.

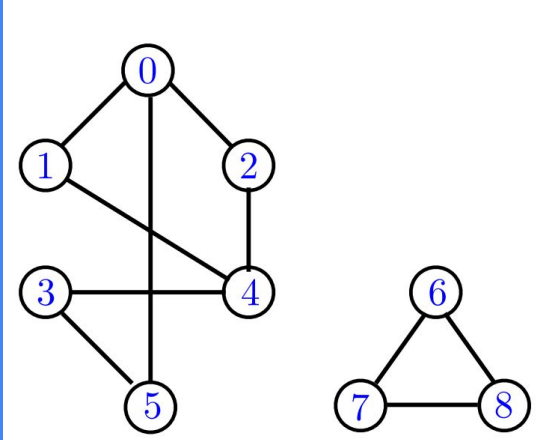
		8	1	2		
		7	0	3		
		6	5	4		

Finding Connected Components

Using the code from the previous exercise, solve this problem.

In a graph, return how many connected components there are, and list the nodes in them.

A connected component is a group of nodes which can reach each other through any number of edges.



So for our example there are two connected components, $\{0,1,2,3,4,5\}$, and $\{6,7,8\}$

We will solve this using *Depth First Search*, which uses a stack to order its traversal.

Knight Moves

Using the properties of how a knight moves in chess (2 units in one direction, one unit in an orthogonal direction), find the fewest number of moves to move from (0,0) to a destination (x,y) on an infinite chess board.

We can represent this as an abstract graph. Each node represents an x-y position on the board, and the edges from that node represent the available moves, leading to new positions.

You have to write the function

```
std::vector<Point> Knight::minKnightMoves(const Point& dest)
```

Returning the smallest set of moves to get from the origin to a destination.

Solve this using *Breadth-first Search*

You also have to write a function to return the path, but I am happy to give that to you.

	X		X		3	2	3	2	3	2	3
					2	3	2	3	2	3	2
X				X	3	4	1	2	1	4	3
					2	1	2	3	2	1	2
					3	2	3	0	3	2	3
X				X	2	1	2	3	2	1	2
					3	4	1	2	1	4	3
	X		X		2	3	2	3	2	3	2