# Applications of linear algebra

## with a couple of examples

# Applications of linear algebra

Linear algebra is one of the fundamental areas of mathematics.

It is required everywhere mathematics is involved or hidden
— and not only where the analysis is explicitly linear:

- Science
  - Mathematics
  - Astronomy
  - Physics
  - Chemistry
  - Biology
  - Statistics
- Engineering (mechanical, electrical, . . . )
- Economics and business
- Transport, logistics, . . .
- "Big Data" analysis, IT, AI, machine learning, . . .

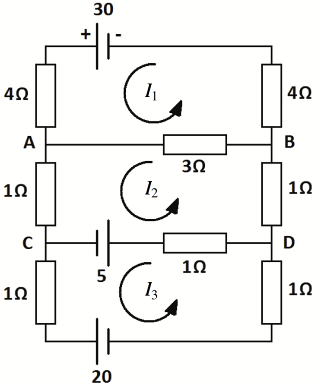
# Applications of linear systems

On a practical level, one of the aspects is solving linear equations.

- Linear systems naturally arise in network analysis
- Network is a set of branches through which something "flows"
  - Electrical wires (electricity flow)
  - Economic linkages (money flow)
  - Pipes through which oil, gas or water flows
  - Fibres through which information flows (Internet)
- Branches meet at nodes or junctions
- A numerical measure is the rate of flow through a branch
- Analysis of networks is based on linear systems

# Example: Electric circuits

Kirchhoff's law: *Algebraic sum of voltage drops* ( $V = RI$ ) *around a loop equals the algebraic sum of the voltage sources in the same direction around the loop:* $\sum R_i I_i = \sum V_i$.




Summing for upper, middle and lower loops:

$$4I_1 + 3I_1 - 3I_2 + 4I_1 = 30$$
$$-3I_1 + 3I_2 + I_2 + I_2 - I_3 + I_2 = 5$$
$$-I_2 + I_3 + I_3 + I_3 = -5 - 20$$

$$\begin{cases} 11I_1 - 3I_2 = 30 \\ -3I_1 + 6I_2 - I_3 = 5 \\ -I_2 + 3I_3 = -25 \end{cases} \qquad \left[\begin{array}{rrr|r} 11 & -3 & 0 & 30 \\ -3 & 6 & -1 & 5 \\ 0 & -1 & 3 & -25 \end{array}\right]$$

System solution yields loop currents: $I_1 = 3$ A, $I_2 = 1$ A, $I_3 = -8$ A

- Total current in branch AB is $I_1 - I_2 = 2$ A
- Total current in branch CD is $I_2 - I_3 = 9$ A

# Example: Balancing chemical equations

$$(x_1)\,\mathrm{C_3H_8} + (x_2)\,\mathrm{O_2} \;\;\rightarrow\;\; (x_3)\,\mathrm{CO_2} + (x_4)\,\mathrm{H_2O}$$

To find $x_i$ such that the amount of each atom is preserved, associate each molecule with a vector, counting $\mathrm{C}$, $\mathrm{H}$, $\mathrm{O}$ atoms:

$$\mathrm{C_3H_8} : \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix} \qquad \mathrm{O_2} : \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} \qquad \mathrm{CO_2} : \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \qquad \mathrm{H_2O} : \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

Then $$x_1 \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{array}{rrrrcl} 3x_1 & & -x_3 & & = & 0 \\ 8x_1 & & & -2x_4 & = & 0 \\ & 2x_2 & -2x_3 & -x_4 & = & 0 \end{array} \qquad \Leftrightarrow \qquad \left[\begin{array}{rrrr|r} 3 & 0 & -1 & 0 & 0 \\ 8 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$$

The solution of this system is $x_1 = 1,\ x_2 = 5,\ x_3 = 3,\ x_4 = 4$:

$$\mathrm{C_3H_8} + 5\,\mathrm{O_2} \;\;\rightarrow\;\; 3\,\mathrm{CO_2} + 4\,\mathrm{H_2O}$$