UNIVERSITY OF TECHNOLOGY SYDNEY

SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES

37233 LINEAR ALGEBRA

Tutorial 1

Question 1

Based on the axioms of linear space, prove that $\forall\, x \in \mathbb{R}: \quad x \cdot \mathbf{0} = \mathbf{0}$

(Hint: re-derive the $0 \cdot \mathbf{u} = \mathbf{0}$ property first, and make use of it).

Question 2

Determine whether or not the following sets are linear spaces:

(a) A set of all arrows in a Cartesian coordinate plane, drawn from the coordinate origin to a point within the first quadrant, with the usual rules for geometric addition and multiplication by a scalar.

(b) A set of all functions $f(x)$ defined for $x \in \mathbb{R}$, having the form $f(x) = \alpha \cos x + \beta \sin x$ (where α, $\beta \in \mathbb{R}$), with the usual rules for addition and multiplication by a scalar.

Question 3

(a) Consider a set X of all solutions to a vector equation $\mathbf{Ax} = \mathbf{0}$ with $m \times n$ matrix $\mathbf{A}$. Determine if X is a subspace of the linear space $\mathbb{R}^n$.

(b) Suppose H is a subspace of vector space V, and $\mathbf{v}_0 \in V$ but $\mathbf{v}_0 \notin H$. Consider a *hyperplane* Γ as a set of all $\mathbf{y} \in V$, such that $\mathbf{y} = \mathbf{v}_0 + \mathbf{x}$ where $\mathbf{x} \in H$. Determine if Γ is a subspace of V.

Question 4

Prove the following property of linear spaces: $\forall\, \mathbf{v} \in \mathbb{R}: \quad (-1) \cdot \mathbf{v} = -\mathbf{v}$

Start with the property $0 \cdot \mathbf{u} = \mathbf{0}$ (see the lecture), but only use the axioms further on.

Question 5

Consider the set of all negative numbers, and define the operation of "addition" of two elements as their arithmetic multiplication, and the operation of "multiplication by a scalar" as an arithmetic operation of taking the element to the power of that scalar. Check if, with such operations, this set is a linear space.

Question 6

(a) Check if the set of all polynomials of a degree equal to 3, is a subspace of the linear space of all polynomials with a degree up to 4.

(b) Check if the set of all functions $f(t) = a \cos t$ is a subspace of the linear space of functions $g(t) = b \sin t + c \cos t$, where a, b, $c \in \mathbb{R}$ and $t \in [0, \pi]$.