

UNIVERSITY OF TECHNOLOGY SYDNEY
SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES
37233 LINEAR ALGEBRA

Tutorial 10

Question 1

Orthogonally diagonalise $\mathbf{A} = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$ and construct its spectral decomposition.

Question 2

Write the matrices of the following quadratic forms:

- (a) $Q(x) = 8x_1^2 + 7x_2^2 - 3x_3^2 - 6x_1x_2 + 4x_1x_3 - 2x_2x_3$
- (b) $Q(x) = 4x_1x_2 + 6x_1x_3 - 8x_2x_3$

Question 3

For the quadratic form $Q(x) = 3x_1^2 - 4x_1x_2 + 6x_2^2$ find a change of variable $\mathbf{x} = \mathbf{P}\mathbf{y}$ for a transformation into quadratic form $Q(y)$ with a diagonal matrix.

Question 4

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$$

- (a) Find the singular values of $\mathbf{A}$.
- (b) Find a unit vector $\mathbf{x}$ for which $\mathbf{A}\mathbf{x}$ attains the maximum length.
- (c) Construct a singular value decomposition of $\mathbf{A}$.