

UNIVERSITY OF TECHNOLOGY SYDNEY
SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES
37233 LINEAR ALGEBRA

Exercises 11

Question 1

Using normal equations, obtain the least-squares solution to the system, and find the error of the approximation.

$$\begin{cases} 2x_1 + x_2 = 3 \\ -x_1 + x_2 = -7 \\ -3x_1 = 2 \\ 1x_1 - x_2 = 3 \end{cases}$$

Question 2

Let

$$\mathbf{A} = \begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$$

- (a) Find an orthogonal basis for $\text{Col } \mathbf{A}$;
- (b) Construct a QR decomposition of \mathbf{A} .

Question 3

Using the QR-decomposition approach, obtain a least-squares solution for system

$$\begin{cases} 3x_1 - 5x_2 + x_3 = 1 \\ x_1 + x_2 + x_3 = -1 \\ -x_1 + 5x_2 - 2x_3 = 4 \\ 3x_1 - 7x_2 + 8x_3 = 1 \end{cases}$$

(use the QR-decomposition obtained with the previous question).

Hint: keep the arising square roots as common multipliers to facilitate calculations.

Question 4

Consider the following system of equations:

$$\begin{aligned} x_1 - x_2 &= -1 \\ 2x_1 + x_2 &= -3 \\ x_1 + x_2 &= 3 \end{aligned}$$

- (a) Find a least-squares solution to this system using normal equations
- (b) Calculate the least-squares error of the solution
- (c) Obtain a QR factorisation of the matrix of this system
- (d) Use the QR factorisation to obtain the least-squares solution