University of Technology Sydney
School of Mathematical and Physical Sciences

37233 Linear Algebra

Exercises 1

Question 1

Find all solutions to the system:

$$\begin{aligned} x_1 - 2x_2 - 4x_3 + 4x_4 &= -1 \\ -3x_1 + 6x_2 + 4x_3 + 3x_4 &= 10 \\ -2x_1 + 4x_2 + 2x_3 + 2x_4 &= 6 \\ -x_1 + 2x_2 - 2x_3 + 2x_4 &= 1 \end{aligned}$$

Question 2

(a) Using row reduction, calculate the determinants of the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 8 & 7 \\ 2 & 9 & 6 \\ 3 & 4 & 5 \end{bmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{bmatrix} 1 & -2 & 2 & 3 & 0 \\ -1 & 3 & -3 & -6 & 5 \\ 5 & -10 & 12 & 0 & -1 \\ -4 & 8 & -8 & -9 & 9 \\ 3 & -6 & 6 & 9 & 5 \end{bmatrix}.$$

(b) For $\mathbf{A}$, compare the ease your calculation to a direct calculation by definition.

(c) Imagine doing a direct calculation for $\mathbf{B}$: how many terms you would need to have?

Question 3

Find the inverse of $\mathbf{A} = \begin{bmatrix} 1 & 0 & 7 \\ -1 & 2 & 0 \\ -2 & 5 & 3 \end{bmatrix}$ if this matrix is not singular.

Question 4

Determine value of x for which the matrix below is singular:

$$\mathbf{A} = \begin{bmatrix} x & 2 & 1 \\ 5 & -1 & 2 \\ -3 & 1 & -1 \end{bmatrix}$$