University of Technology Sydney
School of Mathematical and Physical Sciences

37233 Linear Algebra

Solutions 1

Question 1 [1]

Row-reduction of the augmented matrix of the system yields

$$\begin{bmatrix} 1 & -2 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

implying that x_2 is a free variable whereas

$$x_1 = 2x_2 - 1$$
$$x_3 = x_4 = 1$$

Question 2 [2]

(a) Row-reduction of $\mathbf{A}$ to echelon form without row multiplication yields

$$\begin{bmatrix} 1 & 8 & 7 \\ 0 & -7 & -8 \\ 0 & 0 & 48/7 \end{bmatrix}$$

and requires no row swaps, so the determinant of $\mathbf{A}$ equals to that of the above matrix:

$$\det \mathbf{A} = 1 \cdot (-7) \cdot (48/7) = -48$$

Row-reduction of $\mathbf{B}$ to echelon form without row multiplication yields

$$\begin{bmatrix} 1 & -2 & 2 & 3 & 0 \\ 0 & 1 & -1 & -3 & 5 \\ 0 & 0 & 2 & -15 & -1 \\ 0 & 0 & 0 & 3 & 9 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

and requires no row swaps, so the determinant of $\mathbf{A}$ equals to that of the above matrix:

$$\det \mathbf{A} = 1 \cdot 1 \cdot 2 \cdot 3 \cdot 5 = 30$$

(b) Calculating $\det \mathbf{A}$ directly by definition is

$$\det \mathbf{A} = a_{11}a_{22}a_{33} + a_{13}a_{21}a_{32} + a_{12}a_{23}a_{31} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32} = -48$$

which, of course, gives the same answer and requires approximately the same amount of arithmetic operations as the row-reduction process.

(c) Direct calculation by definition would involve an addition of $5! = 120$ terms with 5 multipliers each, which is overwhelmingly more difficult than the row reduction process, even we take into account that there are two zero elements in the matrix, which eliminates $(2 \cdot 4! - 3!) = 42$ of these terms, but 78 is still quite a number. That was certainly a very nice and quick row-reduction for $\mathbf{B}$, all done with the first row alone; but even if more row operations were required, it still would have been much quicker.

Question 3 **[1]**

The inverse can be found by row-reducing the augmented $\left[\,\mathbf{A}\,|\,\mathbf{I}\,\right]$ matrix, which yields

$$\begin{bmatrix} 1 & 0 & 0 & -6 & -35 & 14 \\ 0 & 1 & 0 & -3 & -17 & 7 \\ 0 & 0 & 1 & 1 & 5 & -2 \end{bmatrix}$$

and therefore

$$\mathbf{A}^{-1} = \begin{bmatrix} -6 & -35 & 14 \\ -3 & -17 & 7 \\ 1 & 5 & -2 \end{bmatrix}$$

Question 4 **[1]**

The matrix is singular if its determinant is zero, or, equivalently, if the row-reduced form does not have pivots in every row. By calculating

$$\det \mathbf{A} = a_{11} \cdot (1-2) - 2 \cdot (-5+6) + (5-3) = -a_{11}$$

we see that for it to be equal to zero (so that $\mathbf{A}$ is singular) we require $a_{11} = 0$.