University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Solutions 2

Question 1

Row-reducing the matrix of the system yields

$$\longrightarrow \begin{bmatrix} 1 & 1 & 4 & 1 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

so the general solution can be written as

$$\mathbf{x} = \begin{bmatrix} x_3 - 2x_4 \\ -5x_3 + x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} \equiv x_3\mathbf{v}_1 + x_4\mathbf{v}_2 \qquad \forall\, x_3,\, x_4$$

and thus the above vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ span the solution space of the system.

Question 2

Row-reducing the matrix of the system yields

$$\longrightarrow \begin{bmatrix} 1 & 1 & -1 & -2 \\ 0 & -3 & -1 & -1 \\ 0 & 0 & -16 & -16 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

so the general solution for the homogeneous system can be written as

$$\mathbf{x} = \begin{bmatrix} x_4 \\ 0 \\ -x_4 \\ x_4 \end{bmatrix} = x_4 \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \equiv t\,\mathbf{v}, \qquad \forall\, t \in \mathbb{R}$$

The same row-reduction sequence over the (inhomogeneous) augmented matrix yields

$$\longrightarrow \begin{bmatrix} 1 & 1 & -1 & -2 & 1 \\ 0 & -3 & -1 & -1 & 5 \\ 0 & 0 & -16 & -16 & -16 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 4 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

and by setting the free variable $x_4 = 0$ a particular solution is obtained as

$$\mathbf{p} = \begin{bmatrix} 4 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \qquad \text{so the complete solution is:} \quad \mathbf{x} = \begin{bmatrix} 4 \\ -2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \qquad t \in \mathbb{R}.$$