UNIVERSITY OF TECHNOLOGY SYDNEY
SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES

37233 LINEAR ALGEBRA

Tutorial 2

Question 1

Let

$$\mathbf{a} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix} 0 & 1 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \end{bmatrix}$$

Calculate by hand, where possible, or identify those calculations which are *not* possible:

$$\mathbf{a}+\mathbf{b}, \quad \mathbf{Aa}, \quad \mathbf{Ab}, \quad \mathbf{Ba}, \quad \mathbf{Bb},$$

$$\mathbf{A}+\mathbf{B}, \quad \mathbf{A}+\mathbf{B}^{\mathsf{T}}, \quad \mathbf{AB}, \quad \mathbf{BA}, \quad \mathbf{AB}^{\mathsf{T}}$$

Question 2

Using Gaussian elimination, find all solutions to the following systems (a)–(d).
Indicate pivot positions, basic variables and free variables if any.

(a)

$$\begin{aligned} 2x_1 - 4x_2 \qquad + x_4 + 7x_5 &= 11 \\ x_1 - 2x_2 - x_3 + x_4 + 9x_5 &= 12 \\ -x_1 + 2x_2 + x_3 + 3x_4 - 5x_5 &= 16 \\ 4x_1 - 8x_2 + x_3 - x_4 + 6x_5 &= -2 \end{aligned}$$

(b)

$$\begin{aligned} x_1 - x_2 - 3x_3 + 8x_4 &= -2 \\ 3x_1 \qquad - 3x_3 + 9x_4 &= -1 \\ x_1 + x_2 + x_3 - 2x_4 &= 1 \end{aligned}$$

(c)

$$\begin{aligned} -x_1 + x_2 + x_3 &= 9 \\ 2x_1 + x_2 - x_3 &= -10 \\ 3x_1 \qquad - 2x_3 &= -19 \\ -x_1 + 2x_2 - 3x_3 &= -10 \end{aligned}$$

Question 3

Consider the system of linear equations $\mathbf{Ax} = \mathbf{b}$ where

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 5 & 5 \\ 3 & 1 & -3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

(a) Find $\mathbf{A}^{-1}$ by row reducing the augmented matrix $\left[\mathbf{A} \,|\, \mathbf{I}\right]$.

(b) Use the inverse matrix to solve the system of equations.

Question 4

Using row-reduction to echelon form, find the determinant of the matrix

$$\mathbf{A} = \begin{bmatrix} -2 & 4 & -6 & 8 \\ 4 & -8 & 2 & -7 \\ -3 & 6 & -9 & 7 \\ 5 & -9 & 18 & -13 \end{bmatrix}$$

Question 5

Assume that $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are different invertible matrices, with some of the relationships as shown in the table below. Based on that information:

(a) Fill in the missing entries in the table to the right.

(b) Identify the identity matrix.

(c) Which matrix is the inverse of the matrix $\mathbf{C}$?

(d) Use the table to obtain the result of $\mathbf{BAC}$.

Product	Result
AA	
AB	
AC	
BA	**B**
BB	**C**
BC	**A**
CA	
CB	
CC	

Question 6

Find 2×2 real matrix with the property $\mathbf{A^2} = -\mathbf{I}$, where $\mathbf{I}$ is an identity matrix.

University of Technology Sydney
School of Mathematical and Physical Sciences

37233 Linear Algebra

Tutorial 2 continuation

Question 1

Describe and compare the solution sets for the homogeneous equation: $x_1 + 9x_2 - 4x_3 = 0$

and the corresponding inhomogeneous equation: $x_1 + 9x_2 - 4x_3 = 4$

Provide a geometric interpretations for the different solution versions.

Question 2

Write the general solution to the following $\mathbf{Ax} = \mathbf{b}$ system: $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 5 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

as the sum of a particular solution to $\mathbf{Ax} = \mathbf{b}$ and the general solution to $\mathbf{Ax} = \mathbf{0}$.