UNIVERSITY OF TECHNOLOGY SYDNEY
SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES

37233 LINEAR ALGEBRA

Exercises 3

Question 1

Find if the following system has a solution for any vector $\mathbf{b} \in \mathbb{R}^3$, and if not, specify the conditions for $\mathbf{b}$ to be a solution:

$$\begin{bmatrix} 2 & -2 \\ 3 & 3 \\ 4 & -4 \end{bmatrix} \mathbf{x} = \mathbf{b}$$

Provide a geometrical interpretation of the result.

Question 2

Determine whether or not the following vectors span $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}.$$

Question 3

Check if the following four vectors are linearly independent. If not, obtain a linear dependence equation for them:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ -3 \\ -4 \\ -4 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 1 \\ 3 \\ 2 \\ 2 \end{bmatrix}.$$

Question 4

Determine if the columns of each of these matrices are linearly independent:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Is it necessary to make any calculations here? Justify the answers.