UNIVERSITY OF TECHNOLOGY SYDNEY
SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES

37233 LINEAR ALGEBRA

Tutorial 3

Question 1

Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix} \qquad \text{and} \qquad \mathbf{y} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$$

Find h such that $\mathbf{y}$ can be written as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

Question 2

Determine whether or not the following vectors span $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix}.$$

Question 3

Determine whether or not the columns of $\mathbf{A}$ (the same matrix as encountered in week 2) are linearly independent, and if not, write a linear dependence equation:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 & 5 \\ 3 & 3 & 6 & 1 & 14 \\ 0 & -1 & 0 & -2 & -9 \end{bmatrix}$$

Question 4

(i) Determine whether or not the columns of the following matrix span $\mathbb{R}^4$:

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 0 & -3 \\ 0 & 12 & -2 \\ 3 & 3 & 4 \end{bmatrix}$$

(ii) Take $\mathbf{A}^\top$ and determine if the columns of $\mathbf{A}^\top$ span $\mathbb{R}^3$.

(iii) Find if the columns of $\mathbf{A}$ are linearly independent; if not, obtain a linear dependence equation.

(iv) Find if the rows of $\mathbf{A}$ are linearly independent; if not, obtain a linear dependence equation.

(Question 5 omitted)

If there is no sufficient time, the remaining questions need to be self-studied:

Question 6

Check the following statements to determine if each of them is true or false:

(a) Any linear combination of vectors in $\mathbb{R}^n$ can always be written in the form $\mathbf{Ax}$.

(b) Matrix $\mathbf{A}$ must have a pivot in every row in order for $\mathbf{Ax} = \mathbf{b}$ to have a solution.

(c) If $\mathbf{Ax} = \mathbf{b}$ is consistent, then $\mathbf{b}$ is in the space spanned by the columns of $\mathbf{A}$.

(d) If the columns of an $n \times m$ matrix $\mathbf{A}$ do not span $\mathbb{R}^n$, then the equation $\mathbf{Ax} = \mathbf{b}$ is inconsistent for some $\mathbf{b} \in \mathbb{R}^n$.

Question 7

Find the value of h such that $\mathbf{b}$ belongs to the span of the columns of $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & -1 \\ 3 & 4 & 1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 2 \\ 5 \\ h \end{bmatrix}.$$

Provide a geometric interpretation of the result.