University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Exercises 4

Question 1

Given the following set of vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 13 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 7 \\ -9 \\ -8 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} -4 \\ 8 \\ -4 \end{bmatrix}, \quad \mathbf{v}_5 = \begin{bmatrix} -6 \\ 7 \\ 9 \end{bmatrix}$$

(a) Specify all the bases for V which can be explicitly constructed out of the above vectors (just the unique sets, regardless of the order of vectors in each set).

(b) Write a representation of vector $\mathbf{v}_3$ as a linear combination in each basis.

Question 2

Consider the following elements of a linear space $\mathbb{M}_2^3$ of all 2×3 matrices:

$$\mathbf{E}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{E}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{E}_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{E}_4 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{E}_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{E}_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Demonstrate that the set $\{\mathbf{E}_i\}$ is a linearly independent set;

(b) Show that any element of $\mathbb{M}_2^3$ can be represented as a linear combination of $\{\mathbf{E}_i\}$;

(c) Figure out if $\{\mathbf{E}_i\}$ is a basis for $\mathbb{M}_2^3$.

Question 3

Find a basis for the set of all vectors defined as follows:

$$H = \left\{ \begin{bmatrix} a + 3c - 5d \\ 2(a + c - d) + 4b \\ b - c + 2d \\ 3(a + d) + 9b \end{bmatrix} \right\}, \qquad \text{where} \quad a,\ b,\ c,\ d\ \in \mathbb{R}.$$

Question 4

Consider the linear space of all positive numbers, where "addition" of two elements is defined as their arithmetic multiplication, and "multiplication by a scalar" is defined as an arithmetic operation of taking the element to the power of that scalar.

(a) Provide an example of a basis for this linear space.

(b) Specify the dimension of this linear space.