UNIVERSITY OF TECHNOLOGY SYDNEY

SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES

37233 LINEAR ALGEBRA

Tutorial 4

Question 1

Consider vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -8 \end{bmatrix}, \quad \text{and} \quad \mathbf{v}_3 = \begin{bmatrix} -3 \\ 7 \end{bmatrix}.$

(a) Do three vectors form a basis for $\mathbb{R}^2$? What are the possible variants of a basis from this set?

(b) For each variant, express $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ via the basis vectors.

Question 2

Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}, \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} 8 \\ 2 \\ 12 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 12 \\ 8 \\ 2 \end{bmatrix}$

(a) Show that $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis for $H = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$. Specify $\dim H$.

(b) Determine whether or not $\mathbf{x} \in H$ and/or $\mathbf{y} \in H$.

Question 3

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix}, \quad \text{and} \quad H = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}.$

(a) Find a basis for H, and specify the dimension of H.

(b) Identify all the subsets of these vectors which form a basis for H.

Question 4

Find the dimension of the subset of all polynomials $\{q(t)\}$ of $\mathbb{P}_3$ such that $q(0) = 0$.

Question 5

Consider the following set $\{\mathbf{W}_i\}$ of matrices from $\mathbb{M}_2^2$ linear space:

$$\mathbf{W}_1 = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, \qquad \mathbf{W}_2 = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}, \qquad \mathbf{W}_3 = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \qquad \mathbf{W}_4 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Specify the dimension of $\mathbb{M}_2^2$. Show that the above set is a basis $\mathcal{W}$ for $\mathbb{M}_2^2$.