University of Technology Sydney
School of Mathematical and Physical Sciences
37233 Linear Algebra

Tutorial 5

Question 1

Given the following bases $\{b_i\}$ and $\{c_i\}$ for $\mathbb{R}^2$:

$$\mathbf{b}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 7 \\ -2 \end{bmatrix}; \qquad \mathbf{c}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad \mathbf{c}_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}.$$

(a) Calculate the $\mathcal{B}$-coordinates of $\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(b) Obtain the change of coordinates matrix $\mathbf{P}_{\mathcal{C}\leftarrow\mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$ and use it to find $[\mathbf{x}]_{\mathcal{C}}$.

(c) Calculate the $\mathcal{C}$-coordinates of $\mathbf{y} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$.

(d) Obtain the change of coordinates matrix $\mathbf{P}_{\mathcal{B}\leftarrow\mathcal{C}}$ from $\mathcal{C}$ to $\mathcal{B}$ and use it to find $[\mathbf{y}]_{\mathcal{B}}$.

Question 2

Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots \mathbf{b}_n\}$ be a basis for a vector space V. Explain why the $\mathcal{B}$-coordinates of vectors $\mathbf{b}_1, \ldots \mathbf{b}_n$ are the columns $\mathbf{e}_1, \ldots \mathbf{e}_n$ of the $n \times n$ identity matrix.

Question 3

Let $\mathcal{B} = \{\mathbf{b}_1,\, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1,\, \mathbf{c}_2\}$ be bases for $\mathbb{R}^2$, which are unknown as such, however we know that $\mathbf{b}_1 = -\mathbf{c}_1 + 3\mathbf{c}_2$ and $\mathbf{b}_2 = 2\mathbf{c}_1 - 4\mathbf{c}_2$.

(a) Find the change-of-coordinate matrices from $\mathcal{B}$ to $\mathcal{C}$, and from $\mathcal{C}$ to $\mathcal{B}$.

(b) Find $[\mathbf{x}]_{\mathcal{C}}$ given that $\mathbf{x} = 5\mathbf{b}_1 + 3\mathbf{b}_2$.

(c) Find $[\mathbf{y}]_{\mathcal{B}}$ given that $\mathbf{y} = 3\mathbf{c}_1 - 5\mathbf{c}_2$.

Question 4

Consider the following basis $\mathcal{W} = \{\mathbf{W}_i\}$ for $\mathbb{M}_2^2$ linear space (see Tutorial 4):

$$\mathbf{W}_1 = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, \qquad \mathbf{W}_2 = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}, \qquad \mathbf{W}_3 = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \qquad \mathbf{W}_4 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Find the coordinates $[\mathbf{Y}]_{\mathcal{W}}$ of matrix $\mathbf{Y} = \begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$ in this basis.

Hint: Make use of the coordinates relative to the standard basis of $\mathbb{M}_2^2$.

Question 5

The Chebyshev polynomials are widely used in calculus and mathematical methods. The first five Chebyshev polynomials (of the first kind) are given by

$$
\begin{aligned}
T_0(x) &= 1 \\
T_1(x) &= x \\
T_2(x) &= 2x^2 - 1 \\
T_3(x) &= 4x^3 - 3x \\
T_4(x) &= 8x^4 - 8x^2 + 1
\end{aligned}
$$

(a) Confirm that the set of the above Chebyshev polynomials is a basis Θ for $\mathbb{P}_4$.

(b) Find the change of coordinates matrix from the standard basis $\mathcal{P}$ in $\mathbb{P}_4$, $\{x^0, x^1, x^2, x^3, x^4\}$, to the Chebyshev basis.

(c) Define f as a polynomial of Maclaurin series for $\cos(4x)$, keeping the terms up to the fourth power in x, and write $[f]_{\mathcal{P}}$.

(d) Using the change of coordinate matrix found in (b), find $[f]_{\Theta}$.