University of Technology Sydney
School of Mathematical and Physical Sciences

37233 Linear Algebra

Self-study problems

Question 1

Consider

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 3 \\ 2 & -1 & 1 \\ 3 & 3 & 5 \end{bmatrix}$$

(a) Construct a sequence of elementary row operation matrices $\mathbf{E}_i$ which reduces the above matrix $\mathbf{A}$ to an upper triangular form: $\mathbf{E}_j \dots \mathbf{E}_1 \mathbf{A} = \mathbf{U}$

(b) Using the elementary matrices constructed in (a), decompose $\mathbf{A} = \mathbf{LU}$.

Question 2

Using LU-factorisation, find the solution of $\mathbf{Ax} = \mathbf{b}$ where

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 3 \\ -6 & -6 & -5 \\ 10 & 11 & 6 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

(a) Use Doolittle's algorithm to obtain an LU-factorisation of $\mathbf{A}$

(b) Use Crout's algorithm to obtain an LU-factorisation of $\mathbf{A}$

(c) Solve the systems as $\mathbf{Ly} = \mathbf{b}$ for $\mathbf{y}$ and then $\mathbf{Ux} = \mathbf{y}$ for $\mathbf{x}$

Question 3

Using Cholesky's algorithm, find the decomposition of

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 2 & -4 \\ -2 & -4 & 14 \end{bmatrix}$$

Is it possible to determine if the decomposition is possible, prior to attempting it?