University of Technology Sydney
School of Mathematical and Physical Sciences

37233 Linear Algebra

# Exercises 7

## Question 1

Let $\widehat{T}$ be a linear transformation with the following matrix $\mathbf{T}$:

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 2 & -2 \end{bmatrix}$$

(a) Specify the domain and codomain of $\widehat{T}$;

(b) Find all the images of $\mathbf{u} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

(c) Describe the range of $\widehat{T}$ and find its basis;

(d) Find all the sources for $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

## Question 2

Consider a linear transformation $\widehat{T}$ with the standard matrix $\mathbf{T} = \begin{bmatrix} 3 & 3 \\ 0 & 1 \end{bmatrix}$.

(a) Specify the domain, codomain and range of $\widehat{T}$.

(b) Illustrate the effect of $\widehat{T}$ by mapping its action on a unit square in the domain of $\widehat{T}$.

(c) Find the vector in the domain of $\widehat{T}$ if its image is $\begin{bmatrix} 3 \\ 1/2 \end{bmatrix}$, and illustrate the result.

## Question 3

Let $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{y}_1 = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{y}_2 = \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix}, \quad \mathbf{y}_3 = \begin{bmatrix} -6 \\ 3 \\ 5 \end{bmatrix}.$

Let $\widehat{T} : \mathbb{R}^3 \mapsto \mathbb{R}^3$ be a linear transformation that maps $\mathbf{e}_i$ to $\mathbf{y}_i$.

(a) Write the standard matrix representation of $\widehat{T}$. (b) Find the image of $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

(c) Find the standard matrix of the reverse transformation from $\mathbf{y}_i$ to $\mathbf{e}_i$.

(d) Using the reverse transformation, obtain the image of $[\mathbf{u}]_{\mathcal{Y}} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$.

## Question 4

Consider matrix

$$\mathbf{A} = \begin{bmatrix} 3 & -3 & 0 \\ 0 & 3 & -3 \\ 0 & 3 & 9 \end{bmatrix}$$

(a) Solve the characteristic equation to find the eigenvalues of $\mathbf{A}$.

(b) Find the bases for eigenspaces of $\mathbf{A}$ corresponding to each eigenvalue.