

UNIVERSITY OF TECHNOLOGY SYDNEY
SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES
37233 LINEAR ALGEBRA

Tutorial 7

Question 1

Illustrate the effect of a linear transformation $\hat{T}$ with the standard matrix

$$\mathbf{T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

by depicting its action on a triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$.

Determine if this is a one-to-one linear transformation.

Question 2

Consider linear transformation $\hat{T}$ with the standard matrix

$$\mathbf{T} = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}$$

- (a) Specify domain, codomain and range of $\hat{T}$; determine if it is one-to-one.
- (b) Illustrate action of $\hat{T}$ on a standard unit square in $\mathbb{R}^2$ (draw a picture).
- (c) Find all the source vectors which produce $\mathbf{y} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as an image.

***Option to Question 2**

Answer the same questions for another matrix, and compare and discuss the results.

$$\mathbf{D} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Question 3

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \quad \lambda = 7$$

- (a) Determine if $\mathbf{v}$ and/or $\mathbf{u}$ are eigenvectors of $\mathbf{A}$
- (b) Determine if λ is an eigenvalue of $\mathbf{A}$
- (c) Use characteristic equation to find eigenvectors and eigenvalues of $\mathbf{A}$

Question 4

For each of the shapes shown in (a)–(e):

- (1) determine if it can be a result of a linear transformation applied to a unit square;
- (2) where applicable, specify the standard matrix of the corresponding transformation.

Unit square:

