University of Technology Sydney
School of Mathematical and Physical Sciences

37233 Linear Algebra

Exercises 8

Question 1

Consider matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 7 & -6 & -1 \\ 1 & -8 & 9 & -3 \\ -2 & -9 & 7 & 1 \\ -2 & 6 & -8 & 3 \end{bmatrix}$$

and vectors

$$\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ -3 \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} 0 \\ 2 \\ -6 \\ 10 \end{bmatrix} \quad \text{and} \quad \mathbf{z} = \begin{bmatrix} -2 \\ -4 \\ 2 \\ 0 \end{bmatrix}$$

(a) Obtain explicit descriptions of $\operatorname{Col}\mathbf{A}$, of $\operatorname{Nul}\mathbf{A}$, of $\operatorname{Row}\mathbf{A}$

(b) Find bases for $\operatorname{Col}\mathbf{A}$, for $\operatorname{Nul}\mathbf{A}$, for $\operatorname{Row}\mathbf{A}$

(c) Determine $\dim(\operatorname{Col}\mathbf{A})$, $\dim(\operatorname{Nul}\mathbf{A})$, $\dim(\operatorname{Row}\mathbf{A})$, and $\operatorname{rank}\mathbf{A}$

(d) For each of $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ check if it belongs: to $\operatorname{Col}\mathbf{A}$; to $\operatorname{Nul}\mathbf{A}$; to $\operatorname{Row}\mathbf{A}$

Question 2

For each of the conditions listed below, construct a 2×2 real matrix if possible, or explain why it is not possible. Where a matrix exists, state what are the dimensions of its column, null and row spaces.

(a) $\operatorname{Col}\mathbf{A} = \operatorname{Row}\mathbf{A}$

(b) $\operatorname{Nul}\mathbf{B} = \operatorname{Col}\mathbf{B}$

(c) $\operatorname{Row}\mathbf{C} = \operatorname{Nul}\mathbf{C}$

(d) $\dim(\operatorname{Col}\mathbf{D}) = \dim(\operatorname{Row}\mathbf{D})$

(e) $\dim(\operatorname{Nul}\mathbf{E}) = \dim(\operatorname{Col}\mathbf{E})$

(f) $\dim(\operatorname{Row}\mathbf{F}) = \dim(\operatorname{Nul}\mathbf{F})$

(g) $\dim(\operatorname{Col}\mathbf{G}) + \dim(\operatorname{Nul}\mathbf{G}) + \dim(\operatorname{Row}\mathbf{G}) = 1$

(h) $\dim(\operatorname{Col}\mathbf{H}) + \dim(\operatorname{Nul}\mathbf{H}) + \dim(\operatorname{Row}\mathbf{H}) = 2$

(i) $\dim(\operatorname{Col}\mathbf{J}) + \dim(\operatorname{Nul}\mathbf{J}) + \dim(\operatorname{Row}\mathbf{J}) = 3$

(j) $\dim(\operatorname{Col}\mathbf{K}) + \dim(\operatorname{Nul}\mathbf{K}) + \dim(\operatorname{Row}\mathbf{K}) = 4$