University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Class test help sheet

$(\mathbf{u}+\mathbf{v}) \in V$	$c\,\mathbf{u} \in V$
(i) $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$	(v) $1 \cdot \mathbf{u}=\mathbf{u}$
(ii) $(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w})$	(vi) $c\,(d\,\mathbf{u})=(c\,d)\,\mathbf{u}$
(iii) $\exists\,\mathbf{0} \ : \ \mathbf{u}+\mathbf{0}=\mathbf{u}$	(vii) $(c+d)\,\mathbf{u}=c\,\mathbf{u}+d\,\mathbf{u}$
(iv) $\forall\,\mathbf{u}\ \exists\,(-\mathbf{u}) \ : \ \mathbf{u}+(-\mathbf{u})=\mathbf{0}$	(viii) $c\,(\mathbf{u}+\mathbf{v})=c\,\mathbf{u}+c\,\mathbf{v}$

$$[\mathbf{x}]_{\mathcal{C}}=\mathbf{P}_{\mathcal{C}\leftarrow\mathcal{B}}\,[\mathbf{x}]_{\mathcal{B}} \qquad \text{using} \qquad \left[\ \mathbf{c}_1\ \ \mathbf{c}_2\ \ \ldots\ \ \mathbf{c}_n\ \mid\ \mathbf{b}_1\ \ \mathbf{b}_2\ \ \ldots\ \ \mathbf{b}_n\ \right] \rightarrow \left[\ \mathbf{I}\ \mid\ \mathbf{P}_{\mathcal{C}\leftarrow\mathcal{B}}\ \right]$$

The invertible matrix theorem — equivalent statements for $n \times n$ matrix $\mathbf{A}$:

- There is an $n \times n$ matrix $\mathbf{A}^{-1}$ such that $\mathbf{A}^{-1}\mathbf{A}=\mathbf{A}\mathbf{A}^{-1}=\mathbf{I}$
- $\mathbf{A}^{\mathsf{T}}$ is invertible
- $\mathbf{A}$ has n pivot positions in the REF form
- $\mathbf{A}\mathbf{x}=\mathbf{0}$ has only the trivial solution
- The columns (rows) of $\mathbf{A}$ form a linearly independent set
- The columns (rows) of $\mathbf{A}$ span $\mathbb{R}^n$
- The columns (rows) of $\mathbf{A}$ form a basis of $\mathbb{R}^n$
- $\widehat{T}:\ \mathbf{x} \mapsto \mathbf{A}\mathbf{x}$ is one-to-one
- $\widehat{T}:\ \mathbf{x} \mapsto \mathbf{A}\mathbf{x}$ maps $\mathbb{R}^n$ onto $\mathbb{R}^n$
- The range of $\widehat{T}:\ \mathbf{x} \mapsto \mathbf{A}\mathbf{x}$ is $\mathbb{R}^n$
- $\operatorname{Col}\mathbf{A}=\operatorname{Row}\mathbf{A}=\mathbb{R}^n$
- $\operatorname{Nul}\mathbf{A}=\{\mathbf{0}\}$ and $\dim(\operatorname{Nul}\mathbf{A})=0$
- $\dim(\operatorname{Col}\mathbf{A})=\dim(\operatorname{Row}\mathbf{A})=n$
- $\operatorname{rank}\mathbf{A}=n$
- The eigenvalues of $\mathbf{A}$ are non-zero

— Additional technical working space —