University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Tutorial 8

Question 1

Let

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & -5 \\ 0 & 0 & 0 & 5 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Write spanning sets for $\operatorname{Col}\mathbf{A}$, $\operatorname{Row}\mathbf{A}$, and $\operatorname{Nul}\mathbf{A}$.

(b) Determine dimensions of $\operatorname{Nul}\mathbf{A}$, $\operatorname{Col}\mathbf{A}$ and $\operatorname{Row}\mathbf{A}$.

(c) Write suitable bases of $\operatorname{Nul}\mathbf{A}$, $\operatorname{Col}\mathbf{A}$ and $\operatorname{Row}\mathbf{A}$.

Question 2

Suppose a system of 9 equations with 10 unknowns is known to have a solution for any right-hand side of the equations. Is it possible to find 2 non-zero solutions of the corresponding homogeneous system that are not multiples of each other?

Question 3

Given vector $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$:

(1) For each of the three conditions below in sub-questions (a), (b), (c):

- either construct a matrix $\mathbf{A}$ having the maximum possible rank in each case,
- or demonstrate that such a matrix is impossible.

(a) $\mathbf{v} \in \operatorname{Col}\mathbf{A}$ and $\mathbf{v} \in \operatorname{Row}\mathbf{A}$ at the same time

(b) $\mathbf{v} \in \operatorname{Row}\mathbf{A}$ and $\mathbf{v} \in \operatorname{Nul}\mathbf{A}$ at the same time

(c) $\mathbf{v} \in \operatorname{Nul}\mathbf{A}$ and $\mathbf{v} \in \operatorname{Col}\mathbf{A}$ at the same time

(2) Suppose that a 3×3 matrix $\mathbf{B}$ is constructed, such that, at the same time:

$$\mathbf{v} \notin \operatorname{Col}\mathbf{B} \quad \text{and} \quad \mathbf{v} \notin \operatorname{Nul}\mathbf{B} \quad \text{and} \quad \mathbf{v} \notin \operatorname{Row}\mathbf{B}$$

(d) Explain what can be concluded about $\operatorname{rank}\mathbf{B}$

(e) Find $\det\mathbf{B}$

Additional questions

Question 4

Let

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 3 & 0 & -1 \\ 2 & -4 & 7 & -3 & 3 \\ 3 & -6 & 8 & 3 & -8 \end{bmatrix}$$

Find bases and dimensions for:

(a) $\operatorname{Row} \mathbf{A}$

(b) $\operatorname{Col} \mathbf{A}$

(c) $\operatorname{Nul} \mathbf{A}$

(d) $\operatorname{Nul} \mathbf{A}^{\mathsf{T}}$, $\operatorname{Col} \mathbf{A}^{\mathsf{T}}$, and $\operatorname{Row} \mathbf{A}^{\mathsf{T}}$.

Question 5

A homogeneous system of 12 equations in 8 unknowns has 2 solutions that are not multiples of each other, whereas any further solutions are linear combinations of those two solutions. Can the set of all the solutions to this system be obtained with less than 12 equations? If so, how many equations are required?

Question 6

Find basis sets for $\operatorname{Nul} \mathbf{A}$ and $\operatorname{Col} \mathbf{A}$ for

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix}$$

Question 7

Let

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & -1 & 1 \\ 3 & 0 & -1 & 2 \\ 3 & 0 & 5 & 3 \end{bmatrix}$$

Find bases and dimensions for:

(a) $\operatorname{Row} \mathbf{A}$

(b) $\operatorname{Col} \mathbf{A}$

(c) $\operatorname{Nul} \mathbf{A}$

(d) $\operatorname{Nul} \mathbf{A}^{\mathsf{T}}$