

UNIVERSITY OF TECHNOLOGY SYDNEY
SCHOOL OF MATHEMATICAL AND PHYSICAL SCIENCES
37233 LINEAR ALGEBRA

Exercises 9

Question 1

Consider $W = \text{Span} \left\{ \begin{bmatrix} -15 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} \right\}$ and vectors $\mathbf{y}_1 = \begin{bmatrix} 5 \\ 3 \\ 5 \end{bmatrix}$, $\mathbf{y}_2 = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$.

For each of the vectors \mathbf{y}_i :

- (a) Decompose $\mathbf{y}_i = \check{\mathbf{y}}_i + \mathbf{z}_i$ with $\check{\mathbf{y}}_i = \text{proj}_W \mathbf{y}_i$ and $\mathbf{z}_i \in W^\perp$
- (b) Compute the distance from \mathbf{y}_i to W

Question 2

Use the Gram-Schmidt process to construct an orthogonal basis for

$$W = \text{Span} \left\{ \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 12 \\ 5 \\ 10 \end{bmatrix} \right\}.$$

Following that, construct an orthonormal basis for W .

Question 3

Consider $W = \text{Span} \left\{ \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 18 \\ 0 \\ 0 \\ 8 \end{bmatrix} \right\}$ and:

- (a) Construct an orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for W
- (b) Obtain an orthonormal basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ for W

Question 4

Extend the basis for W from Q4 of Workshop 9, to an orthonormal basis \mathcal{U} for \mathbb{R}^3 .

Consider another set of vectors, forming matrix

$$\mathbf{Q} = [\mathbf{q}_1 \ \mathbf{q}_2 \ \mathbf{q}_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4/5 & 3/5 \\ 0 & 3/5 & -4/5 \end{bmatrix}$$

- (a) Check if $\{\mathbf{q}_1 \ \mathbf{q}_2 \ \mathbf{q}_3\}$ is an orthonormal basis \mathcal{Q} for \mathbb{R}^3 .
- (b) Calculate $\mathbf{Q}\mathbf{Q}^\top$ and interpret the result in terms of projections.
- (c) Calculate the change of basis matrix $\mathbf{P}_{\mathcal{Q} \leftarrow \mathcal{U}}$ from \mathcal{U} to \mathcal{Q} .
- (d) By theorem, matrix $\mathbf{P}_{\mathcal{Q} \leftarrow \mathcal{U}}$ must be orthogonal. Verify it by analysing its properties.