University of Technology Sydney

School of Mathematical and Physical Sciences

37233 Linear Algebra

Tutorial 9

Question 1

Let

$$\mathbf{y} = \begin{bmatrix} 7 \\ 1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{u} = \begin{bmatrix} 8 \\ -6 \end{bmatrix}$$

(a) Write an orthogonal decomposition of $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$ where $\hat{\mathbf{y}} = \text{proj}_{\mathbf{u}}\, \mathbf{y}$ and $\mathbf{z} \perp \mathbf{u}$.

(b) Compute the distance from $\mathbf{y}$ to the line through $\mathbf{u}$ and the origin.

Question 2

Consider the following vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$$

(a) Find the length of each vector.

(b) Determine if any of these vectors are orthogonal to each other.

(c) Specify if $\mathbf{v}_1,\ \mathbf{v}_2$ form a basis for $W = \text{Span}\{\mathbf{v}_1,\ \mathbf{v}_2\}$, and what kind of basis.

(d) Obtain an orthogonal decomposition of $\mathbf{y} = \hat{\mathbf{y}} + \mathbf{z}$ where $\hat{\mathbf{y}} \in W$ and $\mathbf{z} \in W^{\perp}$.

(e) Find the distance from $\mathbf{y}$ to W.

Question 3

Consider the following basis for $W = \text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$:

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \qquad \mathbf{a}_2 = \begin{bmatrix} 4 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \qquad \mathbf{a}_3 = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

(a) Construct an orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for W using the Gram–Schmidt process.

(b) Obtain an orthonormal basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ from the orthogonal set found in (a).

Question 4

Let

$$\mathbf{U} = [\,\mathbf{u}_1\,\mathbf{u}_2\,] = \begin{bmatrix} 2/3 & -2/3 \\ 1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix} \qquad \text{and} \qquad \mathbf{y} = \begin{bmatrix} 4 \\ 8 \\ 1 \end{bmatrix}$$

(a) Using $\mathbf{U}$ (not by $\mathbf{u}_1 \cdot \mathbf{u}_2$) check if $\mathbf{u}_1$ and $\mathbf{u}_2$ form an orthonormal basis for $W = \operatorname{Col}\mathbf{U}$.

(b) Calculate $\operatorname{proj}_W \mathbf{y}$ using $\mathbf{U}$.

Question 5

Figure out if it is possible for an orthogonal matrix to have all the entries positive.