**37252 Regression and Linear Models**

**Lab 1: Simple Linear Regression I**

This lab is marked out of 20.

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**37252\_Lab1\_Surname\_FirstName**

**Due: 12 noon Wednesday 13 March 2024**

In this week’s lab we build a model to predict university examination scores based on hours spent on revision. The data are taken from 20 students and available in **37252\_Lab1\_data.csv** which can be downloaded from Canvas.

The variables we will consider are summarised in the table below.

|  |  |  |
| --- | --- | --- |
| **Name** | **Role** | **Description** |
| $$score$$ | response | examination score |
| $$hours$$ | predictor | hours spent on revision |

In order to know what type of model to build we need to examine the nature of the relationship between the variables. First we look at this visually with a scatter plot.

> scoredat <- read.csv("~/2024\_37252/Labs/Lab1/37252\_Lab1\_data.csv")

> plot(scoredat$hours, scoredat$score, xlab = "hours", ylab = "score")



1. Describe the direction, type and strength of the relationship between $hours$ and $score$ **[3 marks]**.

Direction – positive (higher values of $hours$ tend to be associated with higher values of $score$) **[1 mark]**.

Type – linear (straight line relationship) **[1 mark]**.

Strength – strong (close clustering about line of best fit) **[1 mark]**.

We can also measure the strength of a relationship statistically. One such measure is Pearson’s correlation coefficient, which measures the strength of a linear relationship.

> cor.test(scoredat$hours, scoredat$score, method = "pearson")

 Pearson's product-moment correlation

data: scoredat$hours and scoredat$score

t = 6.1012, df = 18, p-value = 9.166e-06

alternative hypothesis: true correlation is not equal to 0

95 percent confidence interval:

 0.5944695 0.9268088

sample estimates:

 cor

0.8210109

The **Sig. (2-tailed)** field is the p-value from the test of the hypotheses

$$H\_{0}: ρ=0$$

$$H\_{A}: ρ\ne 0$$

where

$$ρ=corr\left(score,hours\right)$$

is the (unknown) population correlation coefficient.

Note that

$$\hat{ρ}=corr\left(score\_{i},hours\_{i}\right)=0.821$$

is the sample correlation coefficient.

1. Is the population correlation coefficient statistically different from zero at the 0.05 significance level? **[1 marks]**. Compare the sample correlation coefficient to your observations from part (a) **[2 marks]**.

Yes – with p-value reported as 0.000 (less than 0.05) **[1/2 mark]** the null hypothesis can be rejected in favour of the alternative **[1/2 mark]**.

The sample correlation is positive and at 0.821 suggests a strong relationship, as noted in (a) **[2 marks]**.

We now build our first simple linear regression model.

> mod1<-lm(score ~ hours, data = scoredat)

> summary(mod1)

Call:

lm(formula = score ~ hours, data = scoredat)

Residuals:

 Min 1Q Median 3Q Max

-10.6747 -4.1513 0.1568 4.4249 12.2210

Coefficients:

 Estimate Std. Error t value Pr(>|t|)

(Intercept) 22.1640 6.5257 3.396 0.00322 \*\*

scoredat$hours 0.9920 0.1626 6.101 9.17e-06 \*\*\*

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Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.431 on 18 degrees of freedom

Multiple R-squared: 0.6741, Adjusted R-squared: 0.656

F-statistic: 37.22 on 1 and 18 DF, p-value: 9.166e-06

> confint(mod1)

 2.5 % 97.5 %

(Intercept) 8.4540373 35.874044

hours 0.6503953 1.333562

1. Write down the estimated regression equation **[1 mark]** and provide interpretations of the estimated beta coefficients **[2 marks]**.

$$\hat{score}=22.164+0.992×hours$$

**[1 mark]**

The coefficient $\hat{β}\_{0}=22.164$ is the predicted $score$ when $hours=0$ **[1 mark]**.

The coefficient $\hat{β}\_{1}=0.992$ is the predicted change in $score$ for every unit increase in $hours$ **[1 mark]**.

1. What restrictions should be placed on the values that $hours$ can take **[1 mark]**?

As hours spent on revision must be non-negative we should use the bound $hours\geq 0$ **[1/2 mark]**.

Also, $hours\leq \frac{100-22.164}{0.992}=78.464$ as $0\leq score\leq 100$. **[1/2 mark]**

For a model to be useful we need it to be “statistically significant”. For simple linear regression this means we have to show that $β\_{1}\ne 0$, one of which ways to do so is via a T-test.

1. Is the regression model significant at the 0.05 level? Write down the hypotheses **[1 mark]**, the test statistic and p-value **[1 mark]**, the result of the test **[1 mark]** and a conclusion in non-mathematical language **[1 mark]**?

**Hypotheses**

$$H\_{0}: β\_{1}=0$$

$$H\_{A}: β\_{1}\ne 0$$

**[1 mark]**

**Test statistic and p-value**

Test statistic $t=6.101$ with p-value $p=0.000$ **[1 mark]**.

**Test result**

Reject $H\_{0}$ as $p<0.05$ **[1 mark]**.

**Conclusion**

The regression is significant **[1 mark]**.

1. Test whether or not $β\_{0}=36$ at the 0.05 level. Write down the hypotheses **[1 mark]**, the result of the test with reason **[2 marks]** and a conclusion in non-mathematical language **[1 mark]**.

**Hypotheses**

$$H\_{0}: β\_{0}=36$$

$$H\_{A}: β\_{0}\ne 36$$

**[1 mark]**

**Test result**

Reject $H\_{0}$ as 36 is outside the 95% CI for $β\_{0}$ **[2 marks]**.

**Conclusion**

There is significant evidence that $β\_{0}\ne 36$ **[1 mark]**.

We can now use the regression model to predict $score$ when $hours=33$ (or any other non-negative value) and to find a 95% confidence interval for the mean predicted $score$ at this level of $hours$. We can also ask for the 95% confidence interval for the “individual” predicted $score$ at this level of $hours$. This can be done by hand using the formulae in the notes (see pages 32 and 33 Lecture 2 Notes), but we can also get R to do it for us.

> newdata <- data.frame(hours=33)

> predict(mod1, newdata,interval="confidence", level = 0.95)

 fit lwr upr

1 54.89933 51.21958 58.57909

> predict(mod1, newdata,interval="predict", level = 0.95)

 fit lwr upr

1 54.89933 40.89609 68.90258

1. With reference to equations (21) and (23) in Lecture 2, explain why the 95% confidence interval for an individual predicted value is wider than the 95% confidence interval for the mean predicted value **[1 mark]**.

The mean CI is calculated as



while the individual CI is calculated as



These formulae are identical except for the square root term which differs by 1. This makes the square root term larger for (23) which makes the individual CI wider **[1 mark]**.

1. Suppose that the observed value of the $score$ when $hours=33$ was 57. Calculate the residual at this point **[1 mark]**.

$$\hat{ϵ}\_{21}=score\_{21}-\hat{score}\_{21}=57-54.89933$$

$$=2.10067$$

**[1 mark]**