**37252 Regression and Linear Models**

**Lab 9: Simple Logistic Regression**

This lab is marked out of 26.

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**37252\_Lab9\_Surname\_FirstName**

**Due: 12 noon Wednesday 15 May 2024**

In this week’s lab we continue our example from last week. The data are available in **37252\_Lab9\_data.csv** which can be downloaded from Canvas.

The variables we consider are summarised in the table below.

|  |  |  |
| --- | --- | --- |
| **Name** | **Role** | **Description** |
|  | response | successful field goal attempt: 1 (yes), 0 (no) |
|  | predictor | game time quarter (1, 2, 3, 4) |
|  | predictor | kicking distance |

**Simple logistic regression with categorical variable**

Recall from Lab 8 that we found there was no statistically-significant relationship between and . Let’s ignore this for the moment and fit a logistic regression model anyway.

As is a four-state categorical variable, we require three binary dummy variables. We will code them as

We need to specify this variable as a “factor” before fitting the model.

> read.csv("~/2024\_37252/Labs/Lab9/37252\_Lab9\_data.csv")

> NFLdat$qtr <- as.factor(NFLdat$qtr)

> NFLdat$qtr <- relevel(NFLdat$qtr, ref = "4")

> mod1 <- glm(good ~ qtr, family = "binomial", data = NFLdat)

> summary(mod1)

R output is displayed below.

Call:

glm(formula = good ~ qtr, family = "binomial", data = NFLdat)

Deviance Residuals:

Min 1Q Median 3Q Max

-2.1330 0.4658 0.4914 0.5851 0.5851

Coefficients:

Estimate Std. Error z value Pr(>|z|)

(Intercept) 2.0532 0.1972 10.410 <2e-16 \*\*\*

qtr1 0.1132 0.2993 0.378 0.705

qtr2 -0.3750 0.2429 -1.544 0.123

qtr3 -0.3485 0.2848 -1.224 0.221

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Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 810.25 on 1025 degrees of freedom

Residual deviance: 805.10 on 1022 degrees of freedom

AIC: 813.1

Number of Fisher Scoring iterations: 4

1. Write down the fitted logistic regression model in log-odds scale, odds scale and probability scale **[3 marks]**.

Let

**Log-odds scale**

**[1 mark]**

**Odds scale**

**[1 mark]**

**Probability scale**

**[1 mark]**

Recall in Lab 8 we used two-way table analysis to calculate

1. Using the regression model, carry-out the calculations to show that the odds of successful field goal in quarter 1 and in quarter 4 almost match those calculated in Lab 8 **[2 marks]**. Using the regression model (or R output) show that the odds ratio almost matches that from Lab 8 **[1 mark]**.

**[2 marks]**

The last can be obtained directly from the Variables in the Equation table output by R, i.e. .

**[1 mark]**

1. Determined if the model predicts a successful field goal for kicks taken in quarter1 and in quarter 4, i.e. determine if

and

**[3 marks]**

We need to check if and if **[1 mark]**.

The model does predict successful field goal kicks in quarter 1 because

which implies

**[1 mark]**

The model does predict successful field goal kicks in quarter 4 because

which implies

**[1 mark]**

1. Provide interpretations of on the log-odds scale and of on the odds scale **[2 marks]**.

The coefficient is the predicted difference in the log-odds of successful field goal in quarter 1 compared to quarter 4 **[1 mark]**.

The odds ratio is the predicted multiple of the odds of successful field goal in quarter 1 compared to quarter 4 **[1 mark]**.

**Simple logistic regression with continuous predictor**

Recall from Lab 8 that we found there was a statistically-significant relationship between and , where represented kick distance quartile. Let’s infer from this the existence of a statistically-significant relationship between and and build a logistic regression model.

> mod2 <- glm(good ~ distance, family = "binomial", data = NFLdat)

> summary(mod2)

R output is displayed below.

Call:

glm(formula = good ~ distance, family = "binomial", data = NFLdat)

Deviance Residuals:

Min 1Q Median 3Q Max

-2.9500 0.2047 0.3491 0.5846 1.2341

Coefficients:

Estimate Std. Error z value Pr(>|z|)

(Intercept) 6.75473 0.54691 12.351 <2e-16 \*\*\*

distance -0.12083 0.01234 -9.788 <2e-16 \*\*\*

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Signif. codes: 0 ‘\*\*\*’ 0.001 ‘\*\*’ 0.01 ‘\*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 810.25 on 1025 degrees of freedom

Residual deviance: 680.53 on 1024 degrees of freedom

AIC: 684.53

Number of Fisher Scoring iterations: 6

1. Write down the fitted logistic regression model in log-odds scale, odds scale and probability scale **[3 marks]**.

Let

**Log-odds scale**

**[1 mark]**

**Odds scale**

**[1 mark]**

**Probability scale**

**[1 mark]**

1. Interpret the impact of on the log-odds scale and the odds scale **[2 marks]**.

**Log-odds scale**

For a unit increase in , the log-odds of successful field goal are predicted to fall by 0.121 **[1 mark]**.

**Odds scale**

For a unit increase in , the odds of successful field goal are predicted to multiply by **[1 mark]**.

1. Perform a hypothesis test to determine if the regression is significant at the 0.05 level. Write down the hypotheses **[1 mark]**, the test statistic and p-value **[1 mark]**, the result of the test **[1 mark]** and a conclusion in non-mathematical language **[1 mark]**.

**Hypotheses**

**[1 mark]**

**Test statistic and p-value**

Test statistic with p-value **[1 mark]**.

**Test result**

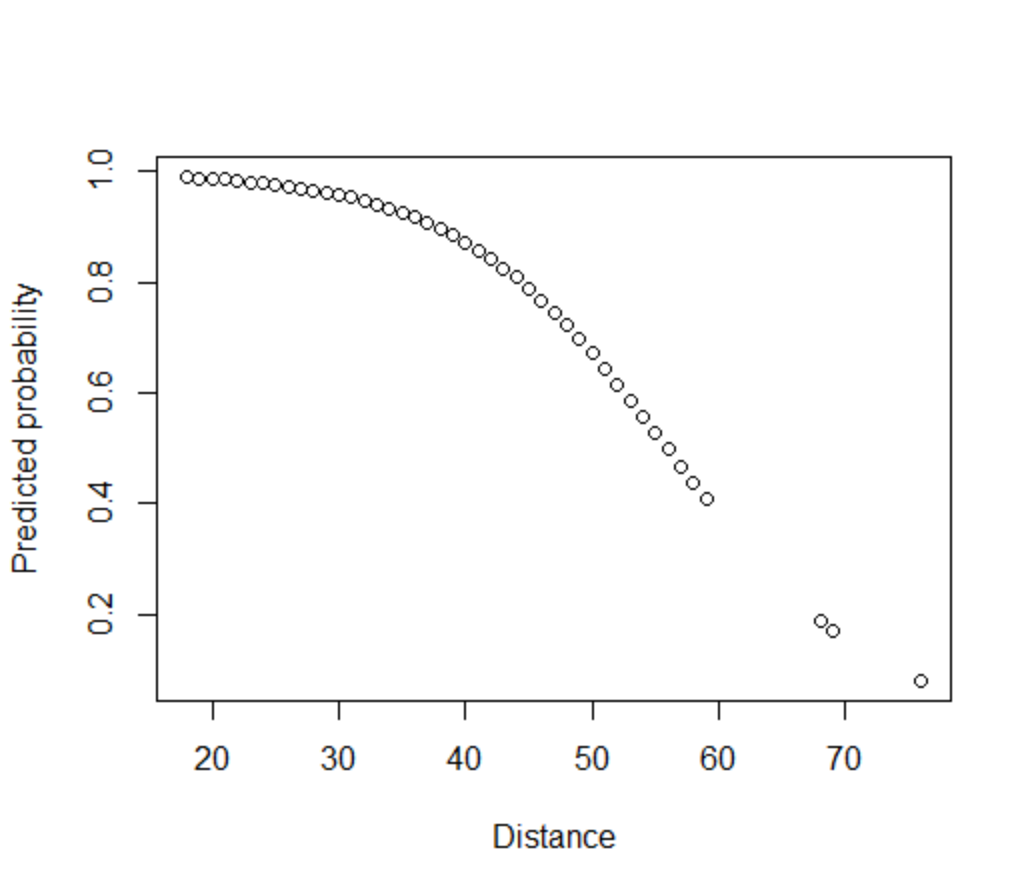
Reject as **[1 mark]**.

**Conclusion**

The regression is significant **[1 mark]**.

1. Use a scatterplot to plot against and describe the relationship **[2 marks]**.

> plot(NFLdat$distance , mod2$fitted.values, xlab = "Distance", ylab = "Predicted probability")



and

**[2 marks]**

1. Calculate the “median effective level” by solving

for **[2 marks]**. Describe the relationship between and **[2 marks]**.

From the equation for the log-odds we have

**[1 mark]**

Making the substitution gives

**[1 mark]**

For we have implying **[1 mark]**.

For we have implying **[1 mark]**.