ANOVA

$$\mathrm{SST} = \sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2 = \left(Y - \bar{Y}\,1\right)^T \left(Y - \bar{Y}\,1\right) \quad (1 \text{ a vector of ones})$$

$$= Y^T Y - \bar{Y}\,Y^T 1 - \bar{Y}\,1^T Y + \bar{Y}^2\,1^T \quad 1$$

$$= Y^T Y - 2\,\bar{Y}\,1^T Y + \bar{Y}^2\,1^T \quad 1$$

$$= Y^T Y - \frac{2}{n} Y^T 1 \;\; 1^T Y + \left(\frac{1}{n} Y^T 1\right)^2 1^T \quad 1$$

$$= Y^T Y - \frac{2}{n} Y^T 1 \;\; 1^T Y + \frac{1}{n^2} Y^T 1 \;\; 1^T Y\, n$$

$$= Y^T Y - \frac{1}{n} Y^T 1 \;\; 1^T Y$$

$$\mathrm{SSR} = \sum_{i=1}^{n} \left(\hat{Y}_i - \bar{Y}\right)^2$$

$$= \hat{Y}^T Y - \frac{1}{n} Y^T 1 \;\; 1^T Y \quad (\text{by above})$$

$$= \left(X\hat{\beta}\right)^T Y - \frac{1}{n} Y^T 1 \;\; 1^T Y$$

$$= \hat{\beta}^T X^T Y - \frac{1}{n} Y^T 1 \;\; 1^T Y$$

$$\mathrm{SSE} = \sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2 = \left(Y - \hat{Y}\right)^T \left(Y - \hat{Y}\right)$$

$$= Y^T Y - Y^T \hat{Y} - \hat{Y}^T Y + \hat{Y}^T \hat{Y}$$

$$= Y^T Y - 2\,\hat{Y}^T Y + \hat{Y}^T \hat{Y}$$

$$= Y^T Y - 2\left(X\hat{\beta}\right)^T Y + \left(X\hat{\beta}\right)^T \left(X\hat{\beta}\right)$$

$$= Y^T Y - 2\,\hat{\beta}^T X^T Y + \hat{\beta}^T X^T X\,\hat{\beta}$$

$$= Y^T Y - 2\,\hat{\beta}^T X^T Y + \hat{\beta}^T X^T X \left(X^T X\right)^{-1} X^T Y \quad \left(\hat{\beta} = \left(X^T X\right)^{-1} X^T Y\right)$$

$$= Y^T Y - 2\,\hat{\beta}^T X^T Y + \hat{\beta}^T I\, X^T Y \quad (I \text{ the identity matrix})$$

$$= Y^T Y - 2\,\hat{\beta}^T X^T Y + \hat{\beta}^T X^T Y$$

$$= Y^T Y - \hat{\beta}^T X^T Y$$