We can write the solution as

$$u(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(z)\, e^{-iz\xi}\, e^{-|\xi|y + ix\xi}\, dz\, d\xi$$

$$\underset{(\text{Fubini})}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\eta)\left(\int_{-\infty}^{\infty} e^{-|\xi|y + i\xi(x-\eta)}\, d\xi\right) d\eta$$

Let's calculate the kernel here

$$\int_{-\infty}^{\infty} e^{-|\xi|y + i\xi(x-\eta)}\, d\xi = \int_{-\infty}^{0} e^{\xi y + i(\xi(x-\eta))}\, d\xi + \int_{0}^{\infty} e^{-\xi(y - i(x-\eta))}\, d\xi$$

$$= \left[\frac{e^{\xi(y+i(x-\eta))}}{y+i(x-\eta)}\right]_{-\infty}^{0} + \left[\frac{e^{-\xi(y-i(x-\eta))}}{y-i(x-\eta)}\right]_{0}^{\infty}$$

$$= \frac{1}{y+i(x-\eta)} + \frac{1}{y-i(x-\eta)} = \frac{2y}{(x-\eta)^2+y^2}$$

Thus if $P(x,y) = \dfrac{2y}{x^2+y^2}$ then

$$u(x,y) = P(\cdot\,, y) * f\ (x)$$

is a solution of the Laplace equation

$P(x,y)$ is called the Poisson kernel