University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Quiz 1
Time allowed: 75 minutes
Question paper MUST NOT BE REMOVED from the room
This is an open-book assessment.
You may use textbooks, subject notes and other offline resources as required. You are not permitted to communicate with other students or to post requests for solutions to online sources

Question 1.

Let X and Y be independent random variables, such that

X has probability mass function $P(X=k)=\begin{cases} k & k\in\{0.1,\ 0.2,\ 0.3,\ 0.4\} \\ 0 & \text{otherwise} \end{cases}$

and $Y \sim \exp(10)$ has probability density function $f(y)=\begin{cases} 10e^{-10y} & y\in[0,\infty) \\ 0 & \text{otherwise} \end{cases}$.

and let $\{u_1,u_2,u_3\}=\{0.744,\ 0.109,\ 0.520\}$ be a set of independent realisations of $U \sim U[0,1]$.

i) Clearly explaining your method, generate $\{x_1,x_2,x_3\}$, three independent realisations of X.

ii) Calculate the cumulative density of Y, $F(y)=P(Y\le y)$.

iii) Hence generate $\{y_1,y_2,y_3\}$, three independent realisations of Y. Clearly explain your method,

(6 Marks)

Question 2.

Let $\{x_1,x_2,x_3,...\}$ each be independent realisations of a standard normal random variable, $X \sim N(0,1)$.

Write down a possible calculation to use the values $\{x_1,x_2,x_3,...\}$ to generate a realisation of each of the following variables:

i) $Q \sim \chi^2(3)$; ii) $T \sim t_2$; iii) $R \sim N(4,1)$;

Note: Your answer could be of a form similar to:
"To generate a realisation of $S \sim N(0,4)$, this could be done by calculating $x_1+x_2+x_3+x_4$."

(6 Marks)

Question 3.

Let X and Y be random variables such that X and Y have joint probability mass function

$$P((X,Y)=(a,b)) = \begin{cases} 0.3 & (a,b)=(0,0) \\ 0.2 & (a,b)=(1,0) \\ 0.4 & (a,b)=(0,1) \\ 0.1 & (a,b)=(1,1) \\ 0 & \text{otherwise} \end{cases}.$$

i) Showing all of your working, show that the marginal distribution of X is $X \sim Bern(0.3)$.

ii) Showing all of your working, find the conditional distribution of $X \mid Y = 1$. Clearly state any parameter(s) needed to define the distribution.

Now, define Z to be a variable which is independent of X which has probability mass function

$$P(Z=c) = \begin{cases} 0.3 & c=-1 \\ 0.4 & c=0 \\ 0.3 & c=1 \\ 0 & \text{otherwise} \end{cases}$$

iii) Write down the joint probability mass function of X and Z.

(8 Marks)

Question 4.

Let X and Y be independent standard normal random variables.

That is, for example, that X has density function

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad \text{for } -\infty < x < \infty .$$

Consider the change of variables $S = X + Y$ and $D = X - Y$.

i) Write down the joint density function of X and Y, $f_{X,Y}(x,y)$.

ii) Write X and Y in terms of S and D.

iii) What are the ranges of S and D? Justify your answers.

iv) Showing all of your working, justify that for this change of variables

$$|\det \boldsymbol{J}| = \left| \det \begin{pmatrix} \frac{\partial X}{\partial S} & \frac{\partial X}{\partial D} \\ \frac{\partial Y}{\partial S} & \frac{\partial Y}{\partial D} \end{pmatrix} \right| = \frac{1}{2} .$$

v) Hence calculate the joint density of $f_{S,D}(s,d)$.

vi) Show that the marginal distribution of S is $S \sim N(0,2)$.

vii) Are S and D independent? Justify your answer.

(10 Marks)

Question 5.

A group of 180 people each enrolled into (exactly) one of three activities. Individuals were classified by age category with everyone being (exactly) one of adult, senior or child.

	Activity A	Activity B	Activity C
Adult	10	10	20
Senior	10	10	40
Child	25	25	30

Originally, it is hypothesised that exactly 20 people from each age category would be expected to enrol into each of the three activities (i.e. 20 adults in Activity A etc.)

i) Calculate the Pearson statistic for this null hypothesis and hence show that the test statistic is 47.5.

ii) At the 5% significance level, test this hypothesis and clearly state your conclusions.

Now, a different null hypothesis is proposed. It is now assumed that the age-related variable and each respondent's rating are independent.

iii) Calculate the Pearson statistic for this null hypothesis and hence show that the test statistic is ≈ 11.67.

iv) At the 5% significance level, test this hypothesis and clearly state your conclusions.

Notes:

The table below gives the 5% critical values for chi-squared variables with 1 to 10 degrees of freedom.

Degrees of freedom, *n*	**5% Critical value of $\chi^2(n)$ (to 3 d.p.)**
1	3.841
2	5.991
3	7.815
4	9.488
5	11.070
6	12.592
7	14.067
8	15.507
9	16.919
10	18.307

(10 Marks)