University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Quiz 1
Time allowed: 75 minutes
Question paper MUST NOT BE REMOVED from the room
This is an open-book assessment.
You may use textbooks, subject notes and online resources as required. You are not permitted to communicate with other students or to post requests for solutions to online sources

Question 1.

Let X and Y be independent random variables.

A realisation x_i of X is generated by setting $x_i = \begin{cases} \lfloor 2u_i \rfloor & 0.2 \le u_i \le 0.7 \\ -1 & u_i < 0.2 \\ 10 & \text{otherwise} \end{cases}$

where u_i is a realisation of $U \sim U[0,1]$.

Y has probability density function $f(y) = \begin{cases} -4y^3 & y \in [-1,0] \\ 0 & \text{otherwise} \end{cases}$.

i) Calculate the probability mass function of X.

ii) Hence calculate $E(X)$.

Let $\{u_1, u_2, \ldots u_5\} = \{0.744,\ 0{,}109,\ 0.520,\ 0.247,\ 0.536\}$ be a set of independent realisations of $U \sim U[0,1]$.

iii) Generate $\{x_1, x_2, \ldots x_5\}$, five independent realisations of X.

iv) Calculate the cumulative probability function of Y, $F(y) = P(Y \le y)$.

v) Hence generate $\{y_1, y_2, \ldots y_5\}$, five independent realisations of Y. Clearly explain your method,

vi) If X and Y are generated from the same set of uniform points (so they are not independent of each other), calculate $P(XY \ge 0)$. Justify your answer.

(10 Marks)

Question 2.

Let *X, Y* and *Z* be random variables with joint probability mass function

$$P((X,Y,Z))=(x,y,z))=\begin{cases} 0.015 & (x,y,z)=(0,0,0) \\ 0.1 & (x,y,z)=(1,0,0) \\ 0.295 & (x,y,z)=(1,1,-1) \\ 0.21 & (x,y,z)=(0,2,-1) \\ 0.16 & (x,y,z)=(-1,2,1) \\ 0.005 & (x,y,z)=(1,2,1) \\ 0.01 & (x,y,z)=(-1,0,0) \\ 0.08 & (x,y,z)=(-1,1,-1) \\ 0.125 & (x,y,z)=(1,3,0) \\ 0 & \text{otherwise} \end{cases}$$

i) Show that $P(XYZ \leq 0) = 0.915$.

ii) Show that the marginal distribution of *Y* is a that of a binomial variable $Y \sim Bin(n,p)$. Clearly state the values of the parameters *n* and *p*.

iii) Calculate the conditional distribution of $Y \mid (X=-1)$

iv) Which common variable describes the distribution of $Y \mid (X=-1)$. Clearly state any parameters needed.

(8 Marks)

Question 3.

Let $\{x_1,x_2,x_3,...\}$ each be independent realisations of a standard normal random variable, $X \sim N(0,1)$.
Of which distribution would each of the following be assumed to be a realisation?:

i) $\dfrac{x_1}{\sqrt{x_2^{\,2}}}$;

ii) $\dfrac{x_1^{\,2}+x_2^{\,2}+x_3^{\,2}+x_4^{\,2}}{x_5^{\,2}+x_6^{\,2}+x_7^{\,2}+x_8^{\,2}}$;

iii) $-x_1+x_2-x_3$.

Note: You do not need to perform any calculations. Your answers can be simple statements similar to "x_1+3 can be assumed to be a realisation of a $N(3,1)$ variable (a normal distribution with mean 3 and variance 1.)"

(6 Marks)

Question 4.

An entertainment centre offers ten pin bowling and a games arcade. The centre monitors ticket sales to both activities for both adults and children.

The centre hypotheses the expected proportions of total tickets sold to each group. These are given in the table below, along with the actual counts of sales.

	Expected proportion	**Count**
Adult Bowling	0.5	61
Child Bowling	0.1	19
Adult Arcade	0.2	14
Child Arcade	0.2	6

For the first analysis, the centre tests whether the observed counts are consistent with the expected proportions.

i) Calculate the Pearson statistic for this null hypothesis.

ii) At the 5% significance level, test this hypothesis. Clearly state your conclusions and the degrees of freedom associated with this test.

Now, a different null hypothesis is proposed. It is now assumed that the age-related factor and the activity type are independent.

iii) Calculate the Pearson statistic for this null hypothesis.

iv) At the 5% significance level, test this hypothesis. Clearly state your conclusions and the degrees of freedom associated with this test.

Notes:

The table below gives the 5% critical values for chi-squared variables with 1 to 6 degrees of freedom.

Degrees of freedom, *n*	**5% Critical value of $\chi^2(n)$ (to 3 d.p.)**
1	3.841
2	5.991
3	7.815
4	9.488
5	11.070
6	12.592

(8 Marks)

Question 5.

Let X and Y be random variables with joint density function

$$f_{X,Y}(x,y) = \begin{cases} 10xy^2 & 0 \le x \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

i) Show that the marginal density of Y is given by $f_Y(y) = \begin{cases} 5y^4 & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$.

ii) Hence calculate $E(Y)$.

iii) Calculate the marginal density of X.

iv) Hence show that the conditional density of $Y \mid (X = x)$ is

$$f_{Y|X=x}(y) = \begin{cases} \dfrac{3y^2}{1-x^3} & x \le y \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

(8 Marks)