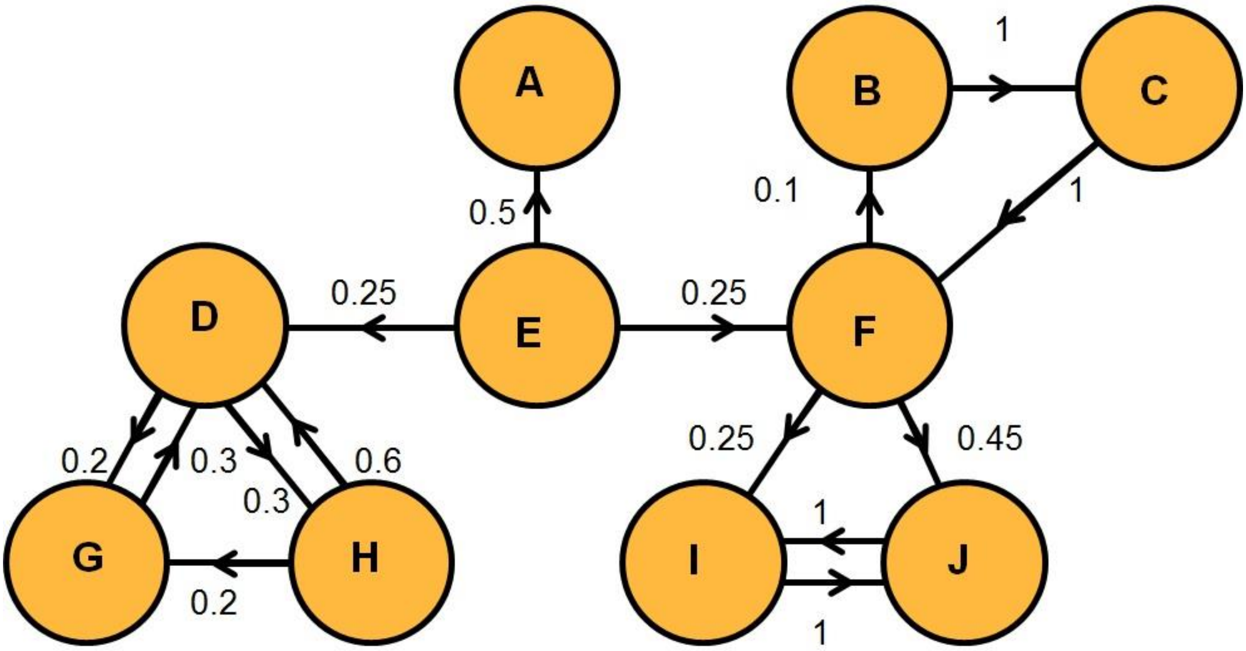
University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Class 10 Preparation Work

1. Let $X_0, X_1, X_2, \ldots$ be a Markov Chain which is represented by the state diagram



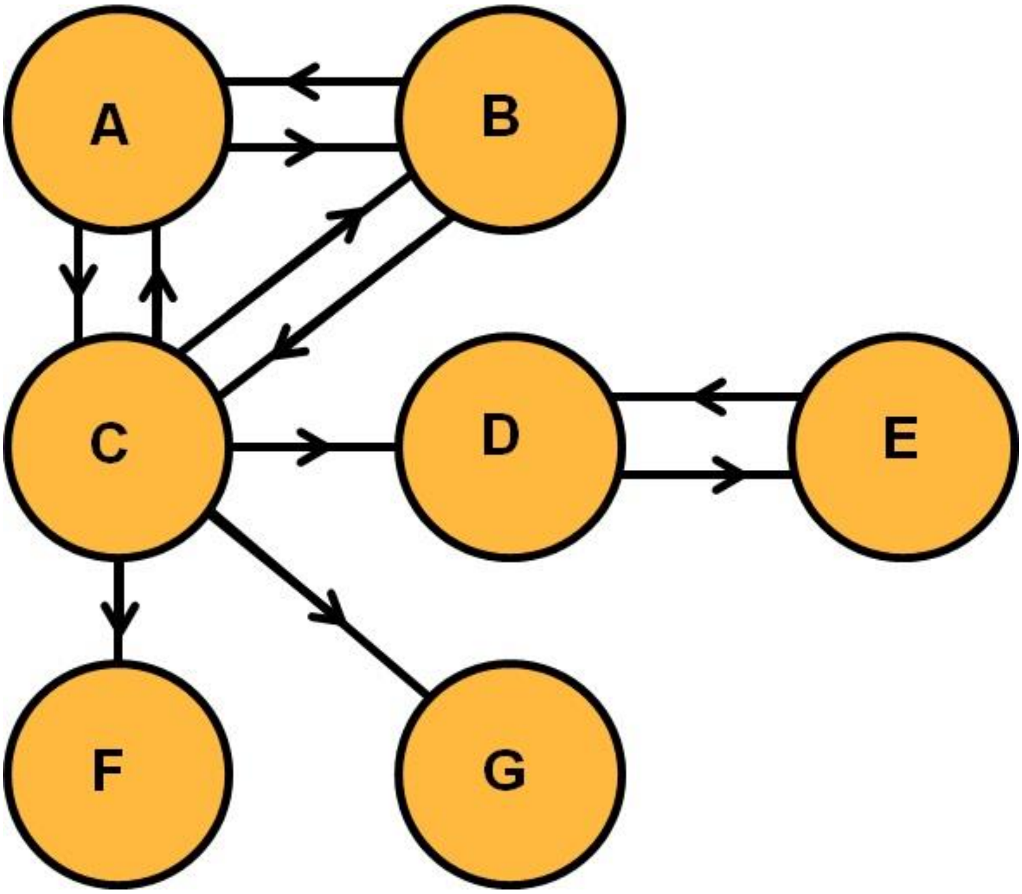
A state i is periodic with period $d > 1$ if d is the smallest positive integer such that $P(X_{n+k} = i \mid X_n = i) = 0$ for all n unless k is divisible by d.

i) Which states are periodic with period $d > 1$? For each of these states, find the period.

Depending on the starting conditions, there are three different possible equilibrium distributions.

ii) Find each of the three possible equilibrium distributions and, for each, state which possible starting positions could lead to that equilibrium distribution.

2. Consider a Markov Chain with the following state diagram.



Assume that all outward arrows from each state are equally weighted. That is, for example, state A has two outward arrows, so the probability of switching from state A to state B in one move is 0.5 and the probability of switching from state A to state C in one move is 0.5.

The Markov Chain is run until it reaches a stable equilibrium distribution $\Pi_{eq} = \begin{pmatrix} \pi_A & \pi_B & \pi_C & \pi_D & \pi_E & \pi_F & \pi_G \end{pmatrix}$ where π_i is the probability that, in the long-run, the system is in state *i*.

Find three possible values of Π_{eq}. For each, for each, state which possible starting positions could lead to that equilibrium distribution.

3. Consider applying the Metropolis-Hastings algorithm to generate 10 realisations $x_1, x_2, ..., x_{20}$ of $X \sim Bin(3,0.5)$.

Use the proposal distribution with probability mass function

$$P(Y = k) = \begin{cases} 0.2 & k \in \{0,1,2,3,4\} \\ 0 & \text{otherwise} \end{cases}.$$

Begin the Markov chain in state $x_0 = 2$ and find the next 10 states.

Below are 20 independent realisations $u_1, u_2, ..., u_{20}$ of a $U[0,1]$ variable. The top row contains the values $u_1, ..., u_{10}$, the second row contains the values $u_{11}, ..., u_{20}$.

0.410	0.777	0.050	0.893	0.912	0.229	0.226	0.529	0.676	0.987
0.370	0.400	0.281	0.406	0.722	0.803	0.092	0.433	0.195	0.118

Using the values $u_1, ..., u_{10}$ in the top row , generate proposed moves $y_1, ..., y_{10}$ by setting $y_j = \lfloor 5u_j \rfloor$ for each *j*.

Where the acceptance probability of the move proposed for x_j is < 1, accept the move only if the acceptance probability is $> u_{10+j}$.