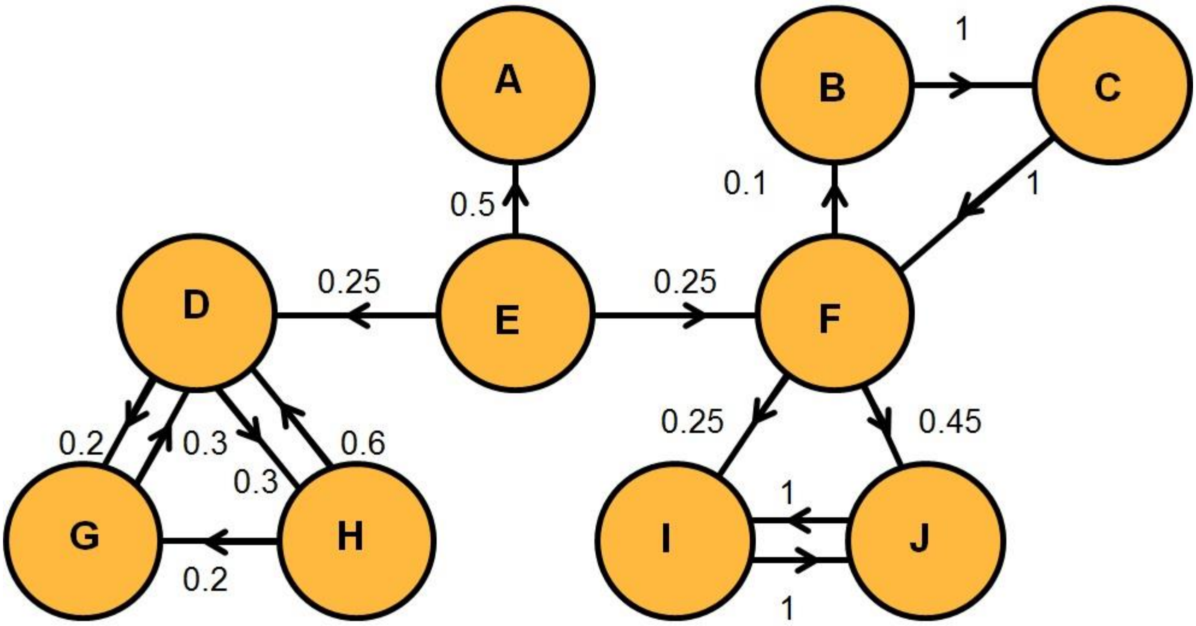
University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Class 10 Preparation Work
SOLUTIONS

1.



i) States I and J both have period 2.
All other states are aperiodic.

ii) The three possible equilibrium distributions
$\Pi_{eq} = (\pi_A \quad \pi_B \quad \pi_C \quad \pi_D \quad \pi_E \quad \pi_F \quad \pi_G \quad \pi_H \quad \pi_I \quad \pi_J)$ are:

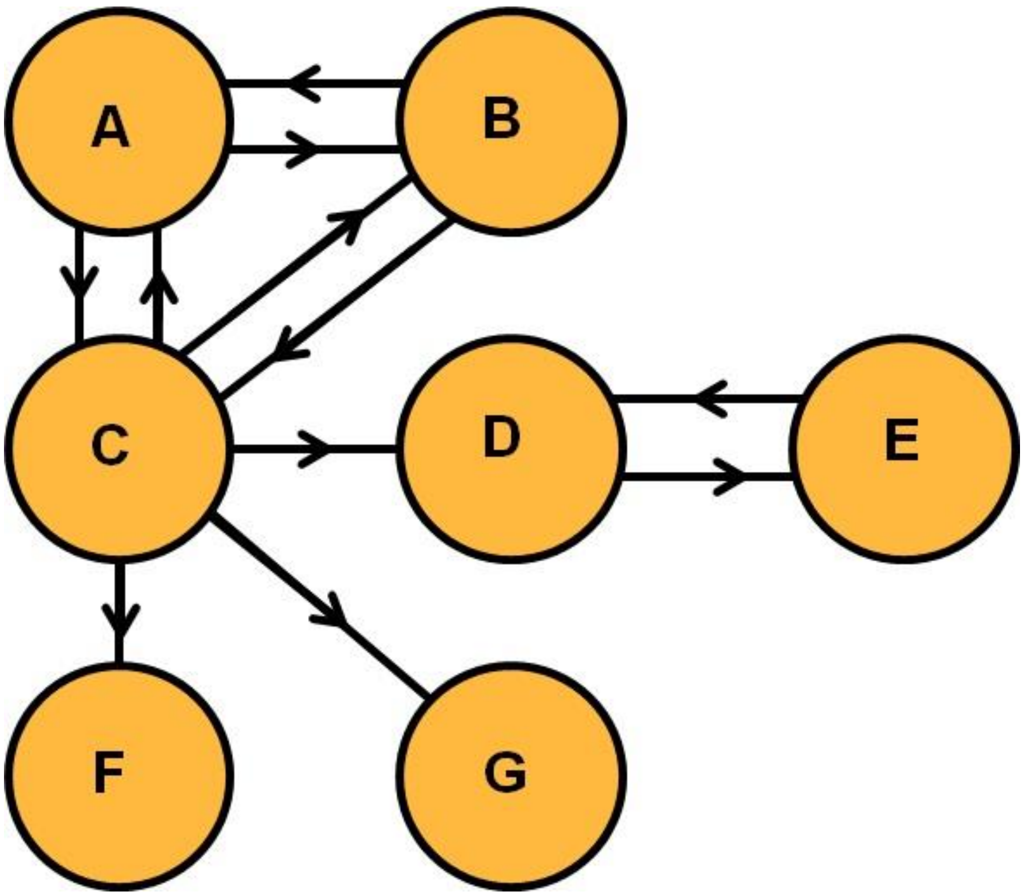
1) $\Pi_{eq} = (1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$. This can be reached if the system begins in A or E.

2) $\Pi_{eq} = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.5 \quad 0.5)$. This can be reached if the system begins in E, B, C, F, I or J.

3) $\Pi_{eq} = \left(0 \quad 0 \quad 0 \quad \frac{24}{55} \quad 0 \quad 0 \quad \frac{2}{5} \quad \frac{9}{55} \quad 0 \quad 0\right)$. This can be reached if the system begins in D, E, G or H.

(This solution is not obvious. Needs eigenvector-eigenvalue methods, but provided here for completeness.)

2.



The three possible equilibrium distributions

$\Pi_{eq} = \begin{pmatrix} \pi_A & \pi_B & \pi_C & \pi_D & \pi_E & \pi_F & \pi_G \end{pmatrix}$ are:

1) $\Pi_{eq} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$. This can be reached if the system begins in A, B, C or G.
2) $\Pi_{eq} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$. This can be reached if the system begins in A, B, C or F.
3) $\Pi_{eq} = \begin{pmatrix} 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 \end{pmatrix}$. This can be reached if the system begins in A, B, C, D or E.

3.

State	Proposal	Acceptance Probability	$A_j > u_{10+j}$?	Accept Move?
$x_0 = 2$	$y_1 = \lfloor 5(0.410) \rfloor = 2$	$A_1 = \min\left\{1, \frac{P(X=2)}{P(X=2)}\right\} = 1$	$1 > 0.370$	Yes
$x_1 = 2$	$y_2 = \lfloor 5(0.777) \rfloor = 3$	$A_2 = \min\left\{1, \frac{P(X=3)}{P(X=2)}\right\} = \frac{(0.5)^3}{3(0.5)^3} = \frac{1}{3}$	$\frac{1}{3} < 0.400$	No
$x_2 = 2$	$y_3 = \lfloor 5(0.050) \rfloor = 0$	$A_3 = \min\left\{1, \frac{P(X=0)}{P(X=2)}\right\} = \frac{(0.5)^3}{3(0.5)^3} = \frac{1}{3}$	$\frac{1}{3} > 0.281$	Yes
$x_3 = 0$	$y_4 = \lfloor 5(0.893) \rfloor = 4$	$A_4 = \min\left\{1, \frac{P(X=4)}{P(X=0)}\right\} = \frac{0}{(0.5)^3} = 0$	$0 < 0.406$	No
$x_4 = 0$	$y_5 = \lfloor 5(0.912) \rfloor = 4$	$A_5 = \min\left\{1, \frac{P(X=4)}{P(X=0)}\right\} = \frac{0}{(0.5)^3} = 0$	$0 < 0.722$	No
$x_5 = 0$	$y_6 = \lfloor 5(0.229) \rfloor = 1$	$A_6 = \min\left\{1, \frac{P(X=1)}{P(X=0)}\right\} = 1$	$1 > 0.803$	Yes
$x_6 = 1$	$y_7 = \lfloor 5(0.226) \rfloor = 1$	$A_7 = \min\left\{1, \frac{P(X=1)}{P(X=1)}\right\} = 1$	$1 > 0.092$	Yes
$x_7 = 1$	$y_8 = \lfloor 5(0.529) \rfloor = 2$	$A_8 = \min\left\{1, \frac{P(X=2)}{P(X=1)}\right\} = 1$	$1 > 0.433$	Yes
$x_8 = 2$	$y_8 = \lfloor 5(0.676) \rfloor = 3$	$A_8 = \min\left\{1, \frac{P(X=3)}{P(X=2)}\right\} = \frac{(0.5)^3}{3(0.5)^3} = \frac{1}{3}$	$\frac{1}{3} > 0.195$	Yes
$x_9 = 3$	$y_{10} = \lfloor 5(0.987) \rfloor = 4$	$A_8 = \min\left\{1, \frac{P(X=4)}{P(X=3)}\right\} = \frac{0}{(0.5)^3} = 0$	$0 < 0.118$	No
$x_{10} = 3$				