University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Class 1 Preparation Work

1. Given realisations $u_1, u_2, ..., u_n$ of a uniform random variable $U \sim U[0,1]$, realisations $x_1, x_2, ..., x_n$ of a random variable X are generated by the following rule:

$$\begin{array}{cc} \text{if } u_i < 0.1 & x_i = -4 \\ \text{if } 0.1 \le u_i < 0.7 & x_i = 0 \\ \text{if } 0.7 \le u_i < 0.85 & x_i = 4 \\ \text{if } u_i \ge 0.85 & x_i = 7 \end{array}.$$

i) Write down the probability mass function of the random variable X.

ii) Given $\{u_1, u_2, ..., u_{10}\} = \{0.511,\ 0.008,\ 0.717,\ 0.333,\ 0.209,\ 0.200,\ 0.173,\ 0.990,\ 0.421,\ 0.571\}$

generate the values of $\{x_1, x_2, ..., x_{10}\}$.

iii) Directly from the probability mass function, calculate $E(X)$, the expected value of X.

iv) Calculate $\bar{X}$, an estimate of the expected value of X using the ten simulated values from part ii).

2. Given a realisation, u_i, of a $U[0,1]$ random variable, a realisation of a discrete random variable *X*, x_i, is generated by the following rule:

If	$u_i < 0.3$	$x_i = 0$
If	$0.3 \le u_i < 0.8$	$x_i = \left\lfloor \frac{1}{u_i} \right\rfloor$
If	$u_i \ge 0.8$	$x_i = 7.5$

Note: $\left\lfloor \frac{1}{u_i} \right\rfloor$ denotes the floor function which, for any $\frac{1}{u_i} \in \mathbb{R}$, takes the value of the largest integer $\le \frac{1}{u_i}$. For example, if $u_i = 0.748$, then $\frac{1}{u_i} \approx 1.337$ and hence $x_i = \left\lfloor \frac{1}{u_i} \right\rfloor = 1$.

i) Write down the probability mass function of *X*.

ii) Given realisations $\{u_1, u_2, ..., u_5\} = \{0.710, 0.119, 0.358, 0.883, 0.504\}$ of a $U[0,1]$ variable, generate five realisations $\{x_1, x_2, ..., x_5\}$ of *X*.

iii) Calculate $\bar{X}$, an estimate of the expected value of *X* from your five realisations $\{x_1, x_2, ..., x_5\}$.

iv) Calculate the expected value of the random variable *X*, $E(X)$ and show that $\bar{X} > E(X)$.