University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Class 1 Preparation Work
SOLUTIONS

1.

i) $$P(X=k)=\begin{cases}0.1 & k=-4\\ 0.6 & k=0\\ 0.15 & k=4\\ 0.15 & k=7\\ 0 & \text{otherwise}\end{cases}$$

ii) Given $\{u_1,u_2,...,u_{10}\}=\{0.511,\ 0.008,\ 0.717,\ 0.333,\ 0.209,\ 0.200,\ 0.173,\ 0.990,\ 0.421,\ 0.571\}$

We obtain $\{x_1,x_2,...,x_{10}\}=\{0,\ -4,\ 4,\ 0,\ 0,\ 0,\ 0,\ 7,\ 0,\ 0\}$.

iii) $$E(X)=\sum kP(X=k)=0.1(-4)+0.6(0)+0.15(4)+0.15(7)=1.25$$

iv) $$\bar{X}=\frac{1}{10}\sum_{i=1}^{10}x_i=0.7$$

2.

i) If $u_i \in [0,0.3)$ then $x_i = 0$.

If $u_i \in [0.3,1/3]$ then $x_i = \left\lfloor \frac{1}{u_i} \right\rfloor = 3$.

If $u_i \in (1/3,0.5]$ then $x_i = \left\lfloor \frac{1}{u_i} \right\rfloor = 2$.

If $u_i \in (0.5,0.8)$ then $x_i = \left\lfloor \frac{1}{u_i} \right\rfloor = 1$.

If $u_i \in [0.8,1]$ then $x_i = 7.5$.

therefore

$$P(X=k) = \begin{cases} \frac{3}{10} & k=0 \\ \frac{3}{10} & k=1 \\ \frac{1}{6} & k=2 \\ \frac{1}{30} & k=3 \\ \frac{1}{5} & k=7.5 \\ 0 & \text{otherwise} \end{cases}$$

ii) $\{x_1, x_2, \ldots, x_5\} = \{1,\ 0,\ 2,\ 7.5,\ 1\}$.

iii) $\bar{X} = \frac{1}{5}(1+0+2+7.5+1) = 2.3$,

iv) $E(X) = \sum k \times P(X=k) = \frac{3}{10}\ 0\ + \frac{3}{10}\ 1\ + \frac{1}{6}\ 2\ + \frac{1}{30}\ 3\ + \frac{1}{5}\ 7.5\ \approx 2.233$

Hence $\bar{X} > E(X)$.