University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Class 2 Preparation Work

1. A random variable Q has probability density function

$$f(q) = \begin{cases} \dfrac{1}{q} & 1 < q \le M \\ 0 & \text{otherwise} \end{cases} \quad \text{where } M \text{ is a real constant.}$$

   i) Show that the maximum value that the variable Q can take, $M = e = \exp(1)$

   ii) Find the cumulative probability function of Q.

2. The cumulative probability function of $Y \sim Weibull(\lambda, k)$ is given by

$$F_Y(y) = P(Y \le y) = \begin{cases} 1 - e^{-\left(\frac{y}{\lambda}\right)^k} & y \ge 0 \\ 0 & y < 0 \end{cases} \quad \text{for } k > 0, \lambda > 0 .$$

   i) Show that the probability density function of Y is given by

$$f_Y(y) = \begin{cases} \dfrac{k y^{k-1} e^{-\left(\frac{y}{\lambda}\right)^k}}{\lambda^k} & y \ge 0 \\ 0 & y < 0 \end{cases}$$

   ii) Calculate the inverse of the cumulative probability function, $F_Y^{-1}(y)$.

   iii) In your own words, clearly explain how the inverse of the cumulative probability function of Y can be used to generate realisations of Y.

The distribution of observed wind speeds is often modelled by a Weibull variable. At a given location, the wind speed (in m/s) $\sim Weibull(6,2)$.

   iv) Given four independent realisations of $U \sim U[0,1]$, $\{u_1, u_2, u_3, u_4\} = \{0.503,\ 0.114,\ 0.760,\ 0.449\}$, generate four independent realisations of $Y \sim Weibull(6,2)$.

3. Let Z be a continuous uniform random variable, $Z \sim U[4,10]$.

i) Write down the probability density function of Z.

ii) Show that the cumulative probability function of Z, $F(z)$ is given by

$$F(z) = \begin{cases} 0 & z < 4 \\ \dfrac{z-4}{6} & z \in [4,10] \\ 1 & z > 10 \end{cases}.$$

iii) Given realisations $\{u_1, u_2, ..., u_5\} = \{0.710, 0.119, 0.358, 0.883, 0.504\}$ of a $U[0,1]$ variable, generate five realisations $\{z_1, z_2, ..., z_5\}$ of Z.

Clearly explain your method and any calculations required.