University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) – Class 4 Preparation Work

1. Let X and Y be independent standard normal random variables,

   Consider the variables defined as $P = X^2$ and $Q = \frac{Y^2}{X^2}$.

   i) For $X = \sqrt{P}$ and $Y = \sqrt{PQ}$, calculate the determinant of the Jacobian matrix $\boldsymbol{J} = \begin{pmatrix} \frac{\partial X}{\partial P} & \frac{\partial X}{\partial Q} \\ \frac{\partial Y}{\partial P} & \frac{\partial Y}{\partial Q} \end{pmatrix}$.

   ii) In order to fully calculate the joint density of P and Q, $f_{P,Q}(p,q)$ using the substitutions from part i), we need to multiply the above determinant by 4 to calculate the joint density $f_{P,Q}(p,q)$ from $f_{X,Y}(x,y)$.

   In your own words, clearly explain why this is the case.

   iii) Hence show that $f_{P,Q}(p,q) = \frac{1}{2\pi\sqrt{q}} e^{-\frac{p(1+q)}{2}}$ for $(p,q) \in [0,\infty)^2$

   iv) Calculate the marginal density of Q, $f_Q(q)$.

   v) Calculate the marginal density of P, $f_P(p)$.

   vi) What is the conditional density function of $Q \mid P = 1$?

   Justify your answer.

2. Let S be a random variable which follows some unknown distribution.

It is known that $E(S) = 50$ and $Var(S) = 64$.

i) A sample of 100 observations of S is made. Using the central limit theorem, calculate the approximate probability that the sample mean is above 53.

ii) Using the central limit theorem calculate the smallest possible sample size such that the sample mean will exceed 51 with probability <5%.

Note: For $Z \sim N(0,1)$, $P(Z > 1.645) \approx 0.05$

3. A number shown on biased six-sided die T is claimed to have probability mass function $P(T = k) = \begin{cases} \frac{k}{21} & k \in \{1,2,3,4,5,6\} \\ 0 & \text{otherwise} \end{cases}$.

The die is rolled 210 times and the following frequency of outcomes was observed.

Outcome	1	2	3	4	5	4
Frequency	11	20	37	44	42	56

Calculate the appropriate Pearson statistic and perform a chi-squared goodness of fit test to assess whether there is reason to reject the hypothesised probability mass function. Perform this test with significance level 0.05.