University of Technology Sydney
School of Mathematical and Physical Sciences

Mathematical Statistics (37262) –
Class 5 Preparation Work

1. i) Clearly explain all the steps you would follow to obtain an estimate of the definite integral $\int_2^{\infty} \frac{3}{\ln(x)\left[\sin(x)+2\right]} dx$ via the Monte Carlo method.

   Note: You do not need to obtain a numerical answer. You should, however clearly state any changes of variable required and derive the resulting equivalent definite integral.

   ii) Using the set of independent realisations of $U \sim U[0,1]$, $\{u_1, u_2, ..., u_5\} = \{0.654, 0.007, 0.238, 0.604, 0.591\}$, apply the Monte Carlo method to estimate the value of the definite integral $\int_{-0.1}^{1} e^{x^4} dx$.
   Clearly explain your method.

   iii) Using the same set of independent realisations of $U \sim U[0,1]$, as in part ii) above, obtain an estimate of the definite integral $\int_{-6}^{-5} \arctan(x) dx$ via the Monte Carlo method.
   Clearly explain your method.

2. Consider the problem of evaluating the value of the definite integral

$$\int_{-\infty}^{0}\left[\frac{10}{1+x^2}\right]dx\,.$$

i) Using the substitution $y=\frac{1}{1+x^2}$ and the same set of independent realisations of $U \sim U[0,1]$, as in question 1, estimate the value of the integral.

Hint: Think carefully when calculating $x(y)$ for the required substitution.

ii) Find an alternative suitable substitution and obtain a second estimate of the value of the integral using the same five realisations of a $U[0,1]$ variable.

Clearly state your change of variable and derive the resulting equivalent definite integral.

3. Let X be a continuous random variable with probability density function

$$f(x)=\begin{cases}\frac{1}{x^2} & x<-4\\ \frac{1}{16} & -4\le x\le 4\\ \frac{1}{x^2} & x>4\end{cases}.$$

Explain briefly why Monte Carlo integration could not be used to estimate the value of $\int_{-\infty}^{\infty} xf(x)dx$.