University of Technology Sydney
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Mathematical Statistics (37262) –
Class 8 Preparation Work

1. Consider independent realisations of $Y \sim N(\mu,\sigma^2)$,

   $\{y_1, y_2, \ldots y_{10}\} = \{-1.5,\ 8.0,\ 3.6,\ 5.1,\ 1.1,\ 4.0,\ 5.3,\ 2.9,\ 2.0,\ 6.4\}$.

   Assume that μ is unknown but you are told that $\sigma^2 = 2^2$.

   i) Calculate the sample mean.

   ii) Hence calculate an unbiased estimate of the population mean μ.

   iii) Calculate a 95% confidence interval for μ.

   iv) Show that a 95% prediction interval for an additional observation of Y is (to 2 decimal places) given by $(-0.42,\ 7.80)$.

   We are now told that the population variance may not be $\sigma^2 = 2^2$ as previously believed. We now assume that both μ and σ^2 are unknown.

   v) Calculate the sample variance $s^2 = \frac{1}{10}\sum_{i=1}^{10}(y_i - \bar{y})^2$.

   vi) Show that $bias(s^2, \sigma^2) \neq 0$.

   vii) Would you expect confidence intervals for the population mean μ to be wider when assuming $Y \sim N(\mu, 2^2)$ or when assuming $Y \sim N(\mu,\sigma^2)$? Justify your answer.

   viii) Calculate a 95% confidence interval for μ.

2. Consider independent realisations $\{x_1, x_2, \ldots x_n\}$ of a continuous uniform random variable $X \sim U[0, 2\theta]$ where $\theta > 0$ is unknown.

   i) Calculate $E(X)$.

   ii) Write down the likelihood function for this set of observations and hence find an estimate $\hat{\theta}_{MLE}$ of θ by maximum likelihood estimation.

   iii) Calculate $\hat{\theta}_{MM}$, an estimate of θ by using the first sample moment.

   iv) Although $\hat{\theta}_{MM}$ is unbiased, it can give estimates of θ which lie outside $[0, 2\theta]$. Give an example of five observations $\{x_1, x_2, \ldots x_5\}$ which would give such a nonsensical estimate for $\hat{\theta}_{MM}$.