University of Technology Sydney
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Mathematical Statistics (37262) –
Tutorial 1

1. Below are 50 reported unordered independent realisations of a $U[0,1]$ variable rounded to 3 decimal places. The top row contains the values $u_1,...,u_{10}$, the second row contains the values $u_{11},...,u_{20}$ etc.

0.876	0.422	0.360	0.428	0.024	0.043	0.111	0.373	0.079	0.251
0.126	0.777	0.666	0.765	0.391	0.275	0.247	0.228	0.491	0.333
0.257	0.222	0.793	0.415	0.146	0.036	0.250	0.552	0.555	0.444
0.000	0.444	0.104	0.134	0.086	0.051	0.195	0.340	0.413	0.335
0.777	0.888	0.930	0.333	0.429	0.257	0.297	0.973	0.232	0.997

i) Using these 50 realisations, generate 50 independent realisations of $T \sim Bern(0.75)$.
Clearly explain your method.

ii) Using one row of values at a time (i.e. $u_1,...,u_{10}$ then $u_{11},...,u_{20}$ etc.) generate 5 independent realisations of $S \sim Bin(10, 0.75)$.
Clearly explain your method.

iii) Using the realisations in order i.e. starting at u_1 and ending at u_{50}, generate independent realisations of $R \sim Geo(0.75)$.
Clearly explain your method.

A realisation of a negative binomial variable $V \sim NegBin(n, p)$ (where $n \in \mathbb{Z}^+$ and $p \in (0,1)$) can be obtained by counting how many independent realisations of a $Bern(p)$ are needed until the nth 1 is observed. For example, if the realisations of the $Bern(p)$ variables are 0, 0, 1, 0, 1, 1, then a realisation of $NegBin(3, p)$ would be 6 since the 3rd 1 was seen on the 6th Bernoulli variable.

iv) What is the range of $V \sim NegBin(n, p)$?
Clearly justify your answer.

2. Let X and Y be independent discrete random variables such that $X \sim Bin(3, 0.5)$ and $Y \sim Bern(0.8)$.

X has probability mass function $P(X = k) = \begin{cases} \dfrac{3!}{k!(3-k)!} 0.5^3 & k \in \{0,1,2,3\} \\ 0 & \text{otherwise} \end{cases}$.

i) Write down the probability mass function of Y.

ii) Calculate the probability mass function of $X + Y$.

iii) Write down a rule which could be applied to realisations of $U \sim U[0,1]$ to generate realisations of $X + Y$.

3. The probability mass function of $R \sim Geo(p)$ is

$$P(R = k) = \begin{cases} p(1-p)^{k-1} & k \in \{1,2,3,...\} \\ 0 & \text{otherwise} \end{cases}. \text{ where } p \in (0,1).$$

Showing all of your working, verify that $E(R) = \dfrac{1}{p}$.

Hint: You may wish to begin with writing out the definition of $E(R)$ as a series and then split the resulting series into a number of geometric series.